

Monetary Policy in Financial Economies

By

Andrew Lee Smith

Submitted to the Department of Economics and the
Graduate Faculty of the University of Kansas
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

Professor John W. Keating,
Chairperson

Professor William A. Barnett

Committee members

Professor Robert DeYoung

Professor Ted Juhl

Professor Shu Wu

Date defended:

May 29, 2014

The Dissertation Committee for Andrew Lee Smith certifies
that this is the approved version of the following dissertation :

Monetary Policy in Financial Economies

Professor John W. Keating, Chairperson

Date approved: May 29, 2014

Abstract

This dissertation explores a critical question that arose naturally as a consequence of the Great Recession: What is the appropriate role for and design of monetary policy in financial economies? In addition to analyzing the unprecedented actions taken by policy makers in the wake of the worst Financial Crisis since the Great Depression, I also examine how conventional monetary policy instruments should be designed and implemented in New-Keynesian economies - the workhorse model of monetary policy for central banks.

Conventional Monetary Policy in Financial Economies

Unconventional monetary policy is designed as a response to financial crises. However, in chapters 1, 2 and 3 I show that even conventional monetary policy must be adapted in financial economies. Specifically, I analyze the performance of both Friedman ‘K-percent’ money growth rules and Taylor rules in New-Keynesian models – the workhorse central bank model for monetary policy. However, unlike standard New-Keynesian models, I examine the theoretical aspects of monetary policy in models with a meaningful financial sector. The cumulative results of this section show the case against using monetary aggregates by central bankers is significantly diminished once the financial sector is included.

Chapter 1: Price vs. Financial Stability: A role for Money in Taylor rules A consensus among central bankers, especially in the U.S., is that money plays no meaningful role in the formation, nor execution of, monetary policy.

This point is theoretically supported by New-Keynesian models in which optimal policy can be described without any reference to money. However in *Price vs. Financial Stability: A role for Money in Taylor rules*, my co-author and I turn this logic on its head. By modeling a financial sector in an otherwise standard New-Keynesian model, we show that augmenting a Taylor rule with a response to the growth rate of money helps to offset the detrimental impacts of financial sector disruptions.

Chapter 2: Determinacy and Indeterminacy in Monetary Policy Rules

with Money In *Determinacy and Indeterminacy in Monetary Policy Rules with Money*, my co-author and I show the most basic of monetary policy rules, Milton Friedman’s ‘k-percent’ rule fails to deliver a unique equilibrium, and therefore creates economic instability. This result is in sharp contrast to the determinacy properties of this rule in economies without a financial sector which feature no bank produced assets. We then show the determinacy properties of this classic monetary policy rule are restored when a Divisia monetary aggregate as formulated by Barnett (1980) is used to measure the aggregate quantity of financial and non-financial assets.

Chapter 3: A Working Solution to Working Capital Indeterminacy

Working capital refers to the financing firms’ require to fund inputs before they receive payments for their output. When this channel is active, changes in interest rates transmit through the usual demand/Euler channel and also a supply/-marginal cost channel. The former dampens demand-pull inflation while the latter will exacerbate cost-push inflation. From a monetary policy standpoint, these dual channels imply *both* a respective lower and an upper bound on the Monetary Authority’s inflation response needed to guarantee the existence of a unique rational expectations equilibrium (determinacy). In *A Working Solution to Working Capital Indeterminacy*, I analytically show that Friedman’s ‘K-Percent’ rule is

determinate in the presence of working capital channels. I additionally provide a simple sufficient condition for the determinacy of interest rate rules reacting to the nominal growth rate of the monetary aggregate: reacting greater than one for one to changes in the growth rate of money guarantees determinacy. All of these results are presented in the framework of a micro-founded New-Keynesian model.

Unconventional Monetary Policy in Financial Economies

In chapters 4 and 5 of this dissertation, I turn the focus to unconventional monetary policy. I use the term unconventional monetary policy to refer to the actions taken by the U.S. Treasury and the Federal Reserve in the wake of the 2008 Financial Crisis. In particular, the TARP legislation resulted in equity injections into the largest U.S. commercial banks. Moreover, the Federal Reserve began large scale asset purchases known as “Quantitative Easing” as a tool to improve the functioning of credit markets. The Federal Reserve has especially focused on purchasing Mortgage Backed Securities to: *(i)* improve the functioning of short-term collateralized debt markets and *(ii)* increase home prices; with the goal that both of these effects will improve the speed and strength of the recovery. The results of this section provide insight into the transmission mechanism of these policies and the role of housing, in both the recession and recovery.

Chapter 4: House Prices, Heterogeneous Banks and Unconventional Monetary Policy Options In *House Prices, Heterogeneous Banks and Unconventional Monetary Policy Options* I answer salient questions in the wake of the 2008 Financial Crisis. Why did a drop in home prices force big banks to withdraw relatively more credit than smaller banks? What is the transmission mechanism of equity injection into “Too Big to Fail” banks and “Quantitative Easing” programs?

To answer these questions I develop a general equilibrium model with fully integrated housing and financial markets. In addition to introducing housing-secured debt into the financial market, the financial sector features heterogeneous banks whereby “Too Big to Fail” institutions emerge in equilibrium. Unlike other models which rely on “Financial Shocks” and quadratic investment adjustment costs to simulate a crisis episode, I take the alternative viewpoint that housing, and housing secured debt, played a critical role in initiating and propagating the crisis. By eschewing these typical assumption, including that all banks are the same, I am able to show how a drop in housing demand can set off a financial fire-sale effect.

Quantitatively, the model matches empirical correlations that the traditional (Bernanke, Gertler, and Gilchrist, 1999) financial accelerator mechanism fails to capture, including the correlation of finance premiums with home prices, investment and output. I then test the model’s qualitative predictions against an estimated VAR. The results of these empirical comparisons support the model’s financial structure. The model provides a framework to examine the ability of QE policies and equity injections into big banks to mitigate a housing generated recession. Although both are effective, the nuances of the policies are important. A prolonged asset purchase program is preferable to a short-term equity injection; however, the model suggests the equity injections may have been necessary to prevent an economic collapse at the acute stage of the 2008 Financial Crisis.

Chapter 5: The Foreclosure Accelerator versus the Financial Accelerator: Housing and Borrower’s Net Worth The seminal work of Bernanke, Gertler, and Gilchrist (1999) highlights how changes in the net worth of borrowers can turn a typical downturn into deep protracted recession. However, this celebrated financial accelerator model is silent with regards to the connection between real-estate prices and borrower’s net worth. In this paper, I

propose an alternative debt-contract in which agent's net worth is largely in the form of real-estate. This financial structure generates amplification and propagation of macroeconomic disturbances through a foreclosure channel as opposed to investment-adjustment costs as in BGG. Contrary to the findings of Christiano, Motto and Rostagno (2013), I estimate the equilibrium model using Bayesian techniques and find that housing demand shocks, not risk shocks, drive finance premiums. Finally, I answer some timely questions in the wake of the great recession including: Did the Federal Reserve's low interest rate policy fuel the housing bubble? How contractionary is the zero lower bound?

Acknowledgements

I gratefully acknowledge fellowship support from the University of Kansas Graduate College and additional funding and resources from the Department of Economics. I would also like to thank: John Keating, my dissertation advisor and coauthor of chapters 1 and 2; William Barnett and the Center for Financial Stability for access to high-quality monetary data which was used extensively in chapters 1, 2 and 3; Robert DeYoung for directing me to the bank holding company data available from the FR Y-9C forms; Ted Juhl and Shu Wu for helpful conversations and suggestions.

Contents

1	Price versus Financial Stability: A role for Money in Taylor rules	1
1.1	Introduction	1
1.2	Model	6
1.3	Calibration and Solution Strategy	19
1.4	Welfare Evaluation of Monetary Policy Rules	22
1.5	Comparative Statics, Robustly Optimal Policy and Determinacy	31
1.6	Conclusion	39
1.A	Appendix: The Equilibrium System	46
1.B	Appendix: Tables	65
2	Determinacy and Indeterminacy in Monetary Policy Rules with Money	67
2.1	Introduction	67
2.2	Model	69
2.3	Friedman k-percent Rules	72
2.4	Interest Rate Rules with Money	76
2.5	Numerical Analysis	79
2.6	Conclusion	88
2.A	Appendix: Proofs	94

3	A Working Solution to Working Capital Indeterminacy	103
3.1	Introduction	103
3.2	DSGE Model	108
3.3	Working Capital Indeterminacy	119
3.4	Adding Real Wage Rigidities	127
3.5	Conclusion: A Working Solution to Working Capital Indeterminacy	132
3.A	Appendix: The Stationary, Non-Linear Model	140
3.B	Appendix: Proofs	142
4	House Prices, Heterogeneous Banks and Unconventional Monetary Policy	
	Options	150
4.1	Introduction	150
4.2	A DSGE Model with Integrated Housing and Financial Markets	153
4.3	VAR Tests of the Model's Amplification Mechanism	178
4.4	Unconventional Monetary Policy	193
4.5	Conclusion	199
4.A	Appendix: VAR Data and Complete Results	204
4.B	Appendix: The Stationary, Non-Linear Model	208
5	The Foreclosure Accelerator, Housing, and Borrower's Net Worth	216
5.1	Introduction	216
5.2	A DSGE Model with Integrated Housing and Financial Markets	218
5.3	Model Inference	236
5.4	Dissecting the Great Recession	246
5.5	Conclusion	259
5.A	Appendix: The Stationary, Non-Linear Model	264

List of Figures

1.1	Inside Money and Financial Distress	4
1.2	IRFs to a Negative Banking Productivity Shock	30
1.3	The Welfare Cost of Responding to Interest Rate Spreads	32
1.4	Determinacy Region Under a Money Growth Targeting Rule	37
1.5	Determinacy Region Under a Simple-Sum Growth Targeting Rule	62
2.1	Determinacy Region Under a Simple-Sum Growth Targeting Rule	81
2.2	Determinacy Region Under a Divisia Growth Targeting Rule	85
2.3	Determinacy Regions Under Inertial Simple-Sum Growth Targeting Rules . .	86
2.4	Determinacy Regions Under Inertial Simple-Sum Growth Targeting Rules . .	87
3.1	The Taylor Principle	105
3.2	Working Capital Indeterminacy	107
3.3	IRFs to a Monetary Contraction with and without Sticky Wages	129
3.4	Determinacy Region Under Inflation Targeting and Sticky Wages	130
3.5	Determinacy Region Under Inertial Inflation Targeting and Sticky Wages . .	131
3.6	Determinacy Region Under Money Growth Targeting and Sticky Wages . . .	133
3.7	Determinacy Region Under Inflation Targeting with a Money Growth Anchor and Sticky Wages	135
4.1	Real Home Prices and Asset Concentration in Big Banks	153

4.2	IRFs from the Equilibrium Model with and without Capital Constraints . . .	175
4.3	Illustrating the Asset Price Spiral from Big Bank Fire Sales	176
4.4	Model A: IRF to a Contractionary Aggregate Shock	185
4.5	Model A: IRF to a Decrease in the Large Bank Share	186
4.6	Model A: IRF to an External Finance Premium Shock	187
4.7	Model A with Alternative Data: IRFs testing the Accelerator Mechanism . .	188
4.8	Model B: IRFs testing the Accelerator Mechanism	189
4.9	Model B with Alternative Data: IRFs testing the Accelerator Mechanism . .	190
4.10	Model C: IRFs testing the Accelerator Mechanism	191
4.11	Model C with Alternative Data: IRFs testing the Accelerator Mechanism . .	192
4.12	IRFs to a Large Housing Demand Shock and Various Unconventional Monetary Policy Responses	193
4.13	Indeterminacy and the Strength of the Asset-Price Spiral	198
4.14	Model A: IRFs testing the DSGE Model's Accelerator Mechanism	206
4.15	Model A: Historical Decompositions	207
5.1	Housing, Finance Premiums and Real GDP	217
5.2	Illustrating Risk Shocks	247
5.3	IRFs to a Contractionary Risk Shock	249
5.4	IRFs to a Contractionary Technology Shock	250
5.5	IRFs to a Contractionary Housing Demand Shock	252
5.6	Historical Decompositions of Risk, Technology and Housing Demand Shocks	253
5.7	Historical Decompositions of the Financial Frictions Output Gap	256
5.8	Historical Decompositions Testing the “too low for too long” Hypothesis . .	257
5.9	Historical Decompositions with and without the Zero Lower Bound	259

List of Tables

1.1	Welfare Results for Optimal Simple Rules	24
1.2	Correlation of Real and Natural Interest Rates	25
1.3	Optimal Simple Rules with Additively Separable Preferences	26
1.4	Optimal Simple Rules when the Efficient Output Gap is observable.	27
1.5	Non-Optimized Rules	28
1.6	Correlation of Real and Natural Interest Rates	28
1.7	Inefficient Resource Costs	31
1.8	Elasticities of Optimal Policy Coefficients	33
1.9	Nash Equilibrium in Γ	35
1.10	Baseline Calibration of Parameters	65
1.11	Equilibrium Variables and System of Equations	66
2.1	Determinacy Regimes Under a Simple-Sum Growth Targeting Rule	80
2.2	Econometric Estimation of the Simple-Sum Error Term	82
2.3	Calibrated Parameters	83
2.4	Money Growth Rates During the Monetarist Experiment	89
4.1	Cyclical Properties of the Model	174
4.2	Equity Injections vs. Quantitative Easing	196
4.3	Bank Mergers and Acquisitions	204

5.1	Model Calibrated Parameters	239
5.2	Model Parameters Priors and Posteriors - Economic Parameters	242
5.3	Model Parameters Priors and Posteriors - Shocks	245
5.4	FEVDs of Risk, Technology and Housing Demand Shocks	254

Chapter 1

Price versus Financial Stability: A role for Money in Taylor rules¹

1.1 Introduction

The New-Keynesian sticky-price (NKSP) framework has become the workhorse model for monetary policy evaluation due to its ability to provide a relevant role for monetary policy while avoiding the well known Lucas critique. The research on optimal monetary policy in these models has reached a clear consensus on two fronts. First, price stability is a paramount policy concern while output should only be stabilized at its flexible price level. Second, money is an inferior policy instrument due to persistent money demand shocks and offers no information regarding the natural rates of output or interest. These conclusions, which are robust across many model specifications, have led both academics and policy makers towards “cashless models” of the economy and the monetary transmission mechanism. The result, which is emphasized in Woodford (2003), is that optimal policy can typically be described by an interest rate rule which reacts to the natural rate of interest, inflation and the deviation of output from its flexible price (or natural) level . Curdia & Woodford (2009)

¹This chapter is coauthored with John W. Keating, Department of Economics, The University of Kansas, Email: jkeating@ku.edu.

show this description of optimal policy extends even to a NKSP model with a financial sector for the appropriately defined natural rate of interest.

However, in practice, policy makers know very little about the natural rate of output and interest in real time. This has led researchers to consider optimal simple policy rules which central banks could actually adopt. A rule is considered *simple* (Gali, 2008) if:

1. It makes the policy instrument a function of only observable variables
2. It does not require knowledge of the correct model
3. It does not require knowledge of model specific parameters

The desire to develop simple rules has led researchers to examine interest rate rules which react to inflation and output growth, variables which are readily available at quarterly frequencies, though sometimes with error. The optimality of such simple rules, usually with no output growth response, has been verified in NKSP models under a wide array of model specifications. Simply put, an interest rate rule responding to inflation appears to be the best available simple rule (Schmitt-Grohe & Uribe, 2007).

Relatively less is known about optimal *simple* rules in models with financial sectors. To date though, the literature on optimal simple rules in such models has often found no need to deviate from inflation targeting interest rate rules. For example, in a model with financial frictions, Faia & Monacelli (2007) find that responding to asset prices offers welfare gains when the inflation coefficient is fixed at Taylor's (1993) original values but these gains are eliminated when the inflation coefficient is increased.² Similarly, Dellas et al. (2010) find that reacting to inflation of non-financial products offers the optimal simple rule in a setting where banks are subject to supply shocks. However, none of these models which feature financial sectors have examined the usefulness of monetary aggregates in simple rules.

²Curdia & Woodford (2010) provide similar evidence that such models may call for augmenting the standard Taylor rule (with output and inflation coefficients fixed at Taylor's (1993) original values) with a reaction to the interest rate spread, although fully optimized simple rules are not analyzed.

If money is to serve a useful role in such contexts it seems most likely to do so as an informational variable regarding the natural rate of interest. The desire for “simple rules” and the readily available data on monetary aggregates at high frequencies suggests that this is a promising chance to expand the set of useful information to policy makers in real time. However, under typical money demand specifications, money is determined by output, inflation, a single interest rate and money demand shocks.

Two things are worth noting under such a specification. First, money provides no additional information to policy makers regarding the natural rates of output and interest not already contained in output, inflation and policy rate data. Second, such a specification is completely at odds with reality for broader monetary aggregates - inside money - whose equilibrium is determined by both *money demand* which depends on a vector of interest rate spreads and *money supply* by financial institutions and the shocks which originate therein. Hence, under more realistic descriptions, movements in broad monetary aggregates are driven by movements in non-policy rates and financial firms ability and willingness to supply monetary assets.

This point is apparent in the graph above which shows the growth rate of Divisia M4 along with the Federal Funds (policy) rate and output growth during the recent financial crisis. Typical money demand specifications described above would have a hard time explaining why money growth slowed while the Fed was cutting interest rates. One possibility is that weak output growth was suppressing the demand for money, however, this story falls apart in 2009 when output growth began rising but money growth weakened further. While suggestive, this graph provides some intuition into the behavior of inside money when the economy is subjected to financial market disturbances, thus motivating the question of what (if any) information can policy makers glean from broad monetary aggregates? In particular we ask, is there an exploitable relationship between broad money growth and non-observable variables such as the natural rate of interest when the economy is subject to financial shocks?

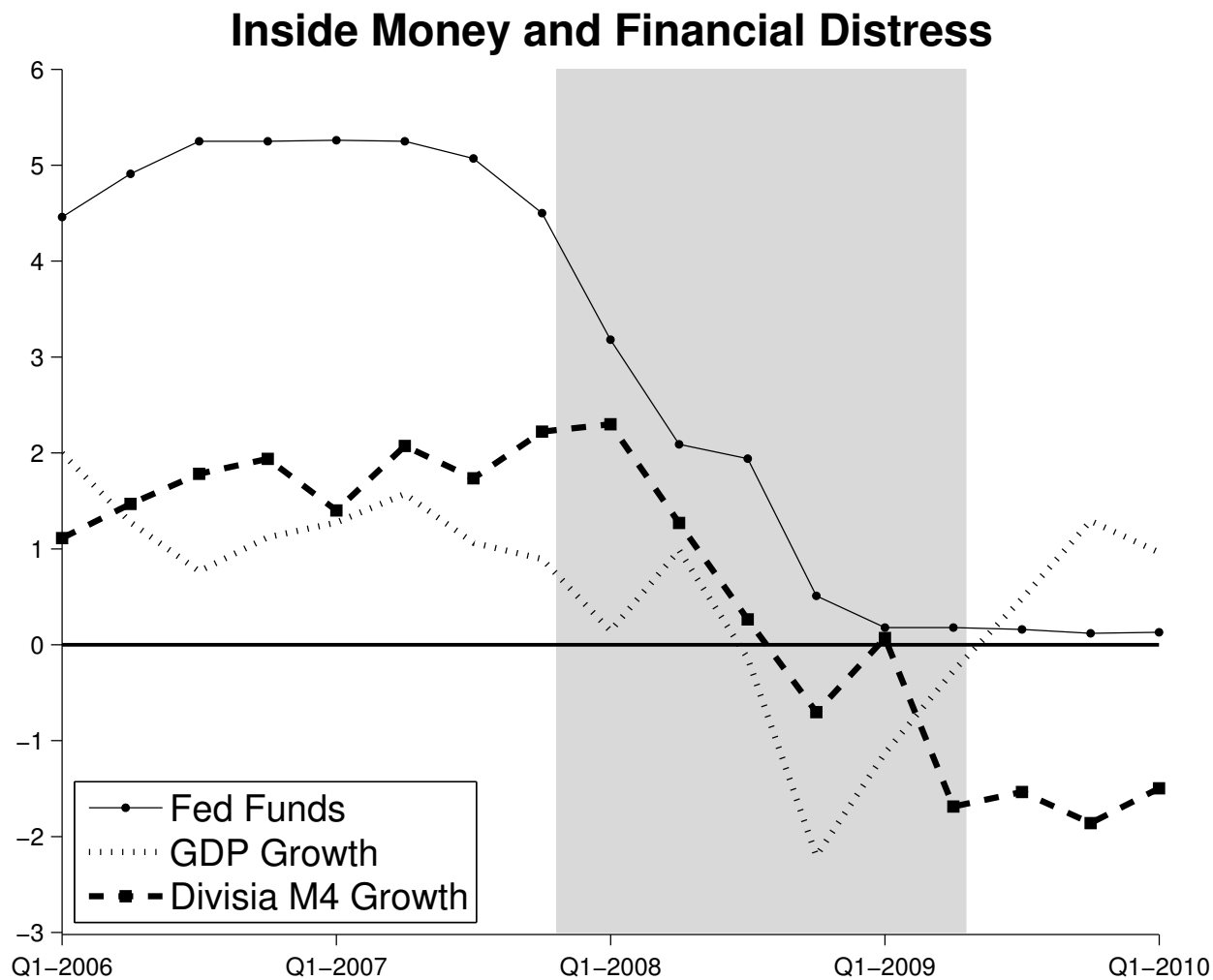


Figure 1.1: The shaded area marks the NBER recession dates. GDP and Divisia M4 growth rates are quarterly percentages while the Fed Funds rate is in annualized percentage points.

1.1.1 Related Literature

We are not the first to explore the usefulness of money in Taylor rules. Berger & Weber (2012) explore the relationship between a variable they define as the money gap (the difference between the equilibrium quantity of money and estimated money demand) and the natural interest rate in a prototypical NKSP model. They find that with noisy output gap information the optimal money gap response is positive. Our work differs from theirs in at least three regards. First, we search for *simple* rules which as defined above avoid the need

to use model specific variables nor estimate any model parameters. This point is especially important since we explore the usefulness of monetary *aggregates* in Taylor rules for which we employ the use of *parameter- and estimation- free* aggregation methods. Second, we examine optimal monetary policy around a largely distorted steady state which requires a second order accurate approximation to the model's equilibrium condition as in Schmitt-Grohe & Uribe (2004). Finally, we perform a complete analysis including numerically examining determinacy regions for Taylor rules which react to Divisia monetary aggregate growth rates and finding a *robustly* optimal policy rule under parametric uncertainty as in Giannoni (2002). A second related paper is by Andres et al. (2009) who explore the empirical linkages between dynamic money demand and the natural rate of interest. They find an empirical relationship and suggest the possibility of exploiting this relationship in optimal monetary policy, although they leave this for future work.

McCallum & Nelson (2011) also examine the link between monetary *aggregates* and the natural rate of interest. They argue that the information in monetary aggregates could be used to draw inference on the natural rate of interest provided the demand for the monetary aggregate depends on a vector of interest rates. For example, assuming aggregate demand depends on these real yields, then movements in these rates will affect the real policy rate consistent with stable prices - the natural rate of interest. As they describe, movements in these nominal yields will reflect movements in real yields aside from the policy rate causing the quantity of money to co-move with the natural rate of interest. This is exactly the path we pursue in this work. We spell out this relationship more carefully and provide an implementable *simple* interest rate rule which could be put to use by central banks without knowledge of model specific variables nor structural parameters.

1.1.2 Outline

The rest of this paper will proceed as follows. Next, we present the New-Keynesian model developed by Belongia & Ireland (2013) to include a role for monetary aggregates and financial firms. Then we define the natural rates of interest and output in the model and show

that the financial market disturbances in the model decreases the natural rate of interest which call for countercyclical monetary policy. We then examine the performance of optimal “simple” interest rate rules using a micro-founded welfare metric and a second order approximation to the model’s equilibrium conditions. With realistic assumptions regarding the monetary authority’s information set we show that reacting to inflation alone, which is typically optimal, fails to respond counter-cyclically to financial market disturbances leading to poor welfare performance.

Instead optimal policy chooses to also react to the growth rate of Divisia monetary aggregates which provide parameter- and estimation- free approximations to the true aggregate. We show that policy rules with a positive Divisia growth response have well-behaved determinacy properties satisfying a novel Taylor principle for monetary aggregates. Moreover, such rules bring about a real policy rate which is highly correlated with the natural rate and maximizes welfare. Interestingly, spread-adjusted Taylor rules are also highly correlated with the natural rate but perform poorly from a welfare standpoint. We offer some insight into this counter-intuitive result by showing that reacting to the interest rate spread fails to provide sufficient liquidity following an adverse financial shock and ultimately induces unwanted volatility in inflation . Finally, we end the paper with a robust policy prescription using a minimax approach, as in Giannoni (2002), given the parameters driving financial and other stochastic shocks remain uncertain.

1.2 Model

The model used in this analysis was developed by Belongia & Ireland (2013). It’s a standard sticky-price New-Keynesian model with the addition of a financial sector comprised of perfectly competitive financial firms. The banks produce interest bearing deposits and loans which requires a varying amount of real resources according to a stochastic process. As is typical in these models, production of the final good requires inputs from intermediate goods producers who have market power and hence can set their price given demand. The presence

of quadratic adjustment costs makes prices sticky which in turn makes monetary policy relevant. Finally, the representative household works for, and holds stock in, the intermediate goods firms. The household also demands loans and deposits from the financial firms which depend on the vector of interest rates the household faces.

1.2.1 The Representative Household

The representative household enters any period $t = 0, 1, 2, \dots$ with a portfolio consisting of 3 assets. The household holds maturing bonds B_{t-1} , shares of monopolistically competitive firm $i \in [0, 1]$ $s_{t-1}(i)$, and currency totaling M_{t-1} . The timing of transactions requires the household to carefully manage its portfolio of these assets. Central to this is the household's interaction with the representative bank with whom it makes deposits and takes loans. The reason the household deposits and borrows money from the bank at the same time is motivated in part by the description of the typical period t household budget constraints as described by Belongia & Ireland (2013). This budgeting can be described by dividing period t into 2 separate periods: first a securities trading session and then a transactions session.

1.2.1.1 Securities Trading Session

In the first part of period t the household purchases new securities which consists of bonds B_t which pay one nominal unit of currency in period t for price $1/r_t$, where r_t is the gross nominal rate of interest, and shares of monopolistically competitive firm i , $s_t(i)$ for a price of $Q_t(i)$ per share. In this first portion of period t the household also acquires the liquidity needed for the transactions period by securing loans from the representative bank totaling L_t . The household ends this securities trading session by allocating its loans and the currency remaining after trading securities between deposits and currency. Since deposits pay interest they dominate currency in return, however the household will hold currency in equilibrium due to the increased liquidity currency offers. The timing of these transactions is summarized

in the securities trading session budget constraint in (1) below.

$$D_t + N_t = M_{t-1} + B_{t-1} - \int_0^1 Q_t(i)(s_t(i) - s_{t-1}(i))di - B_t/r_t + L_t \quad (1.1)$$

1.2.1.2 Transactions Session

In the second portion of period t the household's deposits mature yielding $r_t^D D_t$ units of currency which are then added to the currency the household set aside at the end of the securities trading session - N_t . The household adds to this currency by supplying h_t total hours of labor to intermediate goods producing firm for a nominal wage rate $P_t W_t$. At the same time each intermediate goods producing firm $i \in [0, 1]$ makes a dividend payment of $F_t(i)$ for each share owned by the household. The household must also pay back to the bank all loans with interest totaling $r_t^L L_t$. The household then optimally allocates the remaining currency between consumption goods $P_t C_t$ and currency to be carried into next period M_t . These activities are summarized in the transactions session budget constraint in (2) below.

$$M_t = N_t - P_t C_t + W_t h_t + \int_0^1 F_t(i) s_t(i) di + r_t^D D_t - r_t^L L_t. \quad (1.2)$$

1.2.1.3 Household Preferences

The true monetary aggregate which enters the household's reduced form utility function is given by the CES aggregator

$$M_t^A = \left[\nu^{\frac{1}{\omega}} (N_t)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} (D_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (1.3)$$

where ν calibrates the relative expenditure shares on currency and deposits and ω calibrates the elasticity of substitution between the two monetary assets. In general, we need only assume that the monetary aggregate is block-wise weakly separable within the household's utility function. The approach taken by Belongia & Ireland (2013) is to specify a shopping

time friction of the form

$$h_t^s = \frac{1}{\chi} \left(\frac{v_t P_t C_t}{M_t^A} \right)^\chi \quad (1.4)$$

where v_t is a shock to the demand for monetary services following a first order autoregressive process (in logs)

$$\ln(v_t) = (1 - \rho_v) \ln(v) + \rho_v \ln(v_{t-1}) + \varepsilon_t^v \text{ where } \varepsilon_t^v \sim i.i.d. (0, \sigma_v^2). \quad (1.5)$$

We take this as our baseline calibration however the non additive-separability of the monetary aggregate implies a real balance affect which has been challenged empirically (See for example Ireland (2004a)). To show that our results do not hinge on this feature of the household's preferences we also consider the possibility that the monetary aggregate enters the household's utility function in an additively-separable form so that the term

$$\eta_m v_t \ln \left(\frac{M_t^A}{P_t} \right)$$

is added to the household's utility function over consumption and leisure. In either case we can define the household's preferences recursively by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t a_t [\ln(C_t) - \eta(h_t + h_t^s)]$$

or

$$U = E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[\ln(C_t) - \eta h_t + \eta_m v_t \ln \left(\frac{M_t^A}{P_t} \right) \right]$$

where a_t is a preference shock which follows a first order auto-regressive process (in logs)

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a \text{ where } \varepsilon_t^a \sim i.i.d. (0, \sigma_a^2). \quad (1.6)$$

The representative household faces the problem of maximizing its lifetime utility subject to its budget constraints. The household's optimization problem and the resulting first order

necessary conditions are given in the appendix using Bellman's equation.

1.2.2 The Representative Financial Firm

The representative bank creates demand deposits and originates loans for the representative household in a purely competitive market. Specifically, in period $t = 0, 1, 2, \dots$, the representative bank creates interest bearing deposits in the amount D_t and originates loans in the amount L_t . The pure-competition assumption implies the representative bank takes as given the gross nominal interest rate it pays on deposits r_t^D and the gross nominal interest rate it charges on loans r_t^L . The bank not only pays interest on deposits but also bears a time-varying real cost $c_t(\frac{D_t}{P_t})$ in order to create and service deposits defined by

$$c_t(\frac{D_t}{P_t}) = x_t \frac{D_t}{P_t}.$$

The x_t term is what makes the cost of producing deposits time varying, and in this case stochastic, as this deposit cost function evolves according to the first order auto-regressive process (in logs)

$$\ln(x_t) = (1 - \rho_x)\ln(x) + \rho_x \ln(x_{t-1}) + \varepsilon_t^x \text{ where } \varepsilon_t^x \sim i.i.d. N(0, \sigma_x^2). \quad (1.7)$$

The representative bank is also subject to the balance sheet constraint defined by the identity,

$$L_t = (1 - \tau_t)D_t \quad (1.8)$$

where τ_t represents reserves held by the bank. In this model, the banks demand for reserves varies stochastically according the first order auto-regressive process (in logs)

$$\ln(\tau_t) = (1 - \rho_\tau)\ln(\tau) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_t^\tau \text{ where } \varepsilon_t^\tau \sim i.i.d. N(0, \sigma_\tau^2). \quad (1.9)$$

Taking x_t and τ_t as given, the profit maximization problem facing the representative bank is defined by

$$\max_{D_t, L_t} \Pi_t^B = (r_t^L - 1)L_t - (r_t^D - 1)D_t - P_t x_t \frac{D_t}{P_t} \text{ subject to (1.8)}$$

Substituting (1.8) into the objective function is a simple way to express the bank's problem and leads to the first order necessary condition for profit maximization

$$r_t^D = 1 + (r_t^L - 1)(1 - \tau_t) - x_t. \quad (1.10)$$

or

$$\mathcal{S}_t = 1 + (r_t^L - r_t^D) = 1 + r_t^L(1 - \tau_t) + x_t \quad (1.11)$$

Equation (1.11) shows that the (gross) spread between deposits and loans varies endogenously according to the market determined loan rate and varies exogenously according to the banks demand for reserves and the marginal cost of producing deposits. The exogenous process for reserves demand simulates times of financial distress when banks choose to decrease lending activity and hoard deposits. The deposit cost shock acts to simulate negative banking productivity shocks effectively raising the bank's marginal cost of producing deposits. Just as in Curdia & Woodford (2009), the existence of a time-varying loan-deposit spread has implications for the natural rate of interest³ and ultimately optimal monetary policy.

1.2.3 The Representative Final Goods Producing Firm

The representative final goods producing firm maximizes period t profits for $t = 0, 1, 2, \dots$ using a constant returns CES technology defined by

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad (1.12)$$

³See for example within their paper the natural rate of interest under financial frictions denoted $r_t^{n,FF}$.

where $Y_t(i)$ is an input from intermediate goods producing firm i 's output. As is standard, we assume the final goods market is purely competitive leaving the representative final goods producing firm with no market power. Hence, behaving purely as a price taker in both the output market P_t and the input markets $P_t(i) \forall i \in [0, 1]$ the representative final goods producing firm solves

$$\max_{Y_t, \{Y_t(i)\}_{i \in [0, 1]}} \Pi_t^F = P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad (1.13)$$

subject to (1.12). This constrained maximization problem can easily be transformed into an unconstrained problem by substituting the constraint into the objective function to eliminate the choice variable Y_t . The resulting first order necessary condition defines the factor demand for each input $Y_t(i)$ as

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \quad (1.14)$$

$\forall i \in [0, 1]$.

1.2.4 The Representative Intermediate Goods Producing Firm

Unlike the final goods market the intermediate goods market is not purely competitive. Instead each intermediate goods producing firm $i \in [0, 1]$ produces a differentiated product leading to some degree of market power. To permit aggregation and allow for the consideration of a representative intermediate goods producing firm i , we assume all such firms have the same constant returns to scale technology which implies linearity in the single input labor $h_t(i)$,

$$Y_t(i) = Z_t h_t(i). \quad (1.15)$$

In each period $t = 0, 1, 2, \dots$ the representative intermediate goods producing firm rents $h_t(i)$ units of labor from the representative household for a nominal market determined wage rate,

$P_t W_t$. The Z_t term in (1.15) is an aggregate technology shock that follows a random walk with drift (in logs)

$$\ln(Z_t) = \ln(Z) + \ln(Z_{t-1}) + \varepsilon_t^z \text{ where } \varepsilon_t^z \sim i.i.d. (0, \sigma_z^2). \quad (1.16)$$

The market power of each intermediate goods producing firm i leads to the ability for each firm to set the price $P_t(i)$ of its output $Y_t(i)$ each period t . The price setting ability of each firm is constrained in two ways. First, each intermediate goods producing firm faces a demand for its product from the representative final goods producing firm defined in (1.14). Second, each intermediate goods producing firm faces a convex cost of price adjustment proportional one nominal unit of the final good defined by Rotemberg (1982) to take the form

$$\Phi(P_t(i), P_{t-1}(i), P_t, Y_t) = \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right]^2 Y_t P_t. \quad (1.17)$$

Every intermediate goods producing firm $i \in [0, 1]$ maximizes its period t real price per share denoted by $\frac{Q_t(i)}{P_t}$. Though the firm maximizes period t share price, the costly price adjustment constraint makes the intermediate goods producing firm's problem dynamic (and recursive) as shown in the appendix. Mathematically summarizing, each intermediate goods producing firm solves to following dynamic problem,

$$\max_{\{h_t(i), P_t(i)\}_{t=0}^{\infty}} \frac{Q_t(i)}{P_t}$$

subject to the constraints (1.14), (1.15) and (1.17). In a symmetric equilibrium the log-linearized first order condition of the above problem takes the form of a New-Keynesian Phillips Curve (NKPC) relating current inflation to the average real marginal cost and expected future inflation. The resulting NKPC can be calibrated to match the NKPC

derived from Calvo style price adjustment based on the frequency of price changes⁴.

1.2.5 The Central Bank

We close the model by specifying the general class of monetary policy rules we consider by

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_y (\tilde{Y}_t - \tilde{Y}_{t-1}) + \phi_m \tilde{\mu}_t^{Divisia} - \phi_s \tilde{\mathcal{S}}_t \quad (1.18)$$

in log-deviations from steady state⁵ with ρ_r in $[0, 1]$ and ϕ_π , ϕ_y , ϕ_m and ϕ_s in $[0, \infty)$ and $\mathcal{S} = r_t^L - r_t^D$. The policy rule is restricted to be both linear in logs and react only to observable model non-specific variables. The second restriction is key for the policy rule to be implementable as stressed in Schmitt-Grohe & Uribe (2007) and Faia & Monacelli (2007). For this reason we include the growth rate of output instead of deviations of output from its natural level.⁶ Orphanides (2003) stresses the latter is not available to policy makers in real time without significant measurement error. For robustness, we also examine the case when the efficient level of output is available to policy makers in real time (See section 4.2.3). The implementability restriction also requires that we provide a measure of the monetary aggregate that doesn't require knowledge of its functional form nor the values of its parameters. We discuss this issue below.

1.2.5.1 The Monetary Aggregation Problem

The general problem of tracking an unknown aggregator function without estimation is not new. The solution lies in statistical index number theory as advanced by Diewert (1976) and specifically applied to monetary aggregation by Barnett (1978, 1980). The focus of this field is to provide parameter- and estimation-free aggregates. One such index number performs this task with a known level of accuracy. The Divisia monetary aggregate provides

⁴However, the two pricing assumptions will in general result in different NKPCs up to a second order approximation. This difference will generally lead to different welfare-loss functions when approximated around a distorted steady-state as shown in Lombardo & Vestin (2008).

⁵To be clear, for any variable X_t , $\tilde{X}_t = \ln(X_t) - \ln(\bar{X})$.

⁶This natural level of output is defined in section 3 below.

a second-order accurate approximation to the growth rate of M_t^A .⁷ We now define the Divisia monetary aggregate in this model.

Definition 1.1. The growth rate of the Divisia monetary aggregate is defined by

$$\ln(\mu_t^{Divisia}) = \left(\frac{s_t^N + s_{t-1}^N}{2} \right) \ln \left(\frac{N_t}{N_{t-1}} \right) + \left(\frac{s_t^D + s_{t-1}^D}{2} \right) \ln \left(\frac{D_t}{D_{t-1}} \right) \quad (1.19)$$

where s_t^N and s_t^D are the expenditure shares of currency and interest bearing deposits respectively defined by

$$s_t^N = \frac{u_t^N N_t}{u_t^N N_t + u_t^D D_t} = \frac{(r_t^L - 1)N_t}{(r_t^L - 1)N_t + (r_t^L - r_t^D)D_t} \quad (1.20)$$

and

$$s_t^D = \frac{u_t^D D_t}{u_t^N N_t + u_t^D D_t} = \frac{(r_t^L - r_t^D)D_t}{(r_t^L - 1)N_t + (r_t^L - r_t^D)D_t}. \quad (1.21)$$

Since the definition of the Divisia monetary aggregate only requires knowledge of current and one period lagged monetary component quantities and interest rates our policy rule specified in (1.18) is a simple rule which could actually be implemented by central banks facing limited real time information. For example in the U.S., the St. Louis Fed's MSI series provides Divisia monetary aggregates for M1 and M2 at a monthly frequency⁸.

We embed the Divisia approximation to the true aggregate, as opposed to alternative simple-sum approximations, in the policy rule due to the superiority of the Divisia monetary aggregate in tracking the true aggregate - as shown in this model by Belongia & Ireland (2013).

⁷The ability to track any function which is homogeneous of degree one (as all sensible aggregator functions are) to second order accuracy places the Divisia aggregate in Diewart's (1976) class of *superlative index numbers*.

⁸Private organizations such as the Center for Financial Stability have recently begun providing broader Divisia monetary aggregates at monthly frequencies as well.

⁹ For thoroughness we examine the performance of the more common simple-sum aggregate

$$\ln(\mu^{Simple-Sum}) = \ln\left(\frac{N_t + D_t}{N_{t-1} + D_{t-1}}\right) \quad (1.22)$$

in place of the Divisia aggregate in the above policy rule. The rule results in indeterminacy in most of the parameter space. In the appendix we show the determinacy region and show through linearization the size of the error of the simple-sum aggregate in tracking the true aggregate in this model.

1.2.6 Market Clearing

It is now possible to define the equilibrium conditions which close the model. Market clearing in the labor market requires that labor supply equal labor demand, or

$$h_t = \int_0^1 h_t(i) di. \quad (1.23)$$

Equilibrium in the final goods market requires that the accounting identity

$$Y_t = C_t + x_t \frac{D_t}{P_t} + \frac{\phi}{2} \left[\frac{\Pi_t}{\Pi} - 1 \right]^2 Y_t \quad (1.24)$$

holds as well. Equilibrium in the money market, equity market and bond market requires that at all times

$$M_t = M_{t-1} \quad (1.25)$$

$$s_t(i) = s_{t-1}(i) = 1 \quad (1.26)$$

$$B_t = B_{t-1} = 0 \quad (1.27)$$

⁹For more general research examining the Divisia monetary aggregate's properties relative to alternative simple-sum measures see the following works. At paper length Barnett & Chauvet (2011b); Belongia (1996) and at book length Barnett & Singleton (1987); Belongia & Binner (2000); Barnett & Serletis (2000); Barnett & Chauvet (2011a); Barnett (2012).

respectively. Finally, imposing the symmetry among the intermediate goods producing firms requires that in equilibrium

$$Y_t(i) = Y_t, P_t(i) = P_t, F_t(i) = F_t, \text{ and } Q_t(i) = Q_t. \quad (1.28)$$

1.2.7 Welfare Relevant Natural Rates of Output and Interest

Central to the analysis of optimal monetary policy in NKSP models are the concepts of the natural rate of output and interest. These measures respectively represent the level of output and the real interest rate in an identical economy as the one described without the presence of sticky prices. Woodford (2003) has shown that these concepts play the key role in optimal monetary policy, hence we provide the relevant definitions in this model to show that financial market supply shocks affect these variables and hence impact optimal monetary policy rules - similar to the model of Curdia & Woodford (2009). For cohesiveness, we use Woodford's (2003, pg. 302) definition of the natural rate of interest in a monetary economy.

Definition 1.2. The natural rate of output is the equilibrium level of output at each point in time that would prevail under flexible prices, given a monetary policy that maintains a constant interest rate spread $u_t^A = r_t^L - r_t^A$ between non-monetary (bonds or loans) and monetary riskless short-term assets (currency and deposits).

The aggregate interest rate on monetary riskless short-term assets r_t^A in Definition 2 is the nominal return for holding one unit of the monetary aggregate in period t . This interest rate can be derived from first principles¹⁰ providing a coherent way to think about interest rates in an economy with multiple monetary assets each with different rates of return. Therefore the aggregate user-cost $u_t^A = r_t^L - r_t^A$ provides a natural analogue to the the interest rate spread between bonds and a single monetary asset, the environment considered in Woodford (2003, Ch. 4). Applying this definition to the log-linearized equilibrium conditions results in

¹⁰We carefully define this in the appendix. See Eq. 1.A.18.

a natural rate of output that depends only on the model's stochastic disturbances. As shown in the appendix, the resulting expression for the natural rate of output (in log deviations from steady state¹¹) is given by

$$\tilde{Y}_t^n = \tilde{Z}_t - \Psi_v^y \tilde{v}_t - \Psi_x^y \tilde{x}_t - \Psi_\tau^y \tilde{\tau}_t \quad (1.29)$$

where Ψ_x^y and Ψ_τ^y are positive in all reasonable calibrations¹². As is standard from the real business cycle literature, positive technology shocks increase the productive capacity of the economy under flexible prices. Novel here is the appearance of the financial shocks in this expression. Adverse shocks to the financial intermediary's cost of producing monetary assets and willingness to supply loans will behave as negative technology shocks and lower the natural rate of output.

However, there is a striking difference between financial and goods market supply shocks from a policy standpoint. This difference lies in how these shocks affect the natural rate of interest. The expression for the natural rate - given the above definition of the natural rate of output - is shown below (in log deviations from steady state¹³)

$$\tilde{r}_t^n = (1 - \rho_a) \tilde{a}_t - (1 - \rho_z) \tilde{Z}_t - \Psi_x^r (1 - \rho_x) \tilde{x}_t - \Psi_\tau^r (1 - \rho_\tau) \tilde{\tau}_t \quad (1.30)$$

where $\Psi_x^r > 0$ and $\Psi_\tau^r > 0$ and independent of how money enters the utility function. The implication for monetary policy is that responding to adverse financial supply shocks calls for *countercyclical* policy, meanwhile responding to adverse technology shocks calls for *procyclical* policy. The challenge facing the monetary authority is how to form a policy rule which is optimal in this environment given the lack of information they have on the natural rate. We show the monetary aggregate provides valuable information regarding movements in this key

¹¹To be clear, for any variable X_t , $\tilde{X}_t = \ln(X_t) - \ln(\bar{X})$.

¹²The sign of Ψ_v^y changes depending on the specification of preferences. For additively separable utility it is always positive however for non additively-separable utility it is negative.

¹³To be clear, for any variable X_t , $\tilde{X}_t = \ln(X_t) - \ln(\bar{X})$.

variable.

1.3 Calibration and Solution Strategy

At this point it is useful to proceed by assigning numerical values to the model's parameters. Following the calibration strategy pioneered by Kydland & Prescott (1982) we assign values to parameters in a fashion that allows us to match key features of U.S. data. Since the model used here was developed by Belongia & Ireland (2013) we take many of the values used in their study. Moreover, the model is similar in many regards to the ones estimated by Ireland (2004a,b) providing reliable estimates of the parameters defining the stochastic processes. Table 1.10 in the appendix summarizes the choice of parameter values. We end the section with a brief discussion of the solution procedure.

1.3.1 Calibration

The model is calibrated to the U.S. economy so that one period represents one quarter. We set $\beta = .99$ and we set $\bar{Z} = 1.005$ which is consistent with annual real GDP growth of 2%. These facts imply an annual real interest rate of 6% which is in line with the RBC calibration literature (Kydland & Prescott, 1982). We set $\pi = 1.005$ which matches the official inflation target of the Fed of 2% per year. The disutility of work parameter $\eta = 2.5$ so that one-third of the household's time is spent working. When money is non-separable setting $\chi = 2$ makes the shopping time specification quadratic. As for the CES monetary aggregator M_t^A , the calibrations of $\omega = 1.5$ specifies more substitutability between currency and deposits than the Cobb-Douglas specification and setting $\nu = .225$ pins down the ratio of steady state ratio $\frac{N}{N+D} = .1$, the U.S. average ratio of currency to simple-sum M2 since 1959. Similarly, $\bar{v} = .4$ matches the steady state ratio $\frac{N+D}{PC} = 3.3$, the average of simple-sum M2 to nominal consumption expenditures over the same period.

On the production side of the economy we set the elasticity of substitution between intermediate goods $\theta = 6$ yielding a steady state mark-up of 20% over marginal cost for the monopolistic firm following Ireland's estimates (2000; 2004a; 2004b). The setting for the

cost of price adjustment $\phi = 50$ implies a NKPC that coincides with Calvo style pricing dynamics when the average duration of a price is slightly more than one year (Ireland, 2004b). Regarding the production of financial assets and services setting $\tau = .02$ sets the steady level of reserves to 2%, the average ratio of St. Louis adjusted reserves to the simple-sum deposit components of M2 since 1959 (Ireland, 2011). From the goods market clearing condition, setting $x = .01$ implies in steady-state 3% of output is devoted to banking activities, slightly less than the 3.6% derived from “Federal Reserve banks, credit intermediation, and related activities” on average over the last decade according to the Bureau of Economic Analysis. The fact that our model implies a lower average is accurate considering the limited scope of the activities carried out by the financial sector in this model (Belongia & Ireland, 2006, 2013).¹⁴

Many of the stochastic processes driving the model have been estimated in Ireland (2004a). Specifically, we use Ireland’s estimates of the persistence and standard error of the money demand, preference and technology¹⁵ shocks. However, the parameters driving the financial shocks have not yet been estimated in the literature. Dellas et al. (2010) calibrate a model with reserves demand and bank cost shocks interpreting the calibrated values from a frequentists perspective. Following their approach to calibrating the financial shocks standard errors we set σ_τ and σ_x so that an increase in banks demand for reserves and an increase in the benchmark-M2 aggregate interest rate spread of the magnitude witnessed during the recent financial crisis occurs once every 80 years on average in our model, assuming normality.¹⁶ The time period of 80 years acknowledges the time span between the Great Depression

¹⁴As previously mentioned, the deposit-cost shock also dictates the spread between the benchmark interest rate r_t^L and the deposit rate r_t^D . To confirm the logic of the calibration note the annualized spread between the benchmark rate and the aggregate rate $r^L - r^A$ in the model’s steady-state is about 4.6%, which is very close to the average spread between the benchmark rate and the Divisia M2 aggregate interest rate in U.S. data running from January 1967 to September 2009 equal to 4.2% using data from the Center for Financial Stability. Hence, the ability of the model to emulate the data along this added dimension reconfirms the calibration put forth by Belongia & Ireland (2013).

¹⁵The persistence parameter for the technology shock is set to 1 throughout the paper.

¹⁶More specifically reserves measured using the ratio of St. Louis adjusted reserves to the deposit components of M2 spiked from slightly over .02 in September 2008 to over .21 in June of 2011. Hence we set σ_τ so that $P(\tau > \ln(.21) - \ln(.02)) = 1/320$ where $\tau \sim N(0, \sigma_\tau^2 \sum_{i=0}^{11} \rho_\tau^{2i})$. Similarly, the spread between the benchmark rate and the Divisia M2 aggregate rate stood at just 2.8% in July 2008 and in just one quarter the spread

and the recent financial crisis. This specification of σ_τ is conditional on a value of ρ_τ . When calibrating ρ_τ and ρ_x we once again follow Belongia & Ireland (2013) and set $\rho_\tau = .5$ and $\rho_x = .5$. However since these are only calibrated values and not estimates in section 5.2 we perform a *robust* policy calculation allowing ρ_τ and ρ_x to vary between $[0, .99]$. Moreover, in this section we relax the normality assumption used here to calibrate σ_τ and σ_x to a student's t-distribution with significantly “fatter-tails” as called for by much of the finance literature (Mandelbrot, 1963; Fama, 1965).

1.3.2 Solution Strategy

Solving the full non-linear model is not possible hence we resort to approximation techniques. Two main difficulties are encountered. First, the model as presented does not exhibit a deterministic steady-state due to the unit root in the technology process and the price level. To deal with this we detrend most nominal variables by the price level and the technology shock. Details of this detrending are handled in the appendix. Second, a first order approximation is adequate for questions of local determinacy however for proper welfare rankings we must use (at least) a second order approximation¹⁷ (Schmitt-Grohe & Uribe, 2004). Therefore we analyze determinacy using a linear approximation to the model around the steady state implied by the competitive equilibrium and we evaluate welfare using a second-order approximation around the Ramsey planner's steady state with all distortions in place as in Schmitt-Grohe & Uribe (2007).¹⁸ We discuss this choice and provide more details regarding the welfare evaluation in the section below.

spiked to 4.47%. In order to simulate such liquidity shocks we set σ_x so that $P(x > \ln(4.47) - \ln(2.8)) = 1/320$ where $x \sim N(0, \sigma_x^2)$. The value obtained for σ_x is very close the one obtained by Belongia & Ireland (2006) when they calibrate an RBC version of the model to match their estimated VAR standard errors.

¹⁷An exception to this fact is when the steady state distortions are small. This case is the focus of Woodford (2003) and Curdia & Woodford (2010).

¹⁸It's worth noting, the determinacy results also hold around the Ramsey planner's steady state and the welfare results are qualitatively identical if we approximate the model around the competitive equilibrium's steady state.

1.4 Welfare Evaluation of Monetary Policy Rules

We now present our main results of the paper. Most importantly, optimal monetary policy features a positive response to the growth rate of the Divisia monetary aggregate. We show that policy rules which respond to the growth rate of money result in real policy rates which are highly correlated with the natural rate of interest. However, this condition appears to be only necessary, not sufficient for good policy. Specifically, responding to the interest rate spread increases the correlation between the real and natural rates of interest but fails to deliver good policy. Instead, such rules increase the variance of inflation and the resources allocated to financial intermediation - leading to detrimental welfare effects.

1.4.1 Welfare Evaluation Methodology

The focus of the current literature on optimal simple monetary policy rules has been to use a micro-founded measure of welfare without imposing any upper bound on the size of distortions generated in the competitive equilibrium. Following this line of work we use a second order approximation to the household's utility function as our metric for ranking alternative policy rules. Moreover, the appropriate expectation of welfare is the *conditional* as this measure takes into account welfare gains and losses accrued while transitioning from the deterministic steady state to the stochastic steady state implied by the given policy rule (Kim et al., 2005). This does however give meaning to the initial state at which policy is evaluated. For this reason we evaluate all policies around the Ramsey planner's steady state. Hence, we generally have the following welfare metric, regardless of how preferences over money are specified.

$$\mathcal{W}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t, h_t, \frac{M_t^A}{P_t}, a_t, v_t) \right] \quad (1.31)$$

We take a second order approximation to $u(C_t, h_t, \frac{M_t^A}{P_t}, a_t, v_t)$ around the Ramsey steady state allowing for us to express welfare as the weighted sum of conditional means and covariances

of the arguments in the utility function. We then evaluate these conditional moments using decision rules found from the second order approximation to the equilibrium conditions.¹⁹ We report our results as consumption equivalent shares. Specifically if \mathcal{W}_0^* is the welfare obtained under an optimal policy then Ω is the share of the generic consumption stream the household would need to equate welfare under the sub-optimal policy to the optimal welfare

$$\mathcal{W}_0^* = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t(1+\Omega), h_t, \frac{M_t^A}{P_t}, a_t, v_t) \right]. \quad (1.32)$$

Given this definition, Ω is simply the welfare cost in consumption terms of sub-optimal policies.

1.4.2 Optimal Simple Rules

We now turn our attention to optimal simple rules of the form

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_y (\tilde{Y}_t - \tilde{Y}_{t-1}) + \phi_m \tilde{\mu}_t^{Divisia} - \phi_s \tilde{\mathcal{S}}_t$$

in log-deviations from steady state with ρ_r in $[0, 1]$ and ϕ_π , ϕ_y , ϕ_m and ϕ_s in $[0, \infty)$ and $\mathcal{S}_t = 1 + (r_t^L - r_t^D)$. The most significant departure from policy rules specified in the literature on optimal simple rules is the inclusion of the (growth rate of the) Divisia monetary aggregate and the spread between loans and deposits. We show below that optimal policy calls for a response to the Divisia monetary aggregate but not the interest rate spread. In particular, we highlight the increased correlation of the real interest rate with the natural rate of interest when policy reacts to the Divisia aggregate. Moreover, reacting to Divisia achieves this high correlation at a low inflation volatility compared to rules which react to the interest rate spread. We also examine optimal policy under additively separable preferences over money and under the assumption policy makers have knowledge to the efficient output gap and find

¹⁹To prevent the order of the forecast, and the forecast itself, from exploding we implement the pruning method proposed in Kim et al. (2005). More specifically, we use a first order approximation to the equilibrium conditions to evaluate the conditional variances and then use these conditional variances in the second order approximation to evaluate the means. See Kim et al. (2005) and specifically Section 7 therein.

our results to be robust.

1.4.2.1 Baseline Results

We first present our baseline results assuming policy makers only have knowledge of the growth rate of output, current and past interest rates, inflation and the growth rate of the monetary aggregate. The results of the central bank's optimization are presented in Table 1.1.

Table 1.1: Welfare Results for Optimal Simple Rules

$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_y (\tilde{Y}_t - \tilde{Y}_{t-1}) + \phi_m \tilde{\mu}_t^{Divisia} - \phi_s \tilde{S}_t$						
	<i>Optimal Policy Rule Coefficients</i>					<i>Welfare Cost</i>
Policy Rule	ρ_r^*	ϕ_π^*	ϕ_y^*	ϕ_m^*	ϕ_s^*	Ω^1
Optimal	0	3.234	0	2.366	0	0
Optimal $\phi_\pi = 0$	0	—	0	4.217	0	.0304
Optimal $\phi_m = 0$	0	3.6780	0.8982	—	0	.1551
Optimal $\phi_m = \phi_y = 0$	0	4.070	—	—	0	.1736

¹ Ω is the % of consumption required to equate welfare under any given policy rule to the one under the optimal policy (see Eq. (1.32)). Welfare is calculated as conditional to the initial deterministic Ramsey steady state.

Several points are worth noting. (*i.*) First, responding to the Divisia monetary aggregate is essential to achieve optimal welfare. Notice the welfare cost of not responding to the Divisia aggregate is nearly 5 times as large as the welfare cost of not responding to inflation. One reason for the importance of responding to the Divisia monetary aggregate is due to its ability to provide an indicator in movement of the natural rate. Section 2.7 shows the natural rate of interest falls in response to financial shocks calling for expansionary policy. Only focusing on inflation, or even output growth, fails to provide sufficient expansion. To verify this, Table 1.2 presents the correlation between the natural rate (See Eq. (1.30)) and real interest rate under optimal rules with and without money.

(*ii.*) Second, unlike in Schmitt-Grohe & Uribe (2007) and Faia & Monacelli (2007) - without considering monetary aggregates ($\phi_m = 0$) - responding to output growth is welfare enhancing. The reason for the difference lies in the stochastic rank of our economy versus theirs.

Table 1.2: Correlation of Real and Natural Interest Rates

Policy Rule	$Corr(\tilde{r}_t^n, \tilde{r}_t - E_t \tilde{\pi}_{t+1})$
Optimal	.8281
Optimal $\phi_\pi = 0$.8355
Optimal $\phi_m = 0$.0377

Their economies are driven by aggregate demand and technology shocks, which affect the natural and efficient levels of output symmetrically. In our economy on the other hand, the presence of financial shocks - which affect the natural level of output but not the efficient level - induces policy makers to deviate from strict inflation targeting. Our third point (*iii.*) responding to the interest rate (loan-deposit) spread decreases welfare. We expand on this counter-intuitive point below where we consider non-optimized rules, including a spread-adjusted Taylor rule as advocated by Taylor (2008).

1.4.2.2 Results with Additively Separable Utility

One may wonder if we arrive at the above results due to the specification of non-additively-separable preferences over money and consumption. As Ireland (2004a) highlights this assumption results in money showing up directly in the IS equation and the NKPC. The resulting real-balance effect may lead to a bias to stabilize the monetary aggregate. In this section we directly address this concern. The results below verify that money should enter the policy rule due to the information it conveys to policy makers regarding developments in financial markets, not because of the specification of preferences over money.

Interestingly when money enters the policy rule, it ends up in the IS and NKPC as pointed out by McCallum & Nelson (2011). Hence, Table 1.3 suggests money *should* enter these equations from a normative standpoint, regardless of the empirical motivation for money entering these equations (See e.g. (Ireland, 2004a)). Specifically, under additive separability the optimal coefficients are qualitatively similar to the optimal coefficients under our baseline specification. Qualitatively however, optimal policy continues to respond only to inflation and Divisia. What's more, under these preferences the welfare cost of not responding to

Table 1.3: Optimal Simple Rules with Additively Separable Preferences

$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_y (\tilde{Y}_t - \tilde{Y}_{t-1}) + \phi_m \tilde{\mu}_t^{Divisia} - \phi_s \tilde{S}_t$						
Policy Rule	Optimal Policy Rule Coefficients					Welfare Cost
	ρ_r^*	ϕ_π^*	ϕ_y^*	ϕ_m^*	ϕ_s^*	Ω^1
Optimal	0	3.350	0	2.803	0	0
Optimal $\phi_\pi = 0$	0	—	0	5.012	0	.0290
Optimal $\phi_m = 0$	0	5.7601	0.881	—	0	.2258
Optimal $\phi_m = \phi_y = 0$	0	4.044	—	—	0	.2398

¹ Ω is the % of consumption required to equate welfare under any given policy rule to the one under the optimal policy (see Eq. (1.32)). Welfare is calculated as conditional to the initial deterministic Ramsey steady state.

Divisia continues to be nearly 6 times the cost of not responding to inflation.

1.4.2.3 Results when the Efficient Output Gap is Observable

To this point we have followed the mainstream literature studying optimal *simple* monetary policy rules and assumed that no measure of the output gap is observable. Although this is the most conservative assumption regarding the real time information policy makers, one may argue that assumption is too restrictive. For example, central banks can use standard filtering techniques to generate measures of the trend and cyclical components of output.²⁰ The trend component presumably represents technological changes to which monetary policy should not attempt to affect.²¹ Indeed, there is *some* evidence policy makers have this information. Gali et al. (2003) argue the Volcker-Greenspan Fed's response to technology shocks was consistent with optimal policy. In this section we examine the robustness of our results when we relax this constraint on the policy maker's information set. In particular, we ask, is it still necessary for optimal policy to respond to the monetary aggregate when the efficient

²⁰Woodford (2003 p. 615-616) acknowledges that central bank forecasts of the output gap, "...are usually measures of real GDP relative to some fairly smooth trend." He goes on to say that the measurement appropriate in an optimal policy rule "... is the difference between real GDP and a target level that should vary in response to real disturbances of many sorts..., and it is not obvious these real factors should all be expected to evolve as a smooth trend."

²¹This is the case in our model as well which features a stochastic trend.

output gap²² is observable?

Table 1.4: Optimal Simple Rules when the Efficient Output Gap¹ is observable.

$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_g \tilde{G}_t^e + \phi_m \tilde{\mu}_t^{Divisia} - \phi_s \tilde{S}_t$					
<i>Optimal Policy Rule Coefficients</i>					
	ρ_r^*	ϕ_π^*	ϕ_g^*	ϕ_m^*	ϕ_s^*
Optimal	0	3.234	0	2.366	0

¹ The Efficient Output Gap in levels is given by $G_t^e = \frac{Y_t}{Y_t^e} = \eta \frac{Y_t}{Z_t}$ as shown in the appendix (See Eq. (1.A.32)).

The results in this section show two things. First, when the efficient output gap is observable optimal policy still responds to the Divisia monetary aggregate. Second, and more to the point, knowledge of the efficient output gap has no impact on the optimal policy coefficients - the only variables in the policy rule with a non-zero coefficient are inflation and the growth rate of the Divisia aggregate. This result stresses the importance of financial shocks shaping the optimal simple policy rule. In particular, policy makers must now deal with inefficient resource costs from financial intermediation in addition to inefficient resource cost stemming from price adjustment and inefficient rents due to monopolist's market power.

1.4.3 Non-Optimized Rules

It is insightful at this point to compare the performance of policy rules which have historically provided a description of actual Fed policy with our optimal rules. In particular, we can infer the welfare gains of switching from current policy to our prescribed optimal rule. We examine the performance of the original Taylor rule (Taylor, 1993) and the forward-looking rule estimated in Clarida et al. (2000) which provides a good description of Fed policy during the Volcker-Greenspan era. Not surprisingly, the above rules lack an aggressive enough response to the financial market shocks and hence lack the necessary correlation with the natural rate of interest. Addressing this issue, Taylor (2008) recently suggested allowing the

²²The level of the efficient output gap is given by $G_t^e = \frac{Y_t}{Y_t^e} = \eta \frac{Y_t}{Z_t}$, as shown in the appendix (See Eq. (1.A.32)).

intercept of the Taylor rule to vary with the interest rate spread (SATR). In Table 1.5 we present the welfare performance of the ad-hoc policy rules.

Table 1.5: Non-Optimized Rules

$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi E_t \tilde{\pi}_{t+i} + \phi_g E_t \tilde{G}_{t+i}^e - \phi_s \tilde{S}_t$						
	<i>Policy Rule Coefficients</i>					<i>Welfare Cost</i>
<i>Policy Rule</i> ¹	i	ρ_r	ϕ_π	ϕ_g	ϕ_s	Ω^2
Taylor Rule	0	0	1.5	.125	0	.2084
CGG ³	1	0.79	.4515	.1953	0	.2316
SATR $\phi_s = 2$	0	0	1.5	.125	2	.3834

¹ Here we generalize our previous policy rule allowing for forward (or backward) looking behavior. We use the level of the efficient output gap in this rule due to indeterminacy problems when $\phi_s > 0$ associated with the growth rate rule specified in Eq. (1.18).

² Ω is the % of consumption required to equate welfare under any given policy rule to the one under the optimal policy (see Eq. (1.32)). Welfare is calculated as conditional to the initial deterministic Ramsey steady state.

³ CGG refers to Clarida, Gali and Gertler's (2000) estimated forward-looking Taylor rule.

Table 1.6 shows that policy rules which lack a response to financial variables lack correlation with the natural rate and perform poorly. What is perhaps more shocking is the poor performance of the SATR - despite bringing about a real policy rate which is highly correlated with the natural rate of interest. This reiterates our previous point; good policy rules in this economy are correlated with the natural rate of interest, but correlation with the natural rate of interest is not sufficient to guarantee good policy.

Table 1.6: Correlation of Real and Natural Interest Rates

Policy Rule	$Corr(\tilde{r}_t^n, \tilde{r}_t - E_t \tilde{\pi}_{t+1})$
Taylor	.1644
CGG	.1785
SATR $\phi_s = 2$.7269

1.4.3.1 Creating liquidity without inflation: Why responding to Money Growth (Not Interest Rate Spreads) Is Optimal

This poor welfare performance of spread adjusted Taylor rules (SATR) at first seems counter-intuitive. Table 1.2 suggests countercyclical policy is key to optimally responding to financial shocks, however the point is bit more subtle. Indeed, responding directly to the interest rate spread by lowering the policy rate fails to deliver good policy. The reason for this failure can be seen by examining the impulse responses following a negative banking productivity shock as shown in Figure 1.2 below. The solid lines show the dynamics under the optimal policy which reacts to inflation and the Divisia aggregate while the dotted and diamond lines are the dynamics under various SATR's. Notice the dynamics for inflation are driven primarily by the Fisher equation and the monetary policy rule. That is, inflation dynamics look very similar to the variables in the respective policy rules.

Consider first the interest rate spread rules. A negative banking productivity shock increases the spread between loan and deposit rates resulting in an immediate cut in the policy rate. The real rate will rise by more or less depending on the expected behavior of inflation. Since the rise in the spread has some persistence, the policy rate is expected to remain below steady state and therefore the inflation rate is expected to remain above steady state into the future - causing the real policy rate to fall below the nominal policy rate.²³ This amplification of the real rate results in positive output gaps as long as the spread remains above steady state. More importantly here, the monetary aggregate falls as interest bearing deposits drop, however the expansionary drop in rates is associated with an increase in the monetary base - a liquidity effect.

Compared to the optimal policy rule, this uptick in the monetary base under the SATR is dwarfed. The optimal rule which stabilizes $\tilde{\mu}_t^{Divisia}$, significantly increases the monetary base by lowering the real interest rate by more than even the SATR with $\phi_s = 10$! This

²³This affect is more amplified when the shock has more serial correlation. In fact, for larger values of ρ_x the nominal rate rises on impact due to the large increase in inflation.

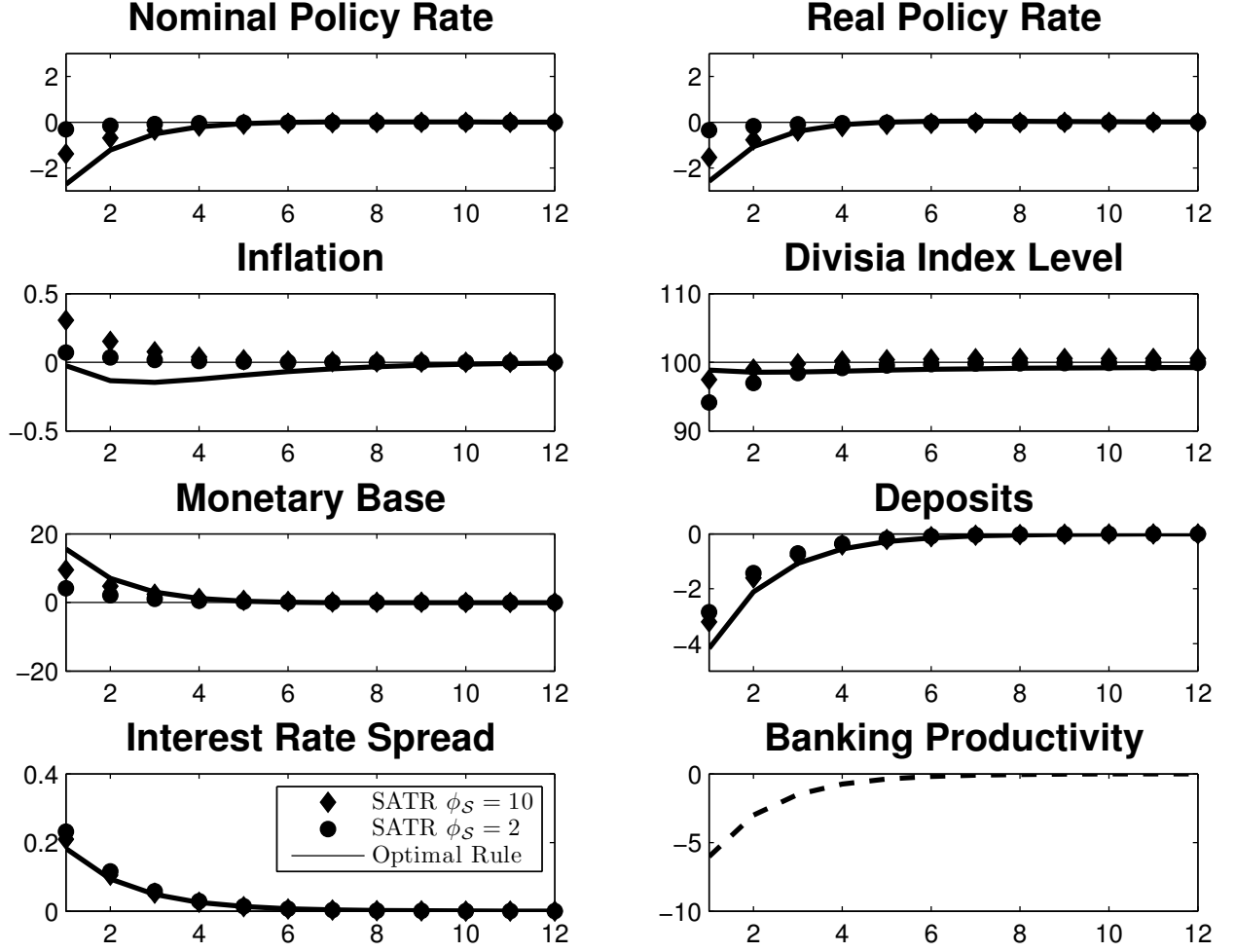


Figure 1.2: Impulse responses to a negative banking productivity shock that raises the annualized spread about 25 basis points. All interest rates, the interest rate spread and inflation are in annualized percentage points. All other variables are in % deviations. The diamond line shows the response under the SATR with $\phi_s = 10$ and the dotted line shows the response under the spread adjusted Taylor rule with $\phi_s = 2$ and the solid line shows the response under the optimal simple rule which features a response to inflation and the growth rate of the Divisia index. (See Table 1.1)

increases welfare by providing a substitute for resource intensive financial assets. Moreover, this increase in the monetary base is accomplished with less inflation volatility than the SATR. By committing to adjust interest rates as needed to stabilize the growth rate of the monetary aggregate the central bank is able to create liquidity without large swings in inflation.

The welfare from the Divisia rule dominates welfare from the spread rule due to the dif-

Table 1.7: Inefficient Resource Costs

<i>PolicyRule</i> ¹	$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\pi}_t^2$	$E_0 \sum_{t=0}^{\infty} \beta^t (\tilde{D}_t + \tilde{x}_t)$
Optimal	.0009	-5.19
SATR $\phi_s = 2$.0016	-1.57

¹ The optimal rule has policy coefficients of $\phi_\pi = 3.234$ and $\phi_m = 2.366$ (see Table 1.1).

ferences described above. The spread rule results in larger resource costs associated with the creation of costly financial assets by failing to provide the proper liquidity needed to substitute away from deposits. In order to generate a similar rise in the monetary base, the SATR with $\phi_s = 10$, inflation becomes more volatile under the SATR compared to the optimal rule. The result is more resources inefficiently spent on producing financial assets and adjusting prices. Meanwhile the Divisia rule is able to stabilize the liquidity flow, and ultimately reduce the resources allocated to creating financial assets, without excessive inflation costs. These differences are summarized in Table 1.7 which highlights the intricacy of optimal monetary policy in this financial/sticky-price economy.

1.5 Comparative Statics, Robustly Optimal Policy and Determinacy

The analysis up to here has been wed to the baseline calibration of the model. Although most of the calibrated values are standard in the literature, the values defining the stochastic processes for the financial sector shocks have not yet been estimated. Therefore in this section we examine the robustness of our results to changes in these key parameters while also examining how the coefficients describing the optimal rule change under such perturbations. First we will perform a simple comparative statics exercise by varying the the parameters defining the stochastic shocks and analyzing the change in the optimal policy coefficients. We then offer a robust policy prescription given the uncertainty surrounding the values of these financial disturbances. The robustly optimal policy features again a positive response to only inflation and the growth rate of the Divisia monetary aggregate. We conclude by

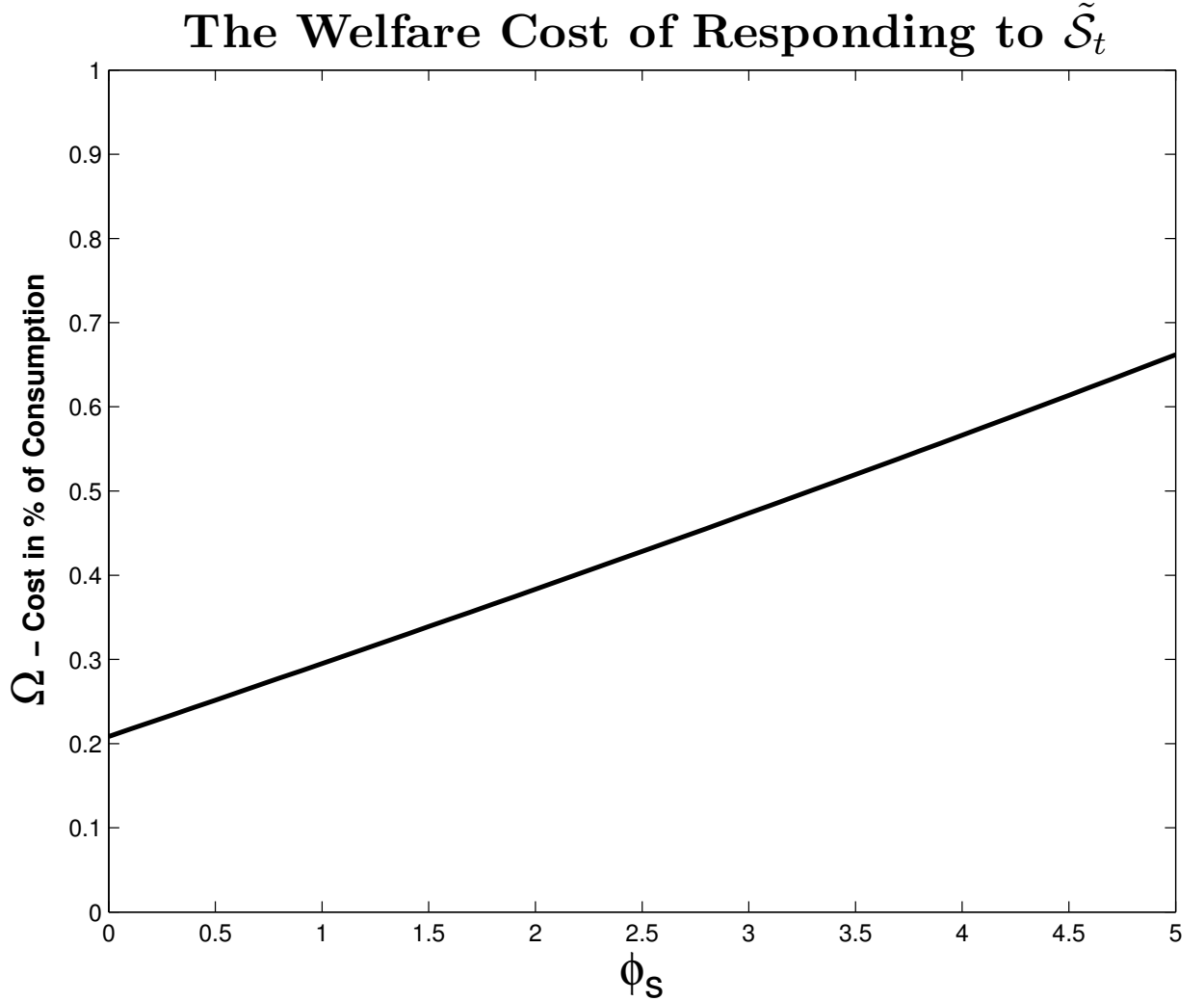


Figure 1.3: $\mathcal{S}_t = 1 + (r_t^L - r_t^D)$ is the gross interest rate spread between loans and deposits (see Eq. (1.11)). The welfare cost Ω are relative to the optimal policy (see Table 1.1). As we vary ϕ_s we keep $\phi_\pi = 1.5$ and $\phi_g = .125$ as in the original Taylor rule (Taylor, 1993).

examining determinacy properties of this rule - a point of practical concern for policy makers.

We show the reaction to Divisia complements the reaction to inflation, so far as determinacy is concerned, resulting in a novel Taylor principle for monetary aggregates.

1.5.1 Comparative Statics

In much the spirit of Poole (1970) we examine how the optimal policy coefficients change when the shocks driving the economy change. We find that when the standard deviation of financial shocks increase, the optimal response to the monetary aggregate increases as

well. This is consistent with Bernanke and Blinder's (1988) suggestions regarding policy when financial and money demand shocks are present; a relative increase in financial shocks should shift the focus to money. Along these same lines, the optimal response to the monetary aggregate decreases when money demand becomes more volatile - the Achilles heel of monetarism. However, the elasticity of ϕ_m^* with respect to σ_v - $\mathcal{E}_{\phi_m^*, \sigma_v}$ - is only 12% (in absolute value) suggesting the tendency to de-emphasize money due to its "instability" has been over stated, at least in regards to its ability to signal movement in the natural rate.

Table 1.8: Elasticities¹ of Optimal Policy Coefficients

$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t + \phi_m \tilde{\mu}_t^{Divisia}$	
ϕ_π^*	ϕ_m^*
$\rho_v = 75$	$\rho_v = -31$
$\rho_a = -193$	$\rho_a = -30$
$\rho_\tau = -11$	$\rho_\tau = -18$
$\rho_x = -103$	$\rho_x = -119$
$\sigma_v = 20$	$\sigma_v = -12$
$\sigma_a = 12$	$\sigma_a = 1$
$\sigma_z = 75$	$\sigma_z = -40$
$\sigma_\tau = -6$	$\sigma_\tau = 8$
$\sigma_x = -104$	$\sigma_x = 45$

¹ All elasticities are arc-elasticities in % computed at the baseline values and a 10% change from baseline values.

With regards to the parameters defining the serial correlation of the shocks, we find that increasing the persistence of the financial shocks calls for a reduction in ϕ_m^* . This may be surprising at first, however looking to the natural rate of interest (see Eq. (1.30)) notice the more persistent the financial shocks become the less they influence the natural rate. However, to the extent that monetary aggregates provide information regarding this variable, we show in the next section the optimal response to the Divisia aggregate is not null, even if the financial shocks are very persistent. Returning momentarily to the money demand shocks notice increasing the persistence of such shocks results in the expected outcome, a decrease

in ϕ_m^* . Again though, this effect is relatively small as $\mathcal{E}_{\phi_m^*, \sigma_v}$ is only about 30%.

1.5.2 Robustly Optimal Policy

The previous section highlights the fact that the exact coefficients that define the optimal policy rule depend on the underlying stochastic processes. Hence, one may wonder then, given our uncertainty over such parameters what is the best policy to follow? We will provide a first attempt to answer this question using a simple 2 player game zero-sum game. In particular, following Giannoni (2002), we set up the zero-sum game Γ between the central bank (CB) and nature (N). The central bank would like to choose policy rule parameters - $\phi \in \Phi$ - to maximize welfare defined by \mathcal{W}_0 (see Eq. (4.2)). At the same time, nature is malevolent and would like to choose shock parameters - $\sigma \in \Sigma$ - which minimize \mathcal{W}_0 . The Nash equilibrium represents the worst set of structural parameters possible, given policy makers choose the best possible policy given the bad state of the world. Intuitively, the Nash equilibrium of this game provides a policy prescription that does the best under the worst case scenario. The nature of zero-sum games allows us to consider solving the minimax problem to find the Nash equilibrium

$$\min_{\phi \in \Phi} \left\{ \max_{\sigma \in \Sigma} \{-\mathcal{W}_0\} \right\}$$

We numerically approach a solution by iterating over nature's problem and over the policy makers problem until all parameters and the objective function converge.

Before solving this problem we must set bounds on the choice space of nature. For the money demand, preference and technology process we assume all parameters lie within 2 standard errors using the estimates from Ireland (2004a) with an upper bound of .99 for the persistence parameters²⁴. Unfortunately, the parameters driving the financial shocks have no comparable estimates to help determine an appropriately bounded set. Therefore we assume that $\rho_x \in [0, .99]$ and $\rho_\tau \in [0, .99]$. As for the standard deviations of these shocks we continue

²⁴As with the rest of the paper, we continue to assume a unit-root in the technology shock

to rely on the observation that large financial disruptions have occurred about every 80 years, however we relax our normality assumption. In particular, to determine a lower bound for the standard deviations of the financial shocks, we allow for the possibility that financial shocks follow a distribution with significantly “fatter-tails” than a normal distribution²⁵. For concreteness, suppose that ε_t^T and ε_t^x are distributed according to a student’s t-distribution with 3 degrees of freedom²⁶ implying we scale our normal standard errors by $\sqrt{\frac{d.o.f.}{d.o.f.-2}} = \sqrt{3}$. The resulting bounds are given by $\sigma_x \in [.0993, .1720]$ and $\sigma_\tau \in (\sqrt{\Sigma_{i=0}^{11} \rho_\tau^{2i}})^{-1} [.4965, .8600]$ where the upper bounds are the standard deviations under the normality assumption and the lower bounds are the upper bounds scaled by $\sqrt{3}$. The resulting Nash Equilibrium, or equivalently, the worst state of the world and the best policy in this state are given in Table 1.9 below.

Table 1.9: Nash Equilibrium in Γ

<i>Nash Equilibrium</i>	
Nature’s Strategy	Central Bank’s Strategy
$\rho_v^* = .99$	$\rho_r^* = 0$
$\rho_a^* = .99$	$\phi_\pi^* = .8940$
$\rho_\tau^* = .99$	$\phi_y^* = 0$
$\rho_x^* = .99$	$\phi_m^* = .5206$
$\sigma_v^* = .0102$	$\phi_s^* = 0$
$\sigma_a^* = .0352$	
$\sigma_z^* = .0116$	
$\sigma_\tau^* = .2621$	
$\sigma_x^* = .1720$	

Nature chooses to maximize the persistence and volatility of all the shocks in the economy, leading to the maximum long-run variance possible in these stochastic processes. In particular nature maximizes the persistence of the financial shocks in the model. How does this minimize welfare? Recall again the equation for the natural rate of interest - reproduced

²⁵See for example Mandelbrot (1963) or Fama (1965).

²⁶The choice of the degrees of freedom is arbitrary but 3 is the smallest integer for which the student’s t-distribution has a finite standard error.

here for convenience.

$$\tilde{r}_t^n = (1 - \rho_a)\tilde{a}_t - (1 - \rho_z)\tilde{Z}_t - \Psi_x^{r^n}(1 - \rho_x)\tilde{x}_t - \Psi_\tau^{r^n}(1 - \rho_\tau)\tilde{\tau}_t$$

By pushing the persistence of the financial shocks to their maximum, nature is trying to mitigate the financial shocks role in driving the natural rate. Why? When financial shocks affect the natural rate policy makers reacting to the Divisia aggregate are able to correlate the real rate with the natural rate. However, by limiting the share of variance in the natural rate explained by financial shocks, while at the same time maximizing the long-run variance of money demand shocks, nature is effectively maximizing the noise to signal ratio of Divisia in signaling movements in the natural rate. Interestingly though welfare is still improved by including a positive - but weakened - reaction to the Divisia aggregate. Therefore, much in line with Brainard's (1967) seminal work on policy under uncertainty we find (in contrast to Giannoni (2002)) uncertainty calls for an attenuated reaction to *both* inflation and the monetary aggregate. However the Divisia response is significant - even in this extreme state of the world.

1.5.3 Determinacy

Thus far we have found the optimal simple *and* robust rules feature a positive response to only inflation and the Divisia monetary aggregate. Clearly if the aforementioned policy rules are followed “to the letter” then the outcome is determinate in the sense that there is a unique path for all nominal and real variables. However, a practical concern for policy makers faced with the policy prescription in this economy is whether reacting to the Divisia monetary aggregate induces undesirable equilibrium outcomes that an inflation only rule would preclude. In other words, if policy deviates slightly from the prescribed rule is the equilibrium outcome still unique? To answer this question we examine the determinacy properties of the optimal rule in this economy. Unfortunately, analytical solutions are not available as in Bullard & Mitra (2002) so instead we examine the determinacy properties

of this rule numerically. We examine determinacy by searching over the values $\phi_\pi \in [0, 5]$ and $\phi_m \in [0, 5]$ by .01 increments and $\rho_r \in [0, 1)$ by .1 increments around both the steady state of the competitive equilibrium and the Ramsey steady state and examining the necessary and sufficient condition for existence and uniqueness as laid out by Blanchard & Kahn (1980). The resulting determinacy region is featured in Figure 1.4.

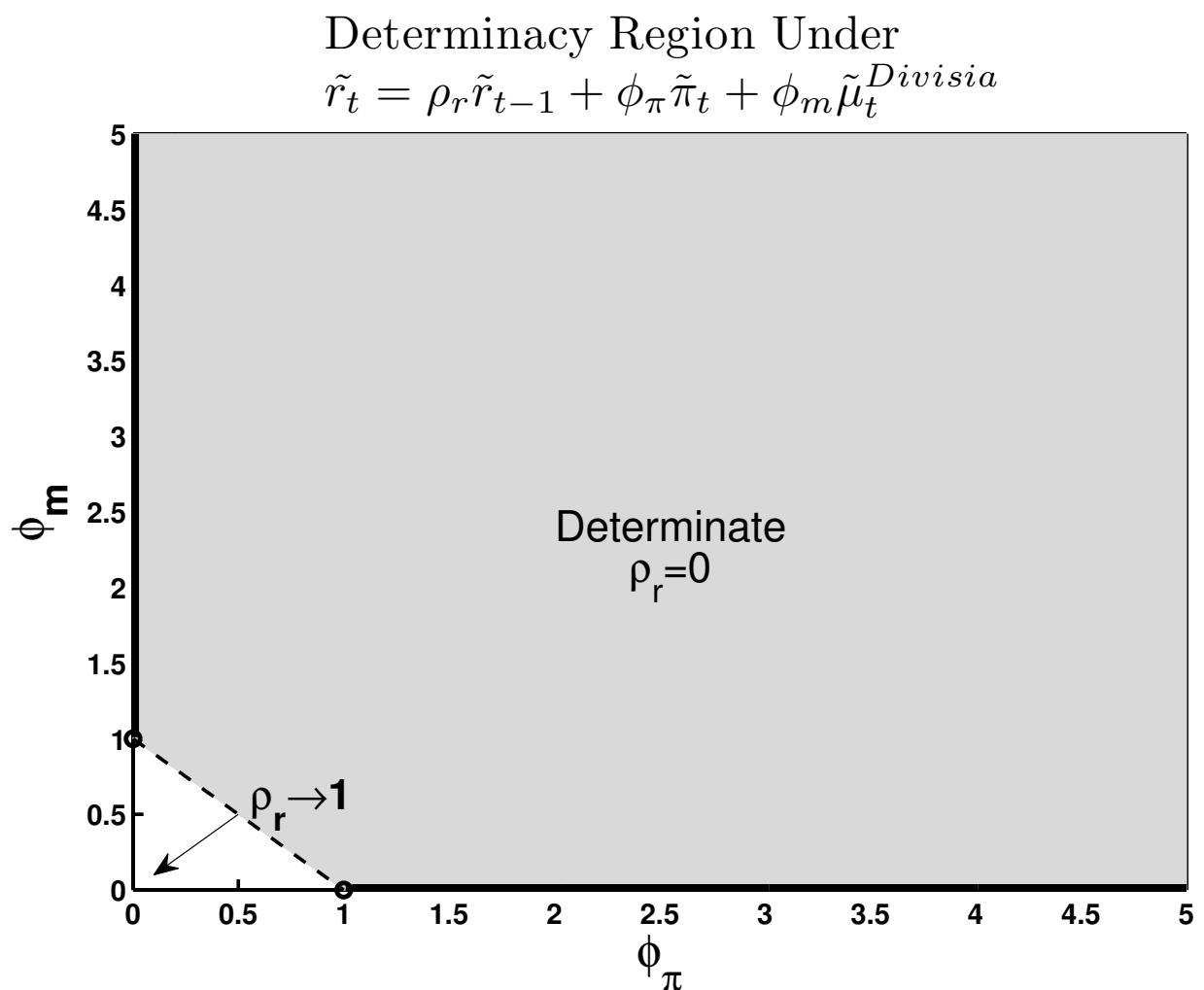


Figure 1.4: The shaded area to the northeast of the dotted line, defined by $\phi_m + \phi_\pi > 1$, is determinate when $\rho_r = 0$. Moreover, if $\rho_r > 0$ then the condition for determinacy under this rule generalizes to $\frac{\phi_m + \phi_\pi}{1 - \rho_r} > 1$, this is a generalized Taylor principle for Divisia monetary aggregates in interest rate rules.

Looking first along the horizontal axis where $\phi_m = 0$, the well known Taylor principle con-

tinues to hold. That is policy makers can guarantee a unique equilibrium by satisfying

$$\phi_\pi > 1$$

Moreover, looking at the vertical axis where $\phi_\pi = 0$, we find that policy makers can guarantee a unique equilibrium outcome by adjusting the policy rate more than one for one with changes in the growth rate of the Divisia monetary aggregate²⁷. That is, the condition that

$$\phi_m > 1$$

is sufficient for determinacy. However, our optimal simple and robust rules feature a positive response to both inflation and the Divisia monetary aggregate. Therefore, focusing our attention on the interior Figure 4, where both responses are positive, we find that the above two conditions are sufficient but clearly not necessary as determinacy is more generally possible by satisfying the condition

$$\phi_\pi + \phi_m > 1 \tag{1.33}$$

Reacting to the growth rate of Divisia actually allows policy makers to achieve determinacy with even weaker inflation responses so long as the reaction to Divisia is increased one for one. Equation (1.33) provides an extension of the well-known Taylor principle to Taylor rules featuring a response to the Divisia monetary aggregate - a *modified Taylor principle* for Divisia monetary aggregates. Most generally, if policy makers introduce lagged interest rates into the policy rule we see that condition (1.33) generalizes to

$$\frac{\phi_\pi + \phi_m}{1 - \rho_r} > 1 \tag{1.34}$$

²⁷We emphasize this result is not generally true for *any* monetary aggregate. In fact, we show in the appendix embedding the simple-sum monetary aggregate (see Eq. (1.22)) tends to result in indeterminacy and no such modified Taylor principle holds.

Equations (1.33) and (1.34) show that there are no more concerns regarding determinacy under optimal rules in this model than there are regarding determinacy in standard NKSP where $\phi_m = 0$ and in fact determinacy is more likely when reacting to the Divisia monetary aggregate.

1.6 Conclusion

We have analyzed optimal monetary policy in a NKSP augmented to include a financial sector. We showed financial disturbances which decrease banks ability and willingness to produce financial assets lowers the natural rate of interest calling for countercyclical monetary policy. The resulting optimal policy deviates from the common inflation only type of Taylor rule common in this literature. Instead, reacting to the growth rate of the Divisia monetary aggregate, in addition to inflation, performs significantly better than reacting to inflation alone. The resulting policy rule produces a real interest rate which is highly correlated with the natural rate of interest. We showed the spread adjusted Taylor rule as advocated by Taylor (2008) actually performs worse than inflation only rules despite its high correlation with the natural rate of interest. This puzzling result is reconciled by the increased inflation and output gap volatility induced by the spread adjusted Taylor rules which is not present in the Divisia growth rules. The optimality of reacting positively to the Divisia growth rate and the inflation rate is shown to hold under different assumption regarding policy maker's information sets and preferences over monetary assets. Finally, we offer a *robust* policy prescription under parametric uncertainty using the minimax approach proposed in Giannoni (2002). The resulting policy rule features an attenuated - but positive and economically significant - response to the growth rate of Divisia and inflation. Moreover, we offer some insight regarding the ability of such rules to bring about a unique equilibrium outcome by examining their determinacy region. The result is a *modified Taylor principle* for Divisia monetary aggregate which is sufficient to guarantee determinacy.

As much as the seminal work of Taylor (1993) marked the beginning of interest rate rules

dominating monetary policy we hope this paper can be part of the beginning of re-examining the usefulness of monetary aggregates in real time policy making. Given the interest in *simple* rules and macro policy under financial distress considerable work lies ahead in better understanding how policy makers can better use the information available in monetary aggregates and other variables available in real time. Specifically, one may wonder in our environment if Divisia level targeting is superior to reacting to the growth rate of the Divisia monetary aggregate. Also, expanding the literature on statistical index number theory to properly track aggregate quantities of credit seems a worthwhile task given the usefulness of properly constructed monetary aggregates in this model. Furthermore, the zero-lower bound on nominal interest rates suggests the usefulness of monetary aggregates may extend beyond informational variables to policy instruments as called for by Taylor (2009). However, an optimal policy prescription for monetary instrument rules is seriously lacking in this modern framework due to the exclusive focus on interest rates. This calls for future work to understand how different aggregates perform as instruments from their determinacy properties to their welfare performance.

References

- Andres, J., Lopez-Salido, J. D., & Nelson, E. (2009). Money and the natural rate of interest: Structural estimates for the united states and the euro area. *Journal of Economic Dynamics and Control*, 33(3), 758–776.
- Barnett, W. A. (1978). The user cost of money. *Economics Letters*, 1(2), 145–149.
- Barnett, W. A. (1980). Economic monetary aggregates an application of index number and aggregation theory. *Journal of Econometrics*, 14(1), 11–48.
- Barnett, W. A. (2012). *Getting it Wrong: How Faulty Monetary Statistics Undermine the Fed, the Financial System, and the Economy*, volume 1 of *MIT Press Books*. The MIT Press.
- Barnett, W. A. & Chauvet, M. (2011a). *Financial Aggregation And Index Number Theory*, volume 2 of *World Scientific Books*. World Scientific Publishing Co. Pte. Ltd.
- Barnett, W. A. & Chauvet, M. (2011b). How better monetary statistics could have signaled the financial crisis. *Journal of Econometrics*, 161(1), 6–23.
- Barnett, W. A. & Serletis, A. (2000). *The Theory of Monetary Aggregation*. Contributions to Economic Analysis. Elsevier.
- Barnett, W. A. & Singleton, K. J. (1987). *The microeconomic theory of monetary aggregation*. Cambridge University Press.

- Belongia, M. T. (1996). Measurement matters: Recent results from monetary economics reexamined. *Journal of Political Economy*, 104(5), 1065–83.
- Belongia, M. T. & Binner, J. M. (2000). *Divisia Monetary Aggregates: Theory and Practice*. Palgrave.
- Belongia, M. T. & Ireland, P. N. (2006). The own-price of money and the channels of monetary transmission. *Journal of Money, Credit and Banking*, 38(2), 429–445.
- Belongia, M. T. & Ireland, P. N. (2013). The barnett critique after three decades: A new keynesian analysis. *Journal of Econometrics*, forthcoming.
- Berger, H. & Weber, H. (2012). *Money as Indicator for the Natural Rate of Interest*. IMF Working Papers 12/6, International Monetary Fund.
- Bernanke, B. S. & Blinder, A. S. (1988). Credit, money, and aggregate demand. *American Economic Review*, 78(2), 435–39.
- Blanchard, O. J. & Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, 48(5), 1305–11.
- Brainard, W. (1967). Uncertainty and the effectiveness of policy. *American Economic Review*, 57(2), 411–425.
- Bullard, J. & Mitra, K. (2002). Learning about monetary policy rules. *Journal of Monetary Economics*, 49(6), 1105–1129.
- Clarida, R., Gali, J., & Gertler, M. (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *The Quarterly Journal of Economics*, 115(1), 147–180.
- Curdia, V. & Woodford, M. (2009). *Credit frictions and optimal monetary policy*. BIS Working Papers 278, Bank for International Settlements.

- Curdia, V. & Woodford, M. (2010). Credit spreads and monetary policy. *Journal of Money, Credit and Banking*, 42(s1), 3–35.
- Dellas, H., Diba, B., & Loisel, O. (2010). *Financial Shocks and Optimal Policy*. Technical report.
- Diewert, W. E. (1976). Exact and superlative index numbers. *Journal of Econometrics*, 4(2), 115–145.
- Faia, E. & Monacelli, T. (2007). Optimal interest rate rules, asset prices, and credit frictions. *Journal of Economic Dynamics and Control*, 31(10), 3228–3254.
- Fama, E. F. (1965). The behavior of stock-market prices. *The Journal of Business*, 38(1), pp. 34–105.
- Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.
- Gali, J., Lopez-Salido, J. D., & Valles, J. (2003). Technology shocks and monetary policy: assessing the fed’s performance. *Journal of Monetary Economics*, 50(4), 723–743.
- Giannoni, M. P. (2002). Does model uncertainty justify caution? robust optimal monetary policy in a forward-looking model. *Macroeconomic Dynamics*, 6(01), 111–144.
- Ireland, P. N. (2000). Interest rates, inflation, and federal reserve policy since 1980. *Journal of Money, Credit and Banking*, 32(3), 417–34.
- Ireland, P. N. (2004a). Money’s role in the monetary business cycle. *Journal of Money, Credit and Banking*, 36(6), 969–83.
- Ireland, P. N. (2004b). Technology shocks in the new keynesian model. *The Review of Economics and Statistics*, 86(4), 923–936.

- Ireland, P. N. (2011). *The Macroeconomic Effects on Interest on Reserves*. Boston College Working Papers in Economics 772, Boston College Department of Economics.
- Kim, H., Kim, J., Schaumburg, E., & Sims, C. A. (2005). *Calculating and Using Second Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models*. Technical report.
- Kydland, F. E. & Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6), 1345–70.
- Lombardo, G. & Vestin, D. (2008). Welfare implications of calvo vs. rotemberg-pricing assumptions. *Economics Letters*, 100(2), 275–279.
- Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36, 394.
- McCallum, B. T. & Nelson, E. (2011). Money and inflation: Some critical issues. In B. M. Friedman & M. Woodford (Eds.), *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics* chapter 3, (pp. 97–153). Elsevier.
- Orphanides, A. (2003). The quest for prosperity without inflation. *Journal of Monetary Economics*, 50(3), 633–663.
- Poole, W. (1970). Optimal choice of monetary policy instruments in a simple stochastic macro model. *The Quarterly Journal of Economics*, 84(2), 197–216.
- Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90(6), 1187–1211.
- Schmitt-Grohe, S. & Uribe, M. (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control*, 28(4), 755–775.
- Schmitt-Grohe, S. & Uribe, M. (2007). Optimal simple and implementable monetary and fiscal rules. *Journal of Monetary Economics*, 54(6), 1702–1725.

- Schmitt-Grohe, S. & Uribe, M. (2012). An ols approach to computing ramsey equilibria in medium-scale macroeconomic models. *Economics Letters*, 115, 128–129.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.
- Taylor, J. B. (2008). *Monetary Policy and the State of the Economy*. Testimony before the committee on financial services u.s. house of representatives.
- Taylor, J. B. (2009). The need to return to a monetary framework. *Business Economics*, 44(2), 63–72.
- Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

1.A Appendix: The Equilibrium System

In this portion of the appendix we derive the model's equilibrium conditions and stationarize the model allowing us to define a recursive (imperfectly) competitive equilibrium. We also derive expressions for the natural rate level of output and the natural interest rate.

1.A.1 The Representative Household's FOCS

Non-Separable Preferences: First consider the case when monetary assets are not additively separable from consumption. In particular, recall from section 2.1.3 that the household faces the following problem when preferences over monetary assets are defined according to a shopping time friction;

$$\max_{\{C_t, h_t, M_t^A, N_t, D_t, L_t, B_{t+1}, M_{t+1}, s_{t+1}(i)\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t a_t [\ln(C_t) - \eta(h_t + h_t^s)]$$

subject to (3.1), (1.2), (1.3) and (1.4) taking B_0 , M_0 and $s_0(i)$ as given. After substituting (1.4) into the objective function, we can form the Bellman equation as follows where all the constraints are expressed in real terms by dividing through by P_t ,

$$\begin{aligned} V(B_{t-1}, M_{t-1}, s_{t-1}(i)) = & \max \left\{ a_t \left[\ln(C_t) - \eta h_t - \frac{\eta}{\chi} \left(\frac{v_t P_t C_t}{M_t^A} \right)^\chi \right] \right. \\ & - \Lambda_t^1 \left(\frac{D_t - M_{t-1} - B_{t-1} + \int_0^1 Q_t(i)(s_t(i) - s_{t-1}(i)) di + B_t/r_t + N_t - L_t}{P_t} \right) \\ & - \Lambda_t^2 \left(\frac{M_t^A}{P_t} - \left[\nu^{\frac{1}{\omega}} \left(\frac{N_t}{P_t} \right)^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} \left(\frac{D_t}{P_t} \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right) \\ & - \Lambda_t^3 \left(\frac{M_t - N_t + P_t C_t - W_t h_t - \int_0^1 F_t(i) s_t(i) di - r_t^D D_t + r_t^L L_t}{P_t} \right) \\ & \left. + \beta E_t[V(B_t, M_t, s_t(i))] \right\} \end{aligned}$$

The first order necessary conditions are given by the following equations. The system of equations (1.A.1)-(1.A.9) is under-determined in the sense that we have introduced various derivatives of the value function. However, we can complement these first order necessary conditions with the Bienveniste-Scheinkman Envelope Conditions to eliminate the value function from the system above. These envelope conditions are given in equations (1.A.10)-(1.A.12) below.

$$a_t \left[1 - \eta \left(\frac{v_t P_t C_t}{M_t^A} \right)^\chi \right] - \Lambda_t^3 C_t = 0 \quad (1.A.1)$$

$$-a_t \eta + \Lambda_t^3 \frac{W_t}{P_t} = 0 \quad (1.A.2)$$

$$\eta a_t \left(\frac{v_t P_t C_t}{M_t^A} \right)^\chi - \frac{\Lambda_t^2 M_t^A}{P_t} = 0 \quad (1.A.3)$$

$$\left(\frac{D_t}{P_t} \right) - (1 - \nu) \left(\frac{M_t^A}{P_t} \right) \left[\frac{\Lambda_t^2}{\Lambda_t^1 - \Lambda_t^3 r_t^D} \right]^\omega = 0 \quad (1.A.4)$$

$$\left(\frac{N_t}{P_t} \right) - \nu \left(\frac{M_t^A}{P_t} \right) \left[\frac{\Lambda_t^2}{\Lambda_t^1 - \Lambda_t^3} \right]^\omega = 0 \quad (1.A.5)$$

$$\Lambda_t^1 - \Lambda_t^3 r_t^L = 0 \quad (1.A.6)$$

$$\frac{-\Lambda_t^1}{r_t} + P_t \beta E_t[V'_{B_t}(B_t, M_t, s_t(i))] = 0 \quad (1.A.7)$$

$$-\Lambda_t^3 + P_t \beta E_t[V'_{M_t}(B_t, M_t, s_t(i))] = 0 \quad (1.A.8)$$

$$-\Lambda_t^1 Q_t(i) + \Lambda_t^3 F_t(i) + P_t \beta E_t[V'_{s_t(i)}(B_t, M_t, s_t(i))] = 0 \quad (1.A.9)$$

Envelope Conditions:

$$V'_{B_{t-1}}(B_{t-1}, M_{t-1}, s_t(i)) = \frac{\Lambda_t^1}{P_t} \quad (1.A.10)$$

$$V'_{M_{t-1}}(B_{t-1}, M_{t-1}, s_t(i)) = \frac{\Lambda_t^1}{P_t} \quad (1.A.11)$$

$$V'_{s_{t-1}(i)}(B_{t-1}, M_{t-1}, s_t(i)) = \frac{\Lambda_t^1 Q_t(i)}{P_t} \quad (1.A.12)$$

Now update (1.A.10)-(1.A.12) and substitute the resulting equations into (1.A.6)-(1.A.8) yielding:

$$\frac{-\Lambda_t^1}{r_t} + \beta E_t \left[\frac{P_t \Lambda_{t+1}^1}{P_{t+1}} \right] = 0 \quad (1.A.13)$$

$$-\Lambda_t^3 + \beta E_t \left[\frac{P_t \Lambda_{t+1}^1}{P_{t+1}} \right] = 0 \quad (1.A.14)$$

$$-\Lambda_t^1 \frac{Q_t(i)}{P_t} + \Lambda_t^3 \frac{F_t(i)}{P_t} + \beta E_t \left[\frac{\Lambda_{t+1}^1 Q_{t+1}(i)}{P_{t+1}} \right] = 0 \quad (1.A.15)$$

The conditions (1.A.1)-(1.A.6) and (1.A.13)-(1.A.15) define the consumers optimal behavior.

Separable Preferences: Under additively separable preferences over monetary assets the household's first order conditions are identical to the above conditions after replacing (1.A.1) and (1.A.3) with

$$\frac{a_t}{C_t} - \Lambda_t^3 = 0 \quad (1.A.16)$$

$$\eta_m a_t v_t - \Lambda_t^2 \frac{M_t^A}{P_t} = 0 \quad (1.A.17)$$

respectively.

1.A.2 Deriving the User Costs of Monetary Assets

As shown in an infinite planning horizon by Barnett & Singleton (1987), we can derive the user costs of all monetary assets in the model from the household's first order necessary conditions. Specifically, the user costs appear naturally as the price of monetary assets according to the familiar optimality condition from microeconomics which dictates, at an optimum, equating the marginal rate of substitution of currency for deposits to the ratio of the price of currency to the price of deposits.

$$\frac{\frac{\partial u_t}{\partial N_t}}{\frac{\partial u_t}{\partial D_t}} = \frac{\frac{\partial u_t}{\partial M_t^A} \frac{\partial M_t^A}{\partial N_t}}{\frac{\partial u_t}{\partial M_t^A} \frac{\partial M_t^A}{\partial D_t}} = \frac{\frac{\Lambda_t^1 - \Lambda_t^3}{\Lambda_t^2}}{\frac{\Lambda_t^1 - \Lambda_t^3 r_t^D}{\Lambda_t^2}} = \frac{\Lambda_t^3 r_t^L - \Lambda_t^3}{\Lambda_t^3 r_t^L - \Lambda_t^3 r_t^D} = \frac{r_t^L - 1}{r_t^L - r_t^D} \equiv \frac{u_t^N}{u_t^D}$$

The second equality follows from equations (1.A.4) and (1.A.5), the household's first order conditions for N_t and D_t . The third equality follows from equation (1.A.6), the household's first order condition for L_t . The resulting ratio defines the relative user costs.

We can also derive the exact price dual to the true quantity aggregator in a similar fashion. Instead of considering the price of each monetary assets individually, consider the optimality condition the monetary aggregate must satisfy. For simplicity, consider the marginal rate of substitution of the monetary aggregate for consumption. Clearly the price of the consumption good is P_t , hence the numerator must denote the price of the monetary aggregate.

$$\begin{aligned} \frac{\frac{\partial u_t}{\partial M_t^A}}{\frac{\partial u_t}{\partial C_t}} &= \frac{\frac{\Lambda_t^2}{P_t}}{\frac{\Lambda_t^3}{P_t}} = \frac{\frac{\Lambda_t^2}{\Lambda_t^3}}{P_t} = \frac{\left[\nu(r_t - 1)^{(1-\omega)} + (1-\nu)(r_t - r_t^D)^{(1-\omega)} \right]^{\frac{1}{1-\omega}}}{P_t} \equiv \frac{r_t^L - r_t^A}{P_t} \\ u_t^A &= r_t^L - r_t^A = \left[\nu(r_t - 1)^{(1-\omega)} + (1-\nu)(r_t - r_t^D)^{(1-\omega)} \right]^{\frac{1}{1-\omega}} \end{aligned} \quad (1.A.18)$$

The first equality follows from equations (1.A.1) and (1.A.3), the household's first order conditions for C_t and M_t^A respectively. The third equality follows from solving equation (1.A.4) for N_t , equation (1.A.5) for D_t and substituting the resulting expressions into equation (1.3). The resulting expression can be solved for $\frac{\Lambda_t^2}{\Lambda_t^3}$ yielding the numerator the follows the third equality. For the last equality we define this expression to be $r_t - r_t^A$, or the opportunity cost of holding the aggregate monetary asset M_t^A . The resulting aggregate user cost (1.A.18) as defined by Barnett (1978) is of the same form as the individual component user costs in equations (1.20) and (1.21). It can be verified that (1.A.18) is in fact the true price dual to the true monetary aggregate in equation (1.3) since it satisfies Fisher's factor reversal test.

$$M_t^A u_t^A = u_t^N N_t + u_t^D D_t \quad (1.A.19)$$

Equation (1.A.19) states that the true quantity index times the true price index equals total expenditures, in this sense, (1.A.18) is the exact price dual to (1.3).

1.A.3 Equilibrium Quantity of the Monetary Aggregate

In this section we derive a condition relating the monetary aggregate to output, a vector of interest rates and financial shocks. This expression is useful for deriving the natural rates of output and interest later and also because it highlights the difference between broad money which depends on a vector of interest rates and the financial sector's productive ability versus typical "money demand" specifications which assume the equilibrium quantity of money depends only agents demand for monetary assets featuring a single interest rate. All variables with a tilde denote the real detrended variable in log-deviations from steady state (with the exception of interest rates which are in nominal terms). Combine (1.A.2), (1.A.3) (or (1.A.16), (1.A.17)) and (1.A.18) to arrive at (up to additive constants)

$$\tilde{M}_t^A = \gamma_m^c \tilde{C}_t + \gamma_m^w \tilde{W}_t - \gamma_m^{u^A} \tilde{u}_t^A + \gamma_m^v \tilde{v}_t \quad (1.A.20)$$

where under additively separable preferences $\gamma_m^c = \gamma_m^{u^A} = \gamma_m^v = 1$, and $\gamma_m^w = 0$ and under non-additively separable preferences $\gamma_m^c = \gamma_m^v = \frac{\chi}{1+\chi}$, and $\gamma_m^w = \gamma_m^{u^A} = \frac{1}{1+\chi}$. Now log-linearize the household's first order condition relating the real wage to the marginal utilities of consumption and leisure and we have that

$$\tilde{W}_t = (1 + \gamma_m) \tilde{C}_t - \gamma_m \tilde{M}_t^A + \gamma_m \tilde{v}_t \quad (1.A.21)$$

where $\gamma_m = \frac{u_{M^A,C}''}{u_C'}$ is zero for additively separable preferences. Now substitute (1.A.21) into (1.A.20) to eliminate the real wage and arrive at

$$\tilde{M}_t^A = \frac{[\gamma_m^c + \gamma_m^w(1 + \gamma_m)]}{[1 + \gamma_m^w \gamma_m]} \tilde{C}_t - \frac{\gamma_m^{u^A}}{[1 + \gamma_m^w \gamma_m]} \tilde{u}_t^A + \frac{[\gamma_m^w \gamma_m + \gamma_m^v]}{[1 + \gamma_m^w \gamma_m]} \tilde{v}_t \quad (1.A.22)$$

We can express consumption in terms of output and the monetary aggregate by substituting equation (1.A.5) into equation (3.28) and log-linearizing to arrive at

$$\tilde{C}_t = \frac{1}{s_c} \left[\tilde{Y}_t - (1 - s_c) \left[\tilde{x}_t + \tilde{M}_t^A + \omega (\tilde{u}_t^A - \tilde{u}_t^D) \right] \right] \quad (1.A.23)$$

where $s_c = \frac{\bar{C}}{\bar{Y}}$. Our equilibrium condition for the monetary aggregate is obtained by substituting (1.A.23) into (1.A.22) and noting that under either specification of preferences $\gamma_m^c + \gamma_m^w = 1$

$$\tilde{M}_t^A = \tilde{Y}_t - \eta_{u^A} \tilde{u}_t^A + \eta_{u^D} \tilde{u}_t^D + \eta_v \tilde{v}_t - \eta_x \tilde{x}_t \quad (1.A.24)$$

where

$$\begin{aligned} \eta_{u^A} &= (1 - s_c) \omega + \frac{\gamma_m^{u^A} s_c}{(1 + \gamma_m \gamma_m^w)} \\ \eta_{u^D} &= \omega (1 - s_c) \\ \eta_v &= s_c \left(1 - \frac{1 - \gamma_m^v}{1 + \gamma_m \gamma_m^w} \right) \\ \eta_x &= (1 - s_c) \end{aligned}$$

and all the coefficients are positive.

1.A.4 The Representative Intermediate Goods Firm's FOCS

Recall from section 2.4, the representative intermediate goods producing firm $i \in [0, 1]$ maximizes its share price in every period $t = 0, 1, 2, 3, \dots$. Mathematically, we have

$$\max_{\{h_t(i), P_t(i)\}_{t=0}^{\infty}} \frac{Q_t(i)}{P_t} \text{ subject to (1.14) and (1.15).}$$

To derive the first order necessary conditions for this problem use the equity pricing relationship (1.A.15) from the representative household's first order condition to solve for period t . Solving (1.A.15) forward and assuming no bubbles yields

$$\frac{Q_t(i)}{P_t} = E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{\Lambda_{t+j}^3 F_{t+j}(i)}{\Lambda_t^1 P_{t+j}} \right]. \quad (1.A.25)$$

Equation (1.A.25) is a familiar asset pricing relationship which states that the market share price of the representative intermediate goods producing firm is proportional to the expected future stream of dividends adjusted for risk. In particular, the stochastic discount factor is given by $\frac{\beta^j \Lambda_{t+j}^3}{\Lambda_t^1}$. In any period $t = 0, 1, 2, 3, \dots$, intermediate goods producing firm $i \in [0, 1]$ pays out all profits as dividends. In real terms, the period t real dividend is given by

$$\frac{F_t(i)}{P_t} = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} h_t(i) - \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right]^2 Y_t. \quad (1.A.26)$$

The first term of the above profit function is period t real revenue, the second term in the firm's period t real wage bill and the third term will be 0 unless the intermediate goods producing firm changes its price from period $t - 1$ to period t an amount different than the steady-state gross rate of inflation rate, π . Substituting (1.A.26) into (1.A.25), we can restate the intermediate goods producing firm's problem as

$$\max_{\{h_t(i), P_t(i)\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \frac{\beta^t \Lambda_t^3}{\Lambda_0^1} \left[\frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} h_t(i) - \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right]^2 Y_t \right]$$

subject to

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \text{ and } Y_t(i) = z_t h_t(i).$$

The problem can be simplified by substituting the inverse of the technology constraint for $h_t(i)$ and then substituting the factor demand into the resulting expression for $Y_t(i)$ so that now the representative intermediate goods producing firm solves the following recursive problem defined by Bellman's equation

$$V(P_{t-1}(i)) = \max_{P_t(i)} \left\{ \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \left[\left[\frac{P_t(i)}{P_t} \right]^{1-\theta} - \frac{W_t}{P_t z_t} \left[\frac{P_t(i)}{P_t} \right]^{-\theta} - \frac{\phi}{2} \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right]^2 \right] + \beta E_t[V(P_t)] \right\}$$

The first order necessary condition for the problem is given by

$$(1-\theta) \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \left[\frac{P_t(i)}{P_t} \right]^{-\theta} \frac{1}{P_t} + \theta \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \frac{W_t}{P_t^2 z_t} \left[\frac{P_t(i)}{P_t} \right]^{-1-\theta} - \phi \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right] \frac{1}{P_{t-1}(i)\pi} + \beta E_t[V'(P_t)] = 0 \quad (1.A.27)$$

Once again invoking the Bienveniste-Scheinkman Envelope Condition we have

$$V'(P_{t-1}(i)) = \frac{\Lambda_t^3 Y_t}{\Lambda_0^1} \phi \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right] \frac{P_t(i)}{(P_{t-1}(i))^2 \pi}. \quad (1.A.28)$$

Updating (1.A.28) one period and substituting into (1.A.27) and then multiplying the resulting equation by $\Lambda_0^1 P_t$ yields

$$(1-\theta) \left[\frac{P_t(i)}{P_t} \right]^{-\theta} + \theta \frac{W_t}{P_t z_t} \left[\frac{P_t(i)}{P_t} \right]^{-1-\theta} - \phi \left[\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right] \frac{P_t}{P_{t-1}(i)\pi} + \beta \phi E_t \left[\frac{\Lambda_{t+1}^3 Y_{t+1}}{\Lambda_t^3 Y_t} \left[\frac{P_{t+1}(i)}{P_t(i)\pi} - 1 \right] \frac{P_t P_{t+1}(i)}{(P_t(i))^2 \pi} \right] = 0 \quad (1.A.29)$$

Finally, when (1.A.29) is log-linearized around a symmetric equilibrium where $P_t(i) = P_t \forall t = 0, 1, 2, 3, \dots$ it takes the form of a New-Keynesian Phillips Curve relating current inflation to the average real marginal cost and expected future inflation.

1.A.5 The Efficient Level of Output

To find the efficient level of output used to define the efficient output gap we solve the benevolent social planner's problem

$$\max_{C_t, Y_t, h_t, \{Y_t(i), h_t(i)\}_{i \in [0,1]}} a_t [\ln(C_t) - \eta h_t]$$

$$s.t. (1.12), (1.15) (1.23) \text{ and } Y_t = C_t$$

We can simplify the above constrained maximization problem, into the unconstrained problem

$$\max_{h_t(i) \}_{i \in [0,1]} a_t \left[\ln \left(Z_t \left[\int_0^1 h_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \right) - \eta \int_0^1 h_t(i) di \right]$$

The first order condition for this problem is given by

$$h_t(i)^{-\frac{1}{\theta}} = \eta \left[\int_0^1 h_t(i)^{\frac{\theta-1}{\theta}} di \right] \quad (1.A.30)$$

Raising each side of (1.A.30) to the $(1-\theta)$ power and integrating each side over i , we have

$$1 = \eta \left[\int_0^1 h_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad (1.A.31)$$

Substituting out $h_t(i)$ in (1.A.31) using (1.15) and then using the definition of Y_t from (1.12) we have the social planner's - or efficient - level of output

$$Y_t^e = \frac{1}{\eta} Z_t$$

Then the gross efficient output gap is simply

$$G_t^e = \frac{Y_t}{Y_t^e} = \eta \frac{Y_t}{Z_t} \quad (1.A.32)$$

1.A.6 The Natural Level of Output

Here we derive the natural level of output in this economy as defined in definition 2 in section 2.7. Assuming prices are flexible, or $\phi = 0$, the monopolistically competitive firm will set its price a constant “mark-up” - $\mathcal{M} = \frac{\theta}{\theta-1}$ - over marginal cost (see Eq. (1.A.29))

$$P_t = \mathcal{M} \frac{W_t}{Z_t} \quad (1.A.33)$$

Rearranging a bit and then expressing (1.A.33) in terms of log-deviations from steady state we have, where all variables with a tilde that follow are real detrended log-deviations from steady state with the exception of interest rates which are in nominal terms.

$$0 = \tilde{W}_t - \tilde{Z}_t$$

Now use the household’s first order condition relating the marginal utility of consumption and leisure to the real wage and log-linearizing we have

$$0 = (1 + \gamma_m) \tilde{C}_t - \gamma_m \tilde{M}_t^A + \gamma_m \tilde{v}_t - \tilde{Z}_t \quad (1.A.34)$$

where $\gamma_m = \frac{u_{MA,C}''}{u_C'}$ is zero for additively separable preferences. Now substitute equation (1.A.23), the log-linearized market clearing condition, into (1.A.34) to obtain

$$\begin{aligned} 0 = & \frac{(1 + \gamma_m) \tilde{Y}_t^n}{s_c} - \frac{(1 - s_c) + \gamma_m}{s_c} \tilde{M}_t^A - \frac{(1 + \gamma_m)(1 - s_c)\omega}{s_c} (\tilde{u}_t^A - \tilde{u}_t^D) \\ & + \gamma_m \tilde{v}_t - \frac{(1 + \gamma_m)(1 - s_c)}{s_c} \tilde{x}_t - \tilde{Z}_t \end{aligned} \quad (1.A.35)$$

Next, substitute equation (1.A.24) into (1.A.35) to eliminate the monetary aggregate and impose the condition that at the natural rate of output $\tilde{u}_t^A = 0$.

$$\begin{aligned}
0 &= \tilde{Y}_t^n - \tilde{Z}_t + \omega(1 - s_c)\tilde{u}_t^D - (1 - s_c)\tilde{x}_t \\
&\quad + \left[\frac{s_c\gamma_m(1 - \eta_v) - (1 - s_c)(1 + \gamma_m)\eta_v}{s_c} \right] \tilde{v}_t \\
&\equiv \tilde{Y}_t^n - \tilde{Z}_t + \omega(1 - s_c)\tilde{u}_t^D - (1 - s_c)\tilde{x}_t + \Psi_v^y \tilde{v}_t
\end{aligned} \tag{1.A.36}$$

Finally we can eliminate \tilde{u}_t^D , and hence \tilde{r}_t , from (1.A.36) by log-linearizing $\tilde{u}_t^D = \tilde{\mathcal{S}}_t = \gamma_{uD}^r \tilde{r}_t + \gamma_{uD}^\tau \tilde{\tau}_t + \gamma_{uD}^x \tilde{x}_t$ and then eliminate \tilde{r}_t from this expression using the condition that $\tilde{u}_t^A = 0$ which implies (using Eq. (1.A.18))

$$0 = \tilde{u}_t^A = \gamma_{uN} \gamma_{uN}^r \tilde{r}_t + \gamma_{uD} \left[\gamma_{uD}^r \tilde{r}_t + \gamma_{uD}^\tau \tilde{\tau}_t + \gamma_{uD}^x \tilde{x}_t \right]$$

which defines the policy rate in terms of stochastic shocks. Clearly to stabilize the aggregate interest rate spread the policy rate must fall when the loan-deposit spread rises. This is confirmed below.

$$\begin{aligned}
\tilde{r}_t &= - \left[\frac{\gamma_{uD} \gamma_{uD}^\tau}{\gamma_{uN} \gamma_{uN}^r + \gamma_{uD} \gamma_{uD}^r} \right] \tilde{\tau}_t - \left[\frac{\gamma_{uD} \gamma_{uD}^x}{\gamma_{uN} \gamma_{uN}^r + \gamma_{uD} \gamma_{uD}^r} \right] \tilde{x}_t \\
&\equiv -s_D^P \frac{\gamma_{uD}^\tau}{\gamma_{uD}^r} \tilde{\tau}_t - s_D^P \frac{\gamma_{uD}^x}{\gamma_{uD}^r} \tilde{x}_t
\end{aligned} \tag{1.A.37}$$

Using this expression in $\tilde{u}_t^D = \tilde{\mathcal{S}}_t = \gamma_{uD}^r \tilde{r}_t + \gamma_{uD}^\tau \tilde{\tau}_t + \gamma_{uD}^x \tilde{x}_t$ we have that

$$\begin{aligned}
\tilde{u}_t^D &= (1 - s_D^P) \gamma_{uD}^\tau \tilde{\tau}_t + (1 - s_D^P) \gamma_{uD}^x \tilde{x}_t \\
&\equiv s_N^P \gamma_{uD}^\tau \tilde{\tau}_t + s_N^P \gamma_{uD}^x \tilde{x}_t
\end{aligned} \tag{1.A.38}$$

Substituting (1.A.38) into (1.A.36) and solving for \tilde{Y}_t^n we have

$$\tilde{Y}_t^n = \tilde{z}_t - \Psi_\tau^y \tilde{\tau}_t - \Psi_x^y \tilde{x}_t - \Psi_v^y \tilde{v}_t \quad (1.A.39)$$

where

$$\begin{aligned} \Psi_\tau^y &= (1 - s_c) \omega \gamma_{u^D}^\tau \\ \Psi_x^y &= (1 - s_c) [\omega s_N^P \gamma_{u^D}^x - 1] \\ \Psi_v^y &= \left[\frac{s_c \gamma_m (1 - \eta_v) - (1 - s_c) (1 + \gamma_m) \eta_v}{s_c} \right] \end{aligned}$$

Although the expression in (1.A.39) is only accurate to first order, it will be useful to define the equivalent expression in terms of stationary variables in levels for uniformness when we define the detrended equilibrium system below. It can be easily verified the resulting expression is proportional to

$$Y_t^n = \left(\frac{Z_t}{Z_{t-1}} \right) v_t^{-\Psi_v^y} x_t^{-\Psi_x^y} \tau_t^{-\Psi_\tau^y} \quad (1.A.40)$$

1.A.7 The Natural Rate of Interest

Combine equations (1.A.1), (1.A.6) and (1.A.13) and log-linearize the resulting expression to obtain the consumption Euler expression which is given by

$$\begin{aligned} \tilde{r}_t - E_t[\tilde{\pi}_{t+1}] &= E_t[(1 + \gamma_m) \Delta \tilde{C}_{t+1} - \gamma_m \Delta \tilde{M}_{t+1}^A - \Delta \tilde{r}_{t+1}^L] \\ &\quad + E_t[\gamma_m \Delta \tilde{v}_{t+1} - \Delta \tilde{a}_{t+1}] \end{aligned} \quad (1.A.41)$$

Following a similar process that was used to arrive at the natural rate of output we can now

substitute the market clearing condition (see (1.A.23)) to eliminate \tilde{C}_t and the equilibrium monetary aggregate condition (see (1.A.24)) to eliminate \tilde{M}_t^A . Then impose the definition of the natural rate of output (see Def. 2, Section 2.7) on the resulting expression using (1.A.39) for the natural rate of output and almost all terms cancel leaving us with just

$$\tilde{r}_t^n = -E_t[\Delta \tilde{a}_{t+1}] + E_t[\Delta \tilde{Z}_{t+1}] - E_t[\Delta \tilde{r}_{t+1}^L] \quad (1.A.42)$$

To eliminate the interest rate from (1.A.42) use equation (1.A.38) to arrive at the expression for the natural rate of interest

$$\tilde{r}_t^n = \tilde{a}_t(1 - \rho_a) - \tilde{z}_t(1 - \rho_z) - \Psi_\tau^r \tilde{r}_t(1 - \rho_\tau) - \Psi_x^r \tilde{x}_t(1 - \rho_x) \quad (1.A.43)$$

where

$$\Psi_\tau^r = \left[\frac{\gamma_{uD} \gamma_{uD}^\tau}{\gamma_{uN} \gamma_{uN}^r + \gamma_{uD} \gamma_{uD}^r} \right] = \frac{\gamma_{uD}^\tau}{\gamma_{uD}^r} (1 - s_N^p)$$

$$\Psi_x^r = \left[\frac{\gamma_{uD} \gamma_{uD}^x}{\gamma_{uN} \gamma_{uN}^r + \gamma_{uD} \gamma_{uD}^r} \right] = \frac{\gamma_{uD}^x}{\gamma_{uD}^r} (1 - s_N^p)$$

Although the expression in (1.A.43) is only accurate to first order, it will be useful to define the equivalent expression in levels and impose the unit root in the technology process for uniformness when we define the equilibrium system below. It can be easily verified the resulting expression is proportional to

$$r_t^n = a_t^{(1-\rho_a)} Z_t^{(1-\rho_z)} \tau_t^{-\Psi_\tau^r(1-\rho_\tau)} x_t^{-\Psi_x^r(1-\rho_x)} \quad (1.A.44)$$

1.A.8 Calibrated Values

In Table 1.10 we summarize the values assigned to the model's parameters in the baseline model. See section 3.1 for a detailed explanation of the calibration. Also, in section 5.2 we analyze *robustly* optimal policy in which case the values assigned to the parameters driving

the model's stochastic shocks (which play a crucial role in shaping optimal policy) are varied over a large range of values.

1.A.9 The Detrended Equilibrium System

At this point it is now possible to define a symmetric equilibrium in the model. First, impose the equilibrium conditions (1.23), (1.25), (1.26), (1.27) and (1.28) on the models equations. As mentioned in the text, the resulting model, in its current form, is not stationary due to the unit roots in the technology process and the price level. Hence, we must next detrend the non-stationary variables and then define an equilibrium in the stationarized model. The resulting model is described in detail in the Table 1.11.

1.A.10 The Simple-Sum Monetary Aggregate

When the Simple-Sum monetary aggregate (see Eq. 1.22) is embedded in the policy rule (see Eq. (1.18)) in place of the Divisia monetary aggregate (see Eq. 3.8) the economy is likely to be indeterminate. In this section we highlight why this occurs. Since issues of local determinacy can be sufficiently analyzed with a linear approximation to the model we linearize the Simple-Sum monetary aggregate to quantify its error in tracking the true aggregate. Begin by substituting (1.A.4) and (1.A.5) into the definition of the growth rate of the simple sum aggregate to eliminate N_t and D_t .

$$\begin{aligned}
\ln(\mu_t^{Simple-Sum}) &= \ln\left(\frac{N_t + D_t}{N_{t-1} + D_{t-1}}\right) \\
&= \Delta \ln(M_t^A) + \omega \Delta \ln(u_t^A) \\
&\quad + \ln\left(\frac{\nu(u_t^N)^{-\omega} + (1-\nu)(u_t^D)^{-\omega}}{\nu(u_{t-1}^N)^{-\omega} + (1-\nu)(u_{t-1}^D)^{-\omega}}\right)
\end{aligned} \tag{1.A.45}$$

The expression for the growth rate of the Simple-Sum aggregate to this point is exact. To gain further insight we now linearize (1.A.45) around the competitive equilibrium.

$$\begin{aligned}
\ln(\mu_t^{Simple-Sum}) &\approx \Delta \ln(M_t^A) + \omega \Delta \left[(\gamma_{uN} \gamma_{uN}^r + \gamma_{uD} \gamma_{uD}^r) \tilde{r}_t + \gamma_{uD} (\gamma_{uD}^\tau \tilde{r}_t + \gamma_{uD}^x \tilde{x}_t) \right] \\
&\quad - \omega \Delta \left[(\gamma_{uN}^{SS} \gamma_{uN}^r + \gamma_{uD}^{SS} \gamma_{uD}^r) \tilde{r}_t + \gamma_{uD}^{SS} (\gamma_{uD}^\tau \tilde{r}_t + \gamma_{uD}^x \tilde{x}_t) \right] \\
&= \Delta \ln(M_t^A) + \omega \left[\gamma_{uN}^r (\gamma_{uN} - \gamma_{uN}^{SS}) + \gamma_{uD}^r (\gamma_{uD} - \gamma_{uD}^{SS}) \right] \Delta \tilde{r}_t \\
&\quad + \omega (\gamma_{uD} - \gamma_{uD}^{SS}) \left[\gamma_{uD}^\tau \Delta \tilde{r}_t + \omega \gamma_{uD}^x \Delta \tilde{x}_t \right] \\
&= \Delta \ln(M_t^A) + \omega (\gamma_{uN}^r - \gamma_{uD}^r) (\gamma_{uN} - \gamma_{uN}^{SS}) \Delta \tilde{r}_t \\
&\quad - \omega (\gamma_{uN} - \gamma_{uN}^{SS}) \left[\gamma_{uD}^\tau \Delta \tilde{r}_t + \omega \gamma_{uD}^x \Delta \tilde{x}_t \right] \\
&\equiv \Delta \ln(M_t^A) + \Psi_r^{SS} \Delta \tilde{r}_t - \Psi_\tau^{SS} \Delta \tilde{r}_t - \Psi_x^{SS} \Delta \tilde{x}_t
\end{aligned} \tag{1.A.46}$$

Its worth noting that $\gamma_{uN} - \gamma_{uN}^{SS} = -(\gamma_{uD} - \gamma_{uD}^{SS}) = 0$ when $\bar{r}^D = 1$. That is, when currency and deposits are perfect one for one substitutes (i.e. their user costs are identical) the simple sum aggregate tracks the true aggregate without error. However, this environment is at odds with reality (and this model).²⁸ To gain further insight into the determinacy properties of Taylor rules reacting to the Simple-Sum aggregate, it is useful to evaluate Ψ_r^{SS} at the models steady state.²⁹ In which case we find that $\Psi_r^{SS} \approx 5$. To see why such a large positive value of Ψ_r^{SS} is troublesome examine the policy rule when reacting to the (growth rate) of the Simple-Sum aggregate.

$$\begin{aligned}
\tilde{r}_t &= \phi_\pi \tilde{\pi}_t + \phi_m \tilde{\mu}_t^{SS} \\
&= \phi_\pi \tilde{\pi}_t + \phi_m \left[\Delta \ln(M_t^A) + \Psi_r^{SS} \Delta \tilde{r}_t \right]
\end{aligned} \tag{1.A.47}$$

Rearranging (1.A.47) so that it can explicitly expressed as an interest rate rule with a unitary

²⁸Meanwhile, carrying out a similar linearization for $\ln(\mu_t^{Divisia})$ one can verify that $\gamma_{uN} - \gamma_{uN}^{Divisia} = -(\gamma_{uD} - \gamma_{uD}^{Divisia}) = 0$ in general in this model. That is, regardless of whether $\bar{r}^D = 1$, $\ln(\mu_t^{Divisia}) \approx \Delta \ln(M_t^A)$.

²⁹The values of Ψ_r^{SS} and Ψ_x^{SS} have no bearing on determinacy.

coefficient on \tilde{r}_t on the left hand side we have:

$$\tilde{r}_t = \begin{cases} -\left[\frac{\phi_m \Psi_r^{SS}}{1-\phi_m \Psi_r^{SS}}\right] \tilde{r}_{t-1} + \left[\frac{\phi_\pi}{1-\phi_m \Psi_r^{SS}}\right] \tilde{\pi}_t + \left[\frac{\phi_m}{1-\phi_m \Psi_r^{SS}}\right] \Delta \ln(M_t^A) & 0 < \phi_m < 1/\Psi_r^{SS} \\ \left[\frac{\phi_m \Psi_r^{SS}}{\phi_m \Psi_r^{SS}-1}\right] \tilde{r}_{t-1} - \left[\frac{\phi_\pi}{\phi_m \Psi_r^{SS}-1}\right] \tilde{\pi}_t - \left[\frac{\phi_m}{\phi_m \Psi_r^{SS}-1}\right] \Delta \ln(M_t^A) & \phi_m > 1/\Psi_r^{SS} \end{cases}$$

Using our baseline calibration, for $\phi_m \approx .2$ the sign of the policy rule's reaction to inflation is flipped so that higher inflation leads to lower nominal rates. Hence, the sizable error in tracking the true aggregate is positively related to the policy rate which leads to this reversed policy rule. Interestingly, the determinacy region displayed in Figure 1.5 shows that the rule is not always indeterminate despite this perverse reaction to inflation. To see why notice that for $\phi_m > 1/\Psi_r^{SS}$ the policy rule is *super-inertial* with a *negative* reaction to inflation. Although the literature on determinacy is scant regarding super-inertial policy rules with a negative inflation reaction (for good reason), numerical analysis in this model appears to bring out a relationship sufficient for determinacy as follows:

$$\text{If } \phi_\pi < 0 \text{ and } \rho_r > 1 \text{ then } \rho_r + \phi_\pi > 1 \text{ is determinate.} \quad (1.A.48)$$

This condition however offers little hope for stabilizing Taylor rules which react to the simple sum aggregate since as $\phi_m \rightarrow \infty$ the policy rule becomes

$$\tilde{r}_t = \tilde{r}_{t-1} - \Delta \ln(M_t^A) \quad (1.A.49)$$

which is always indeterminate.³⁰ Instead, the condition in (1.A.48) only explains why the simple-sum rule is *ever* determinate. Indeed, the small determinacy region in Figure 1.5 is described by the condition:

$$\text{If } \phi_\pi > 0 \text{ and } 1 > \phi_m > \frac{1}{\Psi_r^{SS}} \text{ then } \phi_m + \phi_\pi < 1 \text{ is determinate.}$$

³⁰Interestingly Eq. (1.A.49) is similar to a constant growth rate of Simple-Sum rule, suggesting using Simple-Sum as an instrument is troublesome as well.

Determinacy Region Under $\tilde{r}_t = \phi_\pi \tilde{\pi}_t + \phi_m \tilde{\mu}_t^{Simple-Sum}$

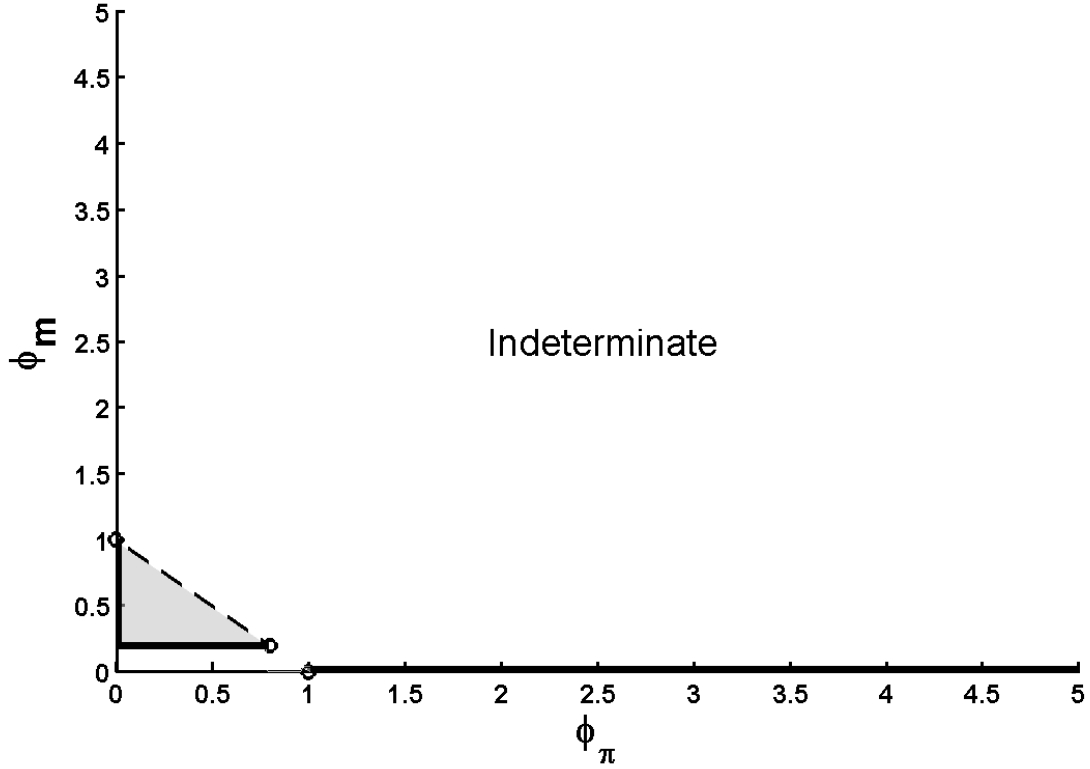


Figure 1.5: The shaded area to the southwest of the dotted line, defined by $\phi_m > 1/\Psi_r^{SS}$ and $\phi_m + \phi_\pi < 1$, is determinate. Here ϕ_m denotes the response to the growth rate simple-sum monetary aggregate.

Not surprisingly, the optimization on this small region of determinacy is not successful hence no policy prescription for policy rules reacting to simple-sum is available. Since reacting to simple-sum limits policy makers reaction to inflation (something central bankers would understandably be adverse to doing), *the most practical policy prescription when reacting to Simple-Sum is to fix $\phi_m = 0$* . This stresses the importance of properly constructing monetary aggregates for use in policy making, extending the point of Belongia (1996) that “measurement matters” to the policy making realm. Indeed when the properly weighted Divisia aggregate is embedded in the policy rule, the determinacy region satisfies a novel Taylor principle for Divisia aggregates.³¹

³¹See Figure 1.4 and Eqs. (1.33) and (1.34).

1.A.11 The Ramsey Problem

In all welfare calculations we approximate the model around the Ramsey planner's steady state. We calculate this steady state using the OLS method described in Schmitt-Grohe & Uribe (2012). Specifically, in terms of detrended variables and equations, define

$$E_t[C(\mathcal{E}_{t+1}, \mathcal{E}_t, S_t, S_{t-1})] = 0 \quad (1.A.50)$$

$$(S_t - S) = \rho(S_{t-1} - S) + \epsilon_t \quad (1.A.51)$$

where (1.A.50) denotes the system of equilibrium conditions defined in Table 1.11 definition of the stochastic processes which are defined in equation (1.A.51). and hence \mathcal{E} denotes the (1x17) vector of endogenous variables³² and the vector S_t denotes the (5x1) vector of exogenous stochastic variables³³ implying ϵ_t is the vector of structural disturbances. The relevant portion of the Ramsey planners Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & \dots - \beta^{t-1} \Gamma_{t-1}^T C(\mathcal{E}_t, \mathcal{E}_{t-1}, S_t, S_{t-1}) \\ & + \beta^t u_t(\mathcal{E}_t, S_t) - \beta^t \Gamma_t^T C(\mathcal{E}_{t+1}, \mathcal{E}_t, S_{t+1}, S_t) + \dots \end{aligned}$$

where $\{\Gamma_t\}_{t=-1}^\infty$ denotes the sequence of (16x1) Lagrange multipliers for the 16 structural equations.³⁴ The relevant first order condition is given by the (1x17) matrix equation³⁵

³²To be explicit the 17 endogenous variables are given by the detrended versions of: $\pi_t, r_t, W_t, \Lambda_t^3, \Lambda_t^1, r_t^L, r_t^D, C_t, \Lambda_t^2, M_t^A, N_t, D_t, L_t, Y_t, h_t, F_t, Q_t$.

³³To be explicit the 5 exogenous stochastic variables are given by the detrended versions of: $v_t, a_t, x_t, \tau_t, Z_t$.

³⁴To be explicit the 16 structural equations are given by: (1.3), (1.8), (1.10), (1.15), (3.28), (1.A.1)-(1.A.6), (1.A.13)-(1.A.15), (1.A.26), (1.A.29).

³⁵All derivatives are computed analytically using MATLAB's symbolic toolbox.

$$\begin{aligned}\frac{\partial u_t(\mathcal{E}_t, S_t)}{\partial \mathcal{E}_t} &= \Gamma_t^T E_t \left[\frac{\partial C(\mathcal{E}_{t+1}, \mathcal{E}_t, S_{t+1}, S_t)}{\partial \mathcal{E}_t} \right] \\ &\quad + \beta^{-1} \Gamma_{t-1}^T \frac{\partial C(\mathcal{E}_t, \mathcal{E}_{t-1}, S_t, S_{t-1})}{\partial \mathcal{E}_t}\end{aligned}\tag{1.A.52}$$

Finding the Ramsey steady state requires finding values of $(\bar{\mathcal{E}}, \bar{\Gamma})$ that solve

$$\begin{aligned}\left. \frac{\partial u_t(\mathcal{E}_t, S_t)}{\partial \mathcal{E}_t} \right|_{(\bar{\mathcal{E}}, \bar{S})} &= \bar{\Gamma}^T \left. \frac{\partial C(\mathcal{E}_{t+1}, \mathcal{E}_t, S_{t+1}, S_t)}{\partial \mathcal{E}_t} \right|_{(\bar{\mathcal{E}}, \bar{S})} \\ &\quad + \beta^{-1} \bar{\Gamma}^T \left. \frac{\partial C(\mathcal{E}_t, \mathcal{E}_{t-1}, S_t, S_{t-1})}{\partial \mathcal{E}_t} \right|_{(\bar{\mathcal{E}}, \bar{S})}\end{aligned}\tag{1.A.53}$$

$$C(\bar{\mathcal{E}}, \bar{\mathcal{E}}, \bar{S}, \bar{S}) = 0\tag{1.A.54}$$

We guess a value of $\bar{\pi}$ and then solve (1.A.54) for the remaining variables in $\bar{\mathcal{E}}$. We then use $\bar{\mathcal{E}}$ to solve (1.A.53) for $\hat{\bar{\Gamma}}_{OLS}$ as proposed in Schmitt-Grohe & Uribe (2012) and define $\delta(\bar{\pi})$ as the resulting residual.³⁶ We then use MATLAB's `fminsearch(-)` to minimize $\delta(\bar{\pi})^T \delta(\bar{\pi})$ where the minimizing argument is the Ramsey steady state value of inflation. Under our baseline preferences this value is $\bar{\pi} = 1.028$ and under additively separable preferences this value is $\bar{\pi} = 1.033$.

³⁶More specifically, $\hat{\bar{\Gamma}}_{OLS}$ is the projection of $\left[\beta^{-1} \frac{\partial C_{t-1}}{\partial \mathcal{E}_t} + \frac{\partial C_t}{\partial \mathcal{E}_t} \right]^T$ onto $\left[\frac{\partial U_t}{\partial \mathcal{E}_t} \right]^T$.

1.B Appendix: Tables

Table 1.10: Baseline^{1,2} Calibration of Parameters

Parameter		Value	Source
Discount Rate:	β	0.99	Kydland & Prescott (1982)
Disutility of Work:	η	2.5	Belongia & Ireland (2013)
Shopping-Time Function:	χ	2	Belongia & Ireland (2013)
CES Monetary Aggregate:	ν	0.225	Belongia & Ireland (2013)
CES Monetary Aggregate:	ω	1.5	Belongia & Ireland (2013)
Final Goods CES:	θ	6	Ireland (2000, 2004a,b)
Cost of Price Adjustment:	ϕ	50	Ireland (2004b)
Steady-State Inflation:	π	1.005	Belongia & Ireland (2013)
Money Demand Shock:	v	0.4	Belongia & Ireland (2013)
Technology Shock:	Z	1.005	Belongia & Ireland (2013)
Reserves Demand Shock:	τ	0.02	Ireland (2011)
Banking Productivity Shock:	x	0.01	Belongia & Ireland (2006, 2013)
Preference Shock:	ρ_a	0.9579	Ireland (2004a)
Money Demand Shock:	ρ_v	0.9853	Ireland (2004a)
Reserves Demand Shock:	ρ_τ	0.5	Belongia & Ireland (2013)
Banking Productivity Shock:	ρ_x	0.5	Belongia & Ireland (2013)
Preference Shock:	σ_a	0.0188	Ireland (2004a)
Money Demand Shock:	σ_v	0.0088	Ireland (2004a)
Reserves Demand Shock:	σ_τ	0.7448	Dellas et al. (2010)
Banking Productivity Shock:	σ_x	0.1720	Belongia & Ireland (2006) Dellas et al. (2010)
Technology Shock:	σ_z	0.0098	Ireland (2004a)

¹ Here the baseline calibrations are presented. In section 5.2 we analyze *robustly* optimal policy and vary the calibrated values of the model's shocks.

² Under additively separable preferences we set $\eta_m = .1$ which matches $\frac{N+D}{PC} = 3.3$ in steady state, the average of Simple-Sum M2 to nominal consumption expenditures since 1959.

Table 1.11: Equilibrium Variables and System of Equations¹

Variable	Detrended Variable	Equation
v_t	v_t	(1.5)
a_t	a_t	(1.6)
x_t	x_t	(1.7)
τ_t	τ_t	(1.9)
Z_t	Z_t/Z_{t-1}	(1.16)
P_t	Π_t	(1.18)
r_t	r_t	(1.A.13)
W_t	$W_t/P_t Z_{t-1}$	(1.A.29)
Λ_t^3	$\Lambda_t^3 Z_{t-1}$	(1.A.2)
Λ_t^1	$\Lambda_t^1 Z_{t-1}$	(1.A.14)
r_t^L	r_t^L	(1.A.6)
r_t^D	r_t^D	(1.10)
\mathcal{S}_t	\mathcal{S}_t	(1.11)
C_t	$C_t Z_{t-1}$	(1.A.1)
M_t^A	$M_t^A/P_t Z_{t-1}$	(1.3)
Λ_t^2	$\Lambda_t^2 Z_{t-1}$	(1.A.3)
N_t	$N_t/P_t Z_{t-1}$	(1.A.4)
D_t	$D_t/P_t Z_{t-1}$	(1.A.5)
L_t	$L_t/P_t Z_{t-1}$	(1.8)
Y_t	Y_t/Z_{t-1}	(3.28)
h_t	h_t	(1.15)
F_t	$F_t/P_t Z_{t-1}$	(1.A.26)
Q_t	$Q_t/P_t Z_{t-1}$	(1.A.15)
r_t^A	r_t^A	(1.A.18)
s_t^N	s_t^N	(1.20)
s_t^D	s_t^D	(1.21)
$\mu_t^{Divisia}$	$\mu_t^{Divisia}$	(3.8)
$\mu_t^{Simple-Sum}$	$\mu_t^{Simple-Sum}$	(1.22)
G_t^e	G_t^e	(1.A.32)
$-^2$	Y_t^n	(1.A.40)
$-^2$	r_t^n	(1.A.44)
$u_t = \ln(C_t) - \eta h_t$	$u_t = \ln(C_t/Z_{t-1}) - \eta h_t$	
$-\eta \frac{1}{\chi} (\frac{v_t P_t C_t}{M_t^A})^\chi$	$-\eta \frac{1}{\chi} (\frac{v_t C_t/Z_{t-1}}{M_t^A/P_t Z_{t-1}})^\chi$	—

¹ The order of the variables correspond to the order in which to solve the system to find the steady state. The corresponding equation number corresponds to the equation used to find the steady state value of the respective variable. (An exception to this is the steady state value for inflation which is calibrated and thus the policy rule is not needed to solve for its steady state value.)

² The natural rate of output and hence the natural rate of interest do not have a non-stationary analogue since these variables are only defined by a linear approximation around the steady state.

Chapter 2

Determinacy and Indeterminacy in Monetary Policy Rules with Money¹

2.1 Introduction

In his classic (1960) work, *A Program for Monetary Stability*, Milton Friedman argued the economy can be stabilized by stabilizing the growth rate of a money at k-percent. This policy proposal has received considerable attention by monetary theorist.² Surprisingly though, the question of whether this rule could be implemented with a broad monetary aggregate in a manner which delivers a unique rational expectations equilibrium remains unanswered. If such a rule fails in this capacity, fixing the growth rate of a monetary aggregate may induce instability by allowing for sunspot equilibria. This paper provides an answer to this question within the preferred framework of modern monetary policy analysis, a New-Keynesian model. We show analytically that Friedman's k-percent rule can deliver a unique rational expectations equilibrium when the true monetary aggregate is used. Since that aggregate depends on deep structural parameters, such a rule is not *simple* in the sense defined by (Gali, 2008).

¹This chapter is coauthored with John W. Keating, Department of Economics, The University of Kansas, Email: jkeating@ku.edu.

²For example, Ireland (1996) has shown that a policy of holding the growth rate of money constant can be optimal from a welfare perspective when prices are set in advance. Additionally, Carlstrom & Fuerst (1995, 2003) explore the welfare and determinacy properties of money growth rules in flexible price models.

A *simple* rule which could actually be used by central banks,³ would be to fix the growth rate of the Divisia monetary aggregate. We show that such a rule inherits the determinacy properties of the k-percent rule based on using the true monetary aggregate. Another well-known non-parametric aggregate is the simple-sum monetary aggregate which was the method used to compute the M2 measure Friedman advocated. Interestingly, if this aggregate is used in place of the true monetary aggregate, we find Friedman's k-percent rule is likely to result in indeterminacy stemming from the simple-sum's error in tracking the true aggregate. We conclude that Friedman was correct in his judgment that fixing the growth rate of a broad monetary aggregate may stabilize the economy. However, we show in the context of a New-Keynesian model with multiple types of assets which provide monetary services that aggregation by simple-sum is in general a flawed approach.

Most similar to our work is Evans & Honkapohja (2003). They analyze determinacy properties of k-percent rules in a New-Keynesian model when money is modeled as a single monetary asset. In this case, money in the model can be interpreted as base money since it earns zero interest. They show numerically that fixing the growth rate of this measure of money is consistent with a unique rational expectations equilibrium under a broad range of values. However Friedman's k-percent rule calls for the central bank to fix the growth rate of a high-level monetary aggregate, not base money. In order to analyze determinacy properties of broad monetary aggregates, we require a model like the one developed by Belongia & Ireland (2014) which provides a role for both currency and deposits as competing sources of monetary services. We use this as our framework to analyze the determinacy properties of monetary policy rules with broad money.

In addition to providing analytic representations of the determinacy regions of various monetary aggregate growth rules, we also analyze interest rate rules reacting to inflation and the growth rate of monetary aggregates. Once again we find that "measurement matters" (Belongia, 1996) from a determinacy standpoint. Interest rate rules reacting to the growth rate of

³Arguably, one reason Friedman advocated a k-percent rule was its simplicity.

either the true monetary aggregate or the Divisia monetary aggregate satisfy a novel Taylor principle for monetary aggregates. The simple-sum monetary aggregate's error in tracking the true aggregate prevents an interest rate rule reacting to simple-sum from having a similar determinacy region. Instead, such rules have very small determinacy regions.

Following this introductory section we present our model which is a standard New-Keynesian framework that allows two different types of assets to provide monetary services. Section 3 examines determinacy properties of k-percent rules and compares outcomes obtained with different monetary aggregates. The fourth section of the paper investigates interest rate rules that allow for a reaction to monetary aggregates. Again we address determinacy properties associated with alternative monetary aggregates. Section 5 performs a numerical analysis with more complicated monetary policy rules that allow policy to react to more information making it impossible to derive analytical results. Section 6 concludes the paper.

2.2 Model

We consider a canonical version of the linearized New-Keynesian (NK) model. Following Woodford's (2003) notation we assume all exogenous disturbances are bounded in amplitude by $\|\xi\|$. All variables with a "-" over them denote non-stochastic steady-state values. Also, variables with a "~" over them denote log-deviations from the non-stochastic steady state. For convenience of notation we assume each stochastic disturbance follows a stationary AR(1) process and is driven by orthogonal white noise shocks. However, this assumption is only made for convenience as all the results presented hold for general stationary processes. As usual, this model consists of 3 non-policy equations when money is included in the model.

$$\tilde{y}_t^e = \mathbb{E}_t\{\tilde{y}_{t+1}^e\} - [\tilde{r}_t - \mathbb{E}_t\{\tilde{\pi}_{t+1}\} - \tilde{r}_t^e] \quad (2.1)$$

$$\tilde{\pi}_t = \kappa \tilde{y}_t^e + \beta \mathbb{E}_t\{\tilde{\pi}_{t+1}\} \quad (2.2)$$

$$\tilde{m}_t - \tilde{p}_t = \tilde{y}_t^e - \tilde{u}_t + \tilde{v}_t + \tilde{z}_t \quad (2.3)$$

Equation (2.1) is the dynamic IS curve, equation (2.2) is the New-Keynesian Phillips Curve (NKPC) and equation (2.3) is the demand for the monetary aggregate. In the above equations $\tilde{y}_t^e = y_t - z_t$ denotes the efficient output gap and $\tilde{r}_t^e = \mathbb{E}_t\{\Delta\tilde{z}_{t+1}\} - \mathbb{E}_t\{\tilde{a}_{t+1}\}$ denotes the efficient interest rate. The variables \tilde{z}_t , \tilde{a}_t , and \tilde{v}_t represent exogenous technology, preference and money-demand disturbances respectively. We depart from the textbook analysis of money. Following Belongia & Ireland (2014) we model broad money so that (2.3) is the demand for the aggregate service flow from currency and interest bearing deposits. For this reason, (2.3) includes the user-cost of the monetary aggregate u_t which in general depends on a vector of interest rates. Modeling money as a broader measure turns out to play a key role in our determinacy results. In addition to examining the effects of monetary aggregate growth rules we will also investigate interest rate rules which react to the growth rate of a monetary aggregate.

We assume a constant elasticity of substitution (CES) function:

$$m_t = \left[\nu^{\frac{1}{\omega}} n_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (2.4)$$

is the true monetary aggregate where n_t and d_t are non interest bearing currency and interest bearing deposits respectively. Meanwhile, $\omega > 0$ is the elasticity of substitution between currency and deposits and $0 < \nu < 1$ governs the steady-state share of currency and deposits. The CES function is homogeneous of degree one in terms of n_t and d_t , an attractive property since that means changing both n_t and d_t by a certain percentage will change the monetary aggregate by exactly that same percentage. Of course, simple-sum has that property. And when $\omega \rightarrow \infty$, $m_t = n_t + d_t$. Hence, the simple-sum monetary aggregate is the appropriate monetary aggregate when currency and deposits are perfect substitutes. More generally, when $\omega < \infty$, currency and deposits are not perfect one-for-one substitutes and then simple-sum will not equal the true aggregate.

The aggregate dual user-cost is correspondingly defined by

$$u_t = \left[\nu \left(\frac{r_t - 1}{r_t} \right)^{1-\omega} + (1-\nu) \left(\frac{r_t - r_t^d}{r_t} \right)^{1-\omega} \right]^{\frac{1}{1-\omega}}. \quad (2.5)$$

Following the structural model of Belongia & Ireland (2014) with profit maximizing banks, the user cost of deposits can be expressed as

$$u_t^d = \frac{r_t - r_t^d}{r_t} = \frac{(r_t - 1)\tau_t + x_t}{r_t} \quad (2.6)$$

where r_t is the gross nominal interest rate on loans, r_t^d is the deposit rate and both τ_t and x_t are exogenous financial disturbances that follow stationary stochastic processes with orthogonal white noise shocks. Specifically, τ_t represents reserves demand disturbances with a mean $\bar{\tau}$ and x_t represents deposit cost disturbances with a mean \bar{x} . To remain empirically relevant, we will assume throughout the paper that the stochastic processes are bounded so that at all times $r_t > r_t^d > 1$ ensuring that deposits are “cheaper” than currency. A log-linear approximation to (2.5) is given by

$$\tilde{u}_t = \eta_r \tilde{r}_t + \eta_\tau \tilde{\tau}_t + \eta_x \tilde{x}_t + \mathcal{O}(\|\xi\|^2) \quad (2.7)$$

where all the coefficients are positive. Substituting (2.7) into (2.3) brings about a five variable dynamic model with three equations (2.1, 2.2 and 2.8).

$$\tilde{m}_t - \tilde{p}_t = \tilde{y}_t^e - \eta_r \tilde{r}_t - \eta_\tau \tilde{\tau}_t - \eta_x \tilde{x}_t + \tilde{v}_t + \tilde{z}_t \quad (2.8)$$

The dimension of the model can be reduced by one variable after defining real balances as $\tilde{l}_t \equiv \tilde{m}_t - \tilde{p}_t$. As is standard, we close the model with a specification of monetary policy.

2.3 Friedman k-percent Rules

In this section we consider the performance of various money growth rules. Initially we close the model with a constant money growth rule for the true monetary aggregate:

$$\Delta \tilde{m}_t = 0. \tag{2.1}$$

Such k-percent rules have been examined in New-Keynesian models by Evans & Honkapohja (2003) and Gali (2008) where they were shown to deliver a unique REE. However, these studies assume the existence of a single monetary asset. This leaves open the question of whether the determinacy properties extend to measures that aggregate different types of monetary assets. Arguably, this is the relevant case to consider. Friedman, for example, argued for fixing the growth rate of a broad aggregate such as M2, not the monetary base. In this section we show three main results. First, the k-percent rule is always determinate when applied to the true monetary aggregate, regardless of the magnitude of the interest semi-elasticity. Second, this result of general determinacy extends to rules in the true aggregate is replaced by the Divisia monetary aggregate. Finally, we show that this strong result does not extend to the simple-sum monetary aggregate. Instead, a constant simple-sum growth rule is indeterminate unless the interest semi-elasticity is large relative to simple-sum's error in tracking the true monetary aggregate. Consider the dynamic system consisting of (2.1), (2.2), (2.8) and (2.1).

Proposition 2.1. *For any $0 < \beta < 1$, for any $\kappa > 0$ and for any $\eta_r \geq 0$ if the central bank follows the policy rule $\Delta \tilde{m}_t = 0$ then there exists a unique REE.*

The proof of this proposition is in the Appendix. Unless stated otherwise, the proof for the remaining propositions, lemma and corollaries will also be found in the Appendix. Proposition 1 shows that a constant monetary aggregate growth rules always deliver a unique REE provided it is based on the true monetary aggregate. This result extends results of Evans &

Honkapohja (2003) to monetary aggregates. This result in of itself is of little use to central banks who do not know the underlying parameters of the monetary aggregate. To circumvent this problem, suppose the central bank fixes the growth rate of the Divisia monetary aggregate instead of the true monetary aggregate developed initially by Barnett (1980).

Definition 2.1. The growth rate of the Divisia monetary aggregate is defined by:

$$\ln(g_t^{divisia}) = \left(\frac{s_t^n + s_{t-1}^n}{2} \right) \ln \left(\frac{n_t}{n_{t-1}} \right) + \left(\frac{s_t^d + s_{t-1}^d}{2} \right) \ln \left(\frac{d_t}{d_{t-1}} \right), \quad (2.2)$$

where s_t^n and s_t^d are the expenditure shares of currency and interest bearing deposits respectively defined by:

$$s_t^n = \frac{(r_t - 1)n_t}{(r_t - 1)n_t + (r_t - r_t^d)d_t} \text{ and } s_t^d = 1 - s_t^n. \quad (2.3)$$

The Divisia monetary aggregate is the expenditure share-weighted growth rate of currency and deposits. In contrast to the true aggregate, the Divisia aggregate is non-parametric and depends only on current and past observable information. Such a rule is more relevant to policy makers as it doesn't require estimating any parameters.⁴

The following lemma shows that for the general CES specification of m_t , the Divisia monetary aggregate (locally) tracks the growth rate of the true monetary aggregate to first order accuracy without error.

Lemma 2.1. *For any $0 < \nu < 1$ and for any $\omega > 0$, the difference between the Divisia monetary aggregate and the true monetary is given by*

$$\ln(g_t^{divisia}) - \Delta \ln(M_t) = \mathcal{O}(\|\xi\|^2).$$

The accuracy properties of the Divisia monetary aggregate are well known in Index number

⁴A central bank could estimate the parameters in equation 4, the monetary aggregator function, but that would induce parameter uncertainty from sampling error, but using Divisia avoids that concern. Moreover, this approach leaves a central bank exposed to questions and criticisms with regards to how they estimate the parameters.

theory. Most notably, Divisia (1926) showed in continuous time, the Divisia monetary aggregate tracks any linearly homogenous function without error. Moreover, Diewert (1976) classified the discrete time Divisia monetary aggregate defined above as superlative - meaning that it can track the growth rate of any twice differentiable linearly homogenous function up to second order accuracy without error. This lemma shows that for the CES function in particular, a *linear approximation* to the Divisia monetary aggregate tracks the true monetary aggregate up to first order accuracy without error. This result is very useful in analyzing local determinacy which requires only a linear approximation to the non-linear model.

Formally, suppose the central bank follows the rule

$$\tilde{g}_t^{divisia} = 0 \quad (2.4)$$

in place of (2.1). The dynamic system consists of (2.1), (2.2), (2.8) and (2.4), and we obtain the following result.

Corollary 2.1. *For any $0 < \beta < 1$, for any $\kappa > 0$ and for any $\eta_r \geq 0$ if the central bank follows the policy rule $\tilde{g}_t^{divisia} = 0$ then there exists a unique REE.*

The proof follows immediately from combining Lemma 1 and Proposition 1. Corollary 1 shows the central bank may replace the true aggregate with the Divisia monetary aggregate without any change in determinacy. This result obtains because determinacy is a local condition and Lemma 1 shows that locally, the Divisia monetary aggregate exactly tracks the growth rate of the true monetary aggregate. Therefore, determinacy properties for growth rules based on the true monetary aggregate are inherited by rules which instead use the Divisia monetary aggregate. However, the same can not be said for the more common simple-sum monetary aggregate.

Definition 2.2. The growth rate of the simple-sum monetary aggregate is defined by

$$\ln(g_t^{simple-sum}) = \ln\left(\frac{n_t + d_t}{n_{t-1} + d_{t-1}}\right). \quad (2.5)$$

The simple-sum aggregate treats currency and interest bearing deposits as one for one perfect substitutes. Standard microeconomic theory dictates in such a case only the cheapest monetary asset would have a positive demand in equilibrium. Since this is not the case in this model, nor reality, it is perhaps not surprising the simple-sum aggregate will locally track the true aggregate with error. The following lemma defines and quantifies this error.

Lemma 2.2. *For any $0 < \nu < 1$, for any $\omega > 0$ and for any $(\bar{\tau}, \bar{x})$ satisfying $(\bar{r} - 1) > (\bar{r} - 1)\bar{\tau} + \bar{x}$ so that $\bar{u}^n > \bar{u}^d > 0$, the difference between the growth rate of the simple-sum monetary aggregate and the growth rate of the true monetary aggregate is given by*

$$\ln(g_t^{simple-sum}) - \Delta \ln(m_t) = \psi_r(\omega)\Delta\tilde{r}_t - \psi_x(\omega)\Delta\tilde{x}_t - \psi_\tau(\omega)\Delta\tilde{\tau}_t + \mathcal{O}(\|\xi\|^2) \quad (2.6)$$

where $\left(\psi_r(\omega), \psi_x(\omega), \psi_\tau(\omega) \right) \in \mathbb{R}_{++}^3$ with $\lim_{\omega \rightarrow \infty} \psi_r(\omega) = \psi_x(\omega) = \psi_\tau(\omega) = 0$.

Intuitively, any change in the relative prices of currency and deposits results in a substitution between these assets from the household. Since the simple-sum monetary aggregate treats these assets as perfect one for one substitutes it is not able to internalize these relative changes in the service flow from the monetary aggregate. Only in the limiting case of perfect substitutes does this error disappear.

From a determinacy standpoint, simple-sum's error in tracking the true monetary aggregate arising from the exogenous financial shocks is irrelevant. However, the error that depends on the endogenous interest rate - $\Delta\tilde{r}_t$ - will influence the determinacy region of a rule utilizing the simple-sum monetary aggregate. In particular, the following corollary summarizes determinacy properties when instead of (2.1) the policy rule is:

$$\tilde{g}_t^{simple-sum} = 0, \quad (2.7)$$

with the dynamic system now consisting of (2.1), (2.2), (2.8) and (2.7).

Corollary 2.2. *For any $0 < \beta < 1$, for any $\kappa > 0$ if the central bank follows the policy rule $\tilde{g}_t^{simple-sum} = 0$ then there exists a unique REE if and only if $\psi_r(\omega) < \eta_r + \frac{1}{2}$.*

Importantly, when using the simple-sum monetary aggregate in place of either the true monetary aggregate or the Divisia monetary aggregate a constant monetary aggregate growth rule may not be determinate. Indeterminacy will occur when the error term $\psi_r(\omega)$ is large relative to the interest semi-elasticity of money demand - η_r . This parameter varies significantly in the empirical literature. For example Ball (2001) estimates $\eta_r = .05$ while Ireland (2009) estimates $\eta_r = 1.9$. Given the wide range of estimates for η_r , central banks may be understandably averse to implementing this Friedman's k-percent rule based on a simple-sum measure. Furthermore, Section 2.5 of this paper presents estimates of $\psi_r(\omega)$ which combined with even the largest plausible estimates of η_r suggest that the determinacy condition in Corollary 2 is unlikely to be satisfied. On the other hand, a k-percent rule with Divisia yields no determinacy concerns whatsoever.

2.4 Interest Rate Rules with Money

In this section we consider interest rate feedback rules which react to lagged interest rates and the growth rate of a monetary aggregate. When Friedman proposed the k-percent rule, he also advocated changes in bank regulation which would make the rule easily implemented. For example, Friedman (1960) argued for increasing the reserve requirements to 100% to make the monetary aggregates more easily controllable. Friedman (1960) went on to say that:

This [constant money growth] is not, under our present System, an easy thing to do. It involves a great many technical difficulties and there will be some deviations from it. If the other changes I suggested were made in the System, it would make the task easier; but even without those changes, it could be done under the present System.

Under a fractional reserve banking system a pragmatic way of targeting the growth rate of money is with an interest rate reaction function whereby the central bank adjusts the interest rate as needed to correct deviations of money growth from the desired k -percent target.

More recently a number of studies have included nominal money growth in interest rate rules such as Canova & Menz (2011), Sims & Zha (2006) and Fahr et al. (2013). Fewer papers have included Divisia monetary aggregates in interest rate feedback rules. One that has is Belongia & Ireland (2012) which estimates structural VAR's with such rules. Keating & Smith (2013) find that interest rate rules that react to Divisia may be optimal from a normative perspective if financial shocks are present and the natural rate of interest is unobservable. This result is consistent with the assertion of McCallum & Nelson (2011) and Andres et al. (2009) that nominal money growth provides valuable real-time information regarding the natural rate of interest. Despite this interest, there are no results on the determinacy properties of such rules to guide policy makers. With this in mind, assume the central bank sets the policy rate according to

$$\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_m \Delta \tilde{m}_t. \quad (2.1)$$

Using (2.1) to close the dynamic model that still contains (2.1), (2.2) and (2.8), we have the following result.

Proposition 2.2. *For any $0 < \beta < 1$, for any $\kappa > 0$ and for any $\eta_r \geq 0$, if the central bank follows the policy rule $\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_m \Delta \tilde{m}_t$ with $\phi_m \geq 0$ and $\phi_r \geq 0$ then there exists a unique REE if and only if $\phi_m + \phi_r > 1$.*

Intuitively, this result extends the Taylor Principle to monetary aggregates. Expectations will remain well-anchored if the central bank reacts more than one for one to nominal aggregate-money growth, in the long-run. Proposition 1 provides a sufficient condition for determinacy that is independent of the magnitude of the interest semi-elasticity of money demand which is important given that the estimates of this parameter vary significantly in the empirical

literature.

Despite the neutrality of the result with regard to η_r , this rule requires that a central bank measure the unobservable true monetary aggregate. To circumvent this issue suppose instead the central bank replaces the true monetary aggregate with the Divisia monetary aggregate

$$\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_d \tilde{g}_t^{divisia}. \quad (2.2)$$

The policy rule in (2.2) could more be easily implemented by a central bank compared to (2.1) as the interest rate is responding to observable information and does not entail the estimation of any deep structural parameters. Ans the following corollary to Proposition 2 shows that a central bank can replace $\Delta \tilde{m}$ with $\tilde{g}_t^{divisia}$ without changing the determinacy condition.

Corollary 2.3. *For any $0 < \beta < 1$, for any $\kappa > 0$ and for any $\eta_r \geq 0$ if the central bank follows the policy rule $\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_d \tilde{g}_t^{divisia}$ with $\phi_d \geq 0$ and $\phi_r \geq 0$ then there exists a unique REE if and only if $\phi_d + \phi_r > 1$.*

The proof of this corollary follows immediately by combining Lemma 1 with Proposition 2. This result is significant in terms of actually implementing interest rate rules reacting to money growth. Most notably, central banks can use the non-parametric Divisia aggregate in interest rate feedback rules like (2.2) and guarantee determinacy by setting $\phi_d > 1$.

The same can not be said, however, for the more common simple-sum monetary aggregate. In particular, the following corollary summarizes determinacy properties when the policy rule in (2.3) is used instead of (2.1) or (2.2)

$$\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_{ss} \tilde{g}_t^{simple-sum}. \quad (2.3)$$

Corollary 2.4. *For any $0 < \beta < 1$, for any $\kappa > 0$ and for any $\eta_r \geq 0$ if the central bank follows the policy rule $\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_{ss} \tilde{g}_t^{simple-sum}$ with $\phi_{ss} \geq 0$ and $\phi_r \geq 0$ then there exists*

a unique REE if and only if $|\phi_r + \phi_{ss} - \phi_{ss}[\psi_r(\omega) - \eta_r]| > |1 - \phi_{ss}[\psi_r(\omega) - \eta_r]|$.

A proof of this derives from Lemma 2 and Proposition 2 and is found in the Appendix. This result highlights how once again simple-sum and Divisia monetary aggregates diverge in terms of determinacy properties. Namely, the determinacy region for interest rate rules reacting to the simple-sum monetary aggregate depend on simple-sum's error in tracking the true monetary aggregate. Intuitively, if $\psi_r(\omega)$ is relatively large compared to the interest semi-elasticity of money demand then an upper and lower bound is placed on ϕ_{ss} for determinacy. Meanwhile, if $\psi_r(\omega)$ is relatively small compared to the interest semi-elasticity of money demand then only a lower bound is placed on ϕ_{ss} for determinacy. For example, in the extreme case where the simple-sum and Divisia aggregates coincide (i.e. $\omega \rightarrow \infty$), Corollary 3 shows that $\phi_{ss} > 1$ is sufficient for determinacy. However, in the more general case, determinacy under the interest rate rule reacting to simple-sum may have a relatively small determinacy region as summarized below for the case in which $\psi_r(\omega) - \eta_r > 1$. The following section of the paper provides an empirical justification for assuming this inequality holds.

To get an idea of how likely determinacy is under an interest rate rule reacting to the simple-sum aggregate Figure 1 plots the determinacy region under the numerical calibration presented below. There are 2 regions of determinacy. Using Leeper's (1991) terminology, one region is in the passive regime while the other is in the active regime. What stands out the most is how unlikely the condition presented in Corollary 4 is satisfied. Under this interest rate rule with simple-sum money determinacy occurs within such a narrow band that it is more the exception than the norm.

2.5 Numerical Analysis

Potentially more realistic policy rules may include not only a response to the growth rate of a monetary aggregate but also a response to inflation. Unfortunately this slightly more complicated rule places analytic results out of reach so we must resort to numerical anal-

Table 2.1: Determinacy Regimes Under
 $\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_{ss} \tilde{g}_t^{simple-sum}$

<i>Regime^a</i>	<i>Determinacy Conditions^b</i>
Passive	$\phi_r + \phi_{ss} < 1$ $\phi_{ss} > \frac{1}{\psi_r(\omega) - \eta_r}$ $\phi_{ss} > \frac{1}{\psi_r(\omega) - \eta_r - 1} \phi_r$ $\phi_{ss} > \frac{1}{2(\psi_r(\omega) - \eta_r) - 1} + \frac{1}{2(\psi_r(\omega) - \eta_r) - 1} \phi_r$
Active I	$\phi_r + \phi_{ss} > 1$ $\phi_{ss} < \frac{1}{\psi_r(\omega) - \eta_r}$ $\phi_{ss} < \frac{1}{\psi_r(\omega) - \eta_r - 1} \phi_r$
Active II	$\phi_r + \phi_{ss} > 1$ $\phi_{ss} > \frac{1}{\psi_r(\omega) - \eta_r}$ $\phi_{ss} < \frac{1}{\psi_r(\omega) - \eta_r - 1} \phi_r$ $\phi_{ss} < \frac{1}{2(\psi_r(\omega) - \eta_r) - 1} + \frac{1}{2(\psi_r(\omega) - \eta_r) - 1} \phi_r$

^a We use the terms "Passive" and "Active" similar to Leeper (1991) to describe a monetary regime where $\phi_r + \phi_{ss} < 1$ or $\phi_r + \phi_{ss} > 1$ respectively.

^b All of these conditions are sufficient for the existence of a unique REE under the conditions stated in Corollary 4 under the additional assumption that $\psi_r(\omega) - \eta_r > 1$. The analysis in the following section shows this to be the empirically relevant case for the U.S. economy.

ysis of determinacy. Many of the parameters in the model are standard so we take their values from the previous literature. We vary the remaining parameters over a reasonable range to understand how determinacy may, or may not, be achieved for the US economy. More precisely, we set $\beta = .99$ and $\kappa = .3$. The two key non-policy parameters relevant for determinacy of the simple-sum rules are η_r - the interest semi-elasticity of money demand and $\psi_r(\omega)$ - simple-sum's endogenous error in tracking the true monetary aggregate. There is a large literature estimating the interest semi-elasticity of money demand. Ball (2001) estimates $\eta_r = .05$ and Ireland (2009) estimates $\eta_r = 1.9$. These important papers provide a reasonable range of parameter values: $\eta_r \in [0.05, 1.9]$.

To gain some insight regarding the size of simple-sum's error term in tracking the true

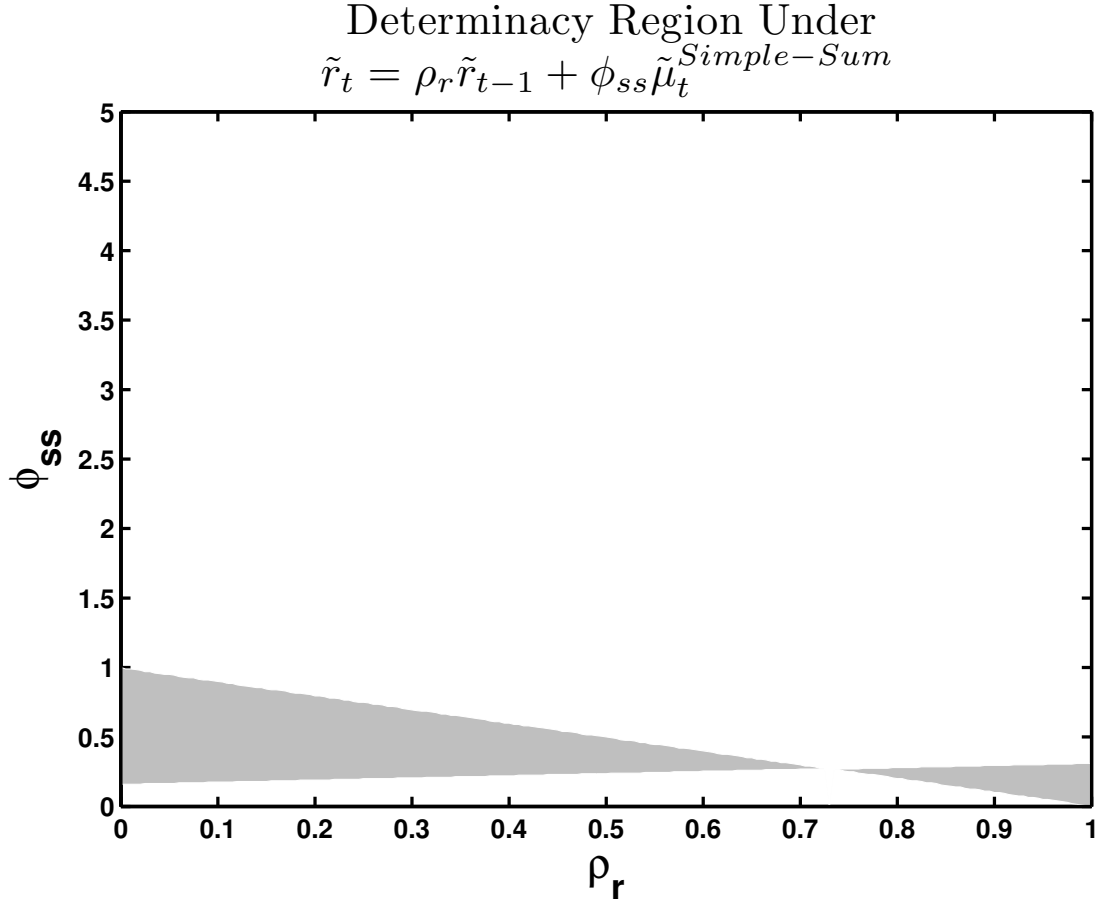


Figure 2.1: The shaded areas is determinate while the white area is indeterminate. The determinacy region above is graphed for $\eta_r = 1.9$ with the other parameters fixed at the values presented in Table 2.3.

monetary aggregate, we interpret

$$\ln(n_t + d_t) - \ln(m_t) = \beta_0 + \beta_1 \tilde{r}_t + \beta_2 \tilde{x}_t + \beta_3 \tilde{\tau}_t + \varepsilon_t \quad (2.1)$$

as a linear econometric relationship with the error in this equation assumed to be a stationary stochastic process. The linear functional relationship is all that is necessary to address determinacy issues. The estimate of β_1 will provide a calibrated value for $\psi_r(\omega)$. We estimate (2.1) using monthly data from 1967:01 to 2012:09 from the St. Louis Fed's FRED database. We use data on simple-sum M2 and the M2 MSI (Divisia) series to form the dependent variable. As for the independent variables we first construct a reserves ratio series - $\tilde{\tau}_t$

- using the St. Louis Adjusted Reserves series and the non-currency components of M2. We then use this and the commercial and industrial loan rate, the M2 own rate and the structural equation in (2.6) to back out a time series for \tilde{x}_t . In this simple model, \tilde{r}_t is simultaneously equal to the loan rate, the benchmark rate and the policy rate. Hence, interpreting \tilde{r}_t empirically is not straightforward. For this reason, we estimate (2.1) twice where first we use the 3-Month Treasury Bill secondary market series for \tilde{r}_t and then we examine the robustness of our estimates by using the commercial and industrial loan rate for \tilde{r}_t . Table 2.2 reveals the point-estimates are surprisingly similar.

Table 2.2: Econometric Estimation of the Simple-Sum Error Term ^a

$\ln(n_t + d_t) - \ln(m_t) = \beta_0 + \beta_1 \tilde{r}_t + \beta_2 \tilde{x}_t + \beta_3 \tilde{\pi}_t + \varepsilon_t$				
1967:01-2012:09	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
	<i>Measuring \tilde{r}_t with the 3-Month T-Bill Rate</i>			
OLS Estimates	-0.2025	11.0377	-0.3444	-0.0509
Standard-Error	0.0164	2.7619	0.0492	0.0164
	<i>Measuring \tilde{r}_t with the C & I Loan Rate</i>			
OLS Estimates	-0.1518	9.8770	-0.3669	-0.0659
Standard-Error	0.0128	2.5236	0.0560	0.0153
1984:01-2007:09	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
	<i>Measuring \tilde{r}_t with the 3-Month T-Bill Rate</i>			
OLS Estimates	-0.0462	5.8694	-0.0589	-0.0289
Standard-Error	0.0019	0.6701	0.0071	0.0079
	<i>Measuring \tilde{r}_t with the C & I Loan Rate</i>			
OLS Estimates	-0.0462	5.6362	-0.0669	-0.0170
Standard-Error	0.0019	0.6427	0.0074	0.0071

^a The standard errors are computed as in Newey & West (1987) using $T^{\frac{1}{4}}$ lags.

The results in Table 2.2 support the theoretical findings in Lemma 2 as the estimated coefficients all have the predicted sign. Moreover, the key parameter for the determinacy of the constant simple-sum growth rule is estimated to be significantly larger than the interest semi-elasticity estimates of Ball (2001) and Ireland (2009) over both the full-sample and

the sub-sample.⁵ To give further credence to the above estimation, the value of $\hat{\beta}_1$ over the 1984:01 - 2007:09 period is nearly identical to the value of $\psi_r(\omega)$ that is implied by the calibration put forth in Belongia & Ireland (2014). For our numerical analysis we use the smaller estimate of obtained from this sample period.

It is immediately clear that over the range of values in Table 2.3 the necessary and sufficient condition for the determinacy of the constant simple-sum growth rule in Corollary 2 is not met. Moreover, Figure 2.1 above plots the determinacy region under the interest rate rule reacting to the lagged interest rate and the growth rate of simple-sum and shows the determinacy region comprises a very small percentage of the parameter space.

Table 2.3: Calibrated Parameters

<i>Parameter</i>	<i>Calibrated Value</i>
β	.99
κ	.30
$\psi_r(\omega)$	5.6
η_r	[0.05, 1.9]
ϕ_r	[0, 1]
ϕ_d	[0, 5]
ϕ_{ss}	[0, 5]
ϕ_π	[0, 5]

However, more realistic policy rules focus on inflation as well. A question of more practical interest may be: How does including a response to the growth rate of money affect the well known determinacy properties of interest rate rules reacting to the lagged interest rate and inflation? Concretely, we consider rules such as

$$\tilde{r} = \phi_r \tilde{r}_{t-1} + \phi_m \Delta \tilde{m}_t + \phi_\pi \tilde{\pi}_t \quad (2.2)$$

⁵The full sample estimates include heterogeneous monetary regimes and the financial crisis. For this reason, we also estimate equation (2.1) over a sub-sample which roughly captures the *Great Moderation* and stops just before the crisis. The Fed's policy over that sub-period can be characterized by a predictable interest rate rule with a low inflation target.

where we will once again focus on implementable rules which replace the growth rate of the true monetary aggregate with either the growth rate of the Divisia monetary aggregate or the growth rate of simple-sum. The main results of this section further emphasize the importance of using the Divisia monetary aggregate in policymaking as the determinacy regions are large and invariant to non-policy parameters such as the interest semi-elasticity of money demand, in contrast rules based on simple-sum money. Importantly, including a response to the growth rate of the Divisia monetary aggregate doesn't destabilize the economy by inducing indeterminacy in an already determinate rule based on the true aggregate. The same can not be said for the simple-sum aggregate.

Interest Rate Rules Reacting to the Lagged Interest Rate, Divisia and Inflation

First consider rules which replace the the true aggregate in (2.2) with the Divisia monetary aggregate.⁶ We find the results in Corollary 3 above extend to a rule reacting to inflation as well. By adjusting the nominal rate more than 1 for 1 to changes in the growth rate of the Divisia monetary aggregate and inflation in the long run, the central bank can bring about a unique REE as Figure 2.2 concisely shows. We summarize this in the result below.

Conjecture 2.1. *For any $\phi_r > 0$, $\phi_d > 0$ and $\phi_\pi > 0$, if the central bank follows the policy rule $\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_d \tilde{g}_t^{divisia} + \phi_\pi \tilde{\pi}_t$ then there exists a unique REE if and only if $\phi_r + \phi_d + \phi_\pi > 1$.*

We present this as a conjecture since we are only able to verify this statement numerically. One interpretation of this result is that including a response to the growth rate of the Divisia monetary aggregate in a policy rule reacting to inflation and lagged interest rates will not disrupt the determinacy properties of the original rule. Keating & Smith (2013) provide reason to consider putting Divisia into a Taylor Rule. They show that adding a response to the growth rate of the Divisia monetary aggregate in that rule is welfare enhancing as it

⁶Of course, Lemma 1 implies the determinacy properties are equivalent under both the true aggregate and the Divisia monetary aggregate.

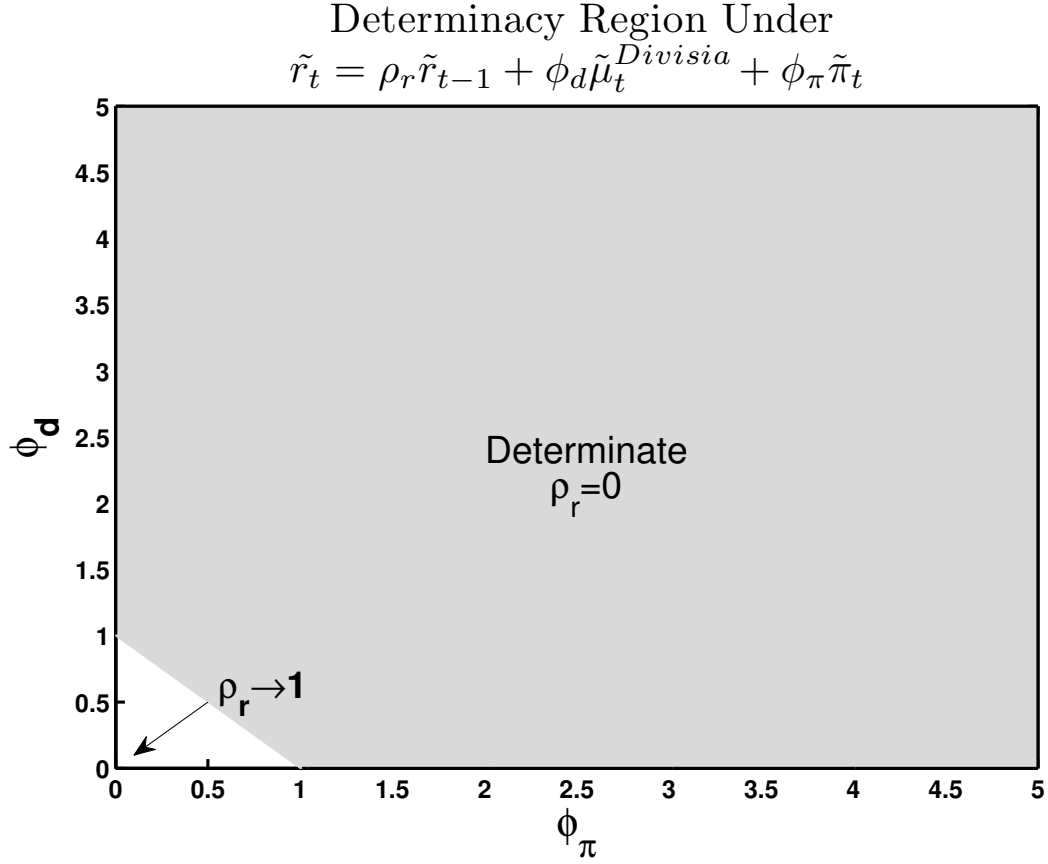


Figure 2.2: The shaded area is determinate while the white area is indeterminate. The determinacy region above is found to hold for all values of $0 < \beta < 1$, $\kappa > 0$ and $\eta_r \geq 0$.

provides information regarding the unobservable natural rate of interest. Here we stress that such a change in policy will not destabilize the economy since we can ensure the existence of a unique REE under this rule. And this determinacy condition represents a modification to the well-known Taylor Principle.

Interest Rate Rules Reacting to the Lagged Interest Rate, simple-sum and Inflation

Now we analyze determinacy properties of nominal interest rate rules reacting to inflation as well as the growth rate of simple-sum and lagged interest rates. Conventional wisdom suggests that reacting aggressively to inflation will stabilize the economy by ensuring a unique REE - the so called Taylor principle. Although this is true in this model, we find that

augmenting an interest rate rule which satisfies the Taylor principle with a positive response to the growth rate of simple-sum may destabilize the economy by inducing indeterminacy. Simply put, reacting to the growth rate of simple-sum is likely to be destabilizing regardless

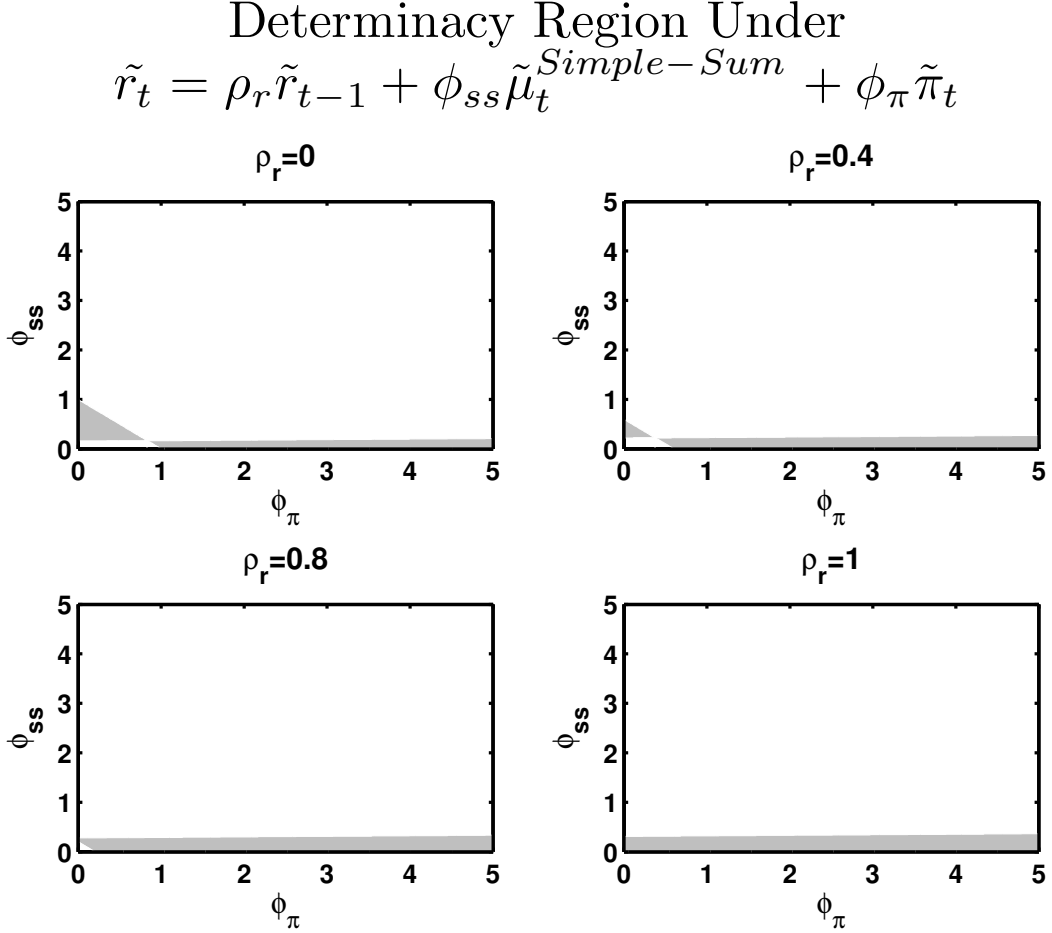


Figure 2.3: The shaded areas are determinate while the white area is indeterminate. The determinacy region above are evaluated for the estimated value of $\psi_r(\omega) = 5.6$.

of the inflation or lagged interest rate response. Figure 2.3 displays this by plotting the determinacy region for the policy rule

$$\tilde{r} = \phi_r \tilde{r}_{t-1} + \phi_{ss} \tilde{g}_t^{simple-sum} + \phi_\pi \tilde{\pi}_t. \quad (2.3)$$

Comparing Figures 2.2 and 2.3 highlights the difference between placing the growth rate of the Divisia aggregate and the simple-sum aggregate in an interest rate reaction function.

Most noticeably, when the Divisia aggregate is embedded in the interest rate rule the determinacy regions are large and independent of non-policy parameters such as the interest semi-elasticity of money demand. On the other hand, when the simple-sum aggregate is placed in the interest rate reaction function the economy is more likely to be indeterminate than determinate. Moreover, the determinacy regions depend critically on non-policy parameters including the interest semi-elasticity of money demand and the portion of simple-sum's error related to the policy rate - $\psi_r(\omega)$. To emphasize this point, Figure 2.4 plots the determinacy regions under the policy defined by (2.3) and $\psi_r(\omega) = 3$, which is nearly three standard deviations below the estimated value.

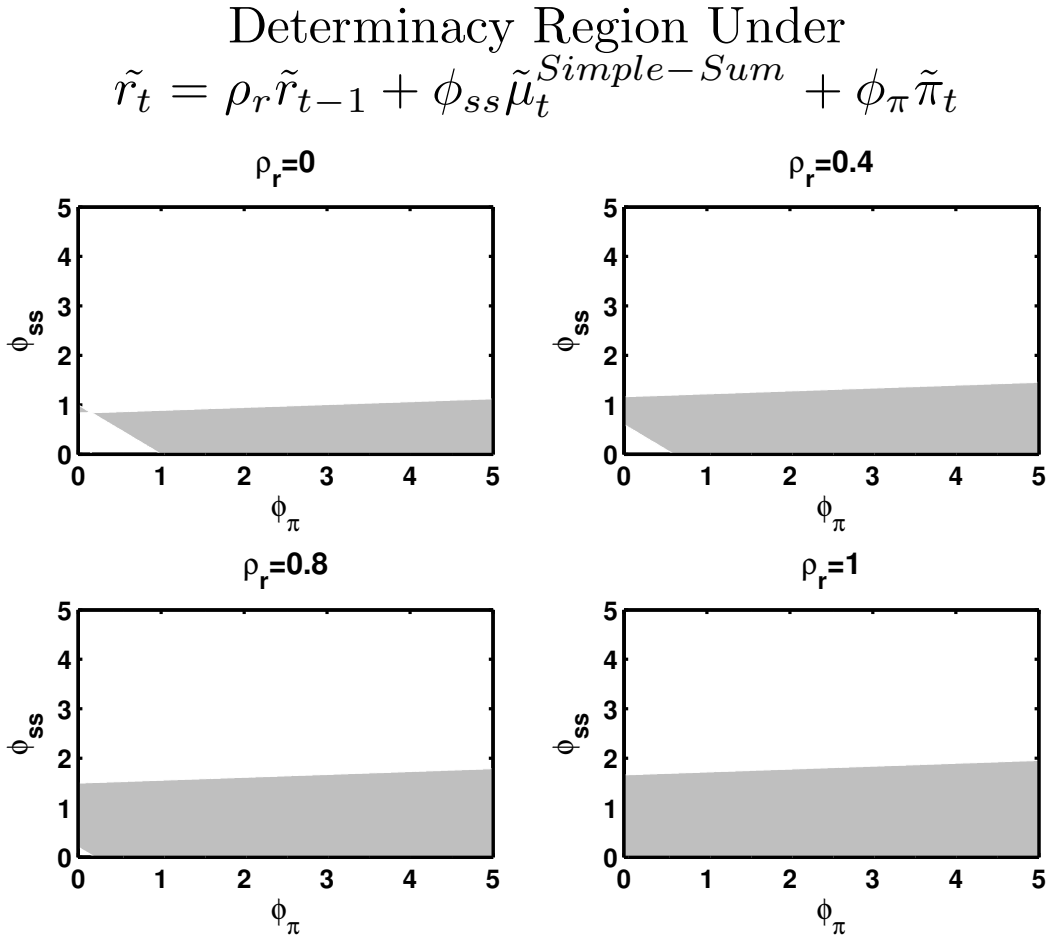


Figure 2.4: The shaded areas are determinate while the white area is indeterminate. The determinacy region above are evaluated for $\psi_r(\omega) = 3$, nearly three standard deviations below the estimated value.

Figure 2.4 highlights how sensitive the determinacy regions are to this parameter. Very similar results are found if we again set $\psi_r(\omega) = 5.6$ and analyze determinacy under a much larger value of $\eta_r = 3.8$, twice the estimated value from Ireland (2009). This suggests what matters most for the determinacy properties of simple-sum oriented policy rules is not the policy parameters, but the size of that aggregate's error $\psi_r(\omega)$ relative to the interest semi-elasticity η_r . Also, obeying the Taylor Principle when reacting to the simple-sum will frequently not result in a determinate equilibrium.

2.6 Conclusion

The ability to achieve a unique rational expectations equilibrium is a fundamental issue for any policy rule. We show that Friedman's k-percent rule is determinate so long as the monetary aggregate is measured accurately. Ironically if the recommended aggregate of Friedman, a broad simple-sum measure, is used in his k-percent rule the economy is likely to be unstable due to self-fulfilling expectations. This problem can be remedied in practice by using the non-parametric Divisia monetary aggregate in the k-percent rule. To be fair to Friedman, he was aware of flaws in the simple-sum aggregation method and even hinted at something akin to the Divisia index in Friedman & Schwartz (1971), pp. 151-152:

This [simple-summation] procedure is a very special case of the more general approach. In brief, the general approach consists of regarding each asset as a joint product having different degrees of 'moneyness,' and defining the quantity of money as the weighted sum of the aggregated value of all assets, the weights for individual assets varying from zero to unity with a weight of unity assigned to that asset or assets regarded as having the largest quantity of 'moneyness' per dollar of aggregate value. The procedure we have followed implies that all weights are either zero or unity.

In Taylor-type rules in which the interest rate may react Divisia, we find that determinacy arises under a variant of the Taylor Principle. On the other hand when the interest rate

may respond to simple-sum the Taylor Principle no longer guarrentess determinacy and the regions of determinacy are substantially smaller.

The paper’s results provide an alternative perspective on how "measurement matters" (Belongia, 1996). Our contribution is to show that the choice of monetary aggregate in a policy rule may have a substantive effect on the likelihood that monetary policy will be able to elicit a unique equilibrium outcome for an economy. Placed in the larger literature stressing the differences between Divisia and simple-sum monetary aggregates,⁷ the case against the use of monetary aggregates in policy making rests on the failures of simple-sum monetary aggregates.

As an example, Barnett & Chauvet (2011b) highlight how the growth rate of simple-sum and Divisia monetary aggregates differed during the Monetarist Experiment.

Table 2.4: Money Growth Rates During the Period November 1979 to August 1982

Monetary Aggregate	Mean Growth Rate
Divisia M2	4.5
simple-sum M2	9.3
Divisia M3	4.8
simple-sum M3	10.0

The growth rates of simple-sum aggregates were above Fed’s targets, while the more accurate Divisia measures were growing at substantially lower rates.⁸ Inflation fell quickly, but over-tightening led to a more severe recession than was likely needed or desired. When combined with the theoretical results in this paper, a natural question arises: Had accurate measures of broad money been used (such as the Divisia quantity aggregates) during the Monetarist Experiment, would monetary aggregates have fallen from favor among central bankers?

⁷For more general research examining the Divisia monetary aggregate’s properties relative to alternative simple-sum measures see the following works. At paper length Barnett & Chauvet (2011b); Belongia (1996), Barnett et al. (1984) and at book length Barnett & Singleton (1987); Belongia & Binner (2000); Barnett & Serletis (2000); Barnett & Chauvet (2011a); Barnett (2012).

⁸The Federal Reserve Bulletin reported target ranges.

References

- Andres, J., Lopez-Salido, J. D., & Nelson, E. (2009). Money and the natural rate of interest: Structural estimates for the united states and the euro area. *Journal of Economic Dynamics and Control*, 33(3), 758–776.
- Ball, L. (2001). Another look at long-run money demand. *Journal of Monetary Economics*, 47(1), 31–44.
- Barnett, W. A. (1980). Economic monetary aggregates an application of index number and aggregation theory. *Journal of Econometrics*, 14(1), 11–48.
- Barnett, W. A. (2012). *Getting it Wrong: How Faulty Monetary Statistics Undermine the Fed, the Financial System, and the Economy*, volume 1 of *MIT Press Books*. The MIT Press.
- Barnett, W. A. & Chauvet, M. (2011a). *Financial Aggregation And Index Number Theory*, volume 2 of *World Scientific Books*. World Scientific Publishing Co. Pte. Ltd.
- Barnett, W. A. & Chauvet, M. (2011b). How better monetary statistics could have signaled the financial crisis. *Journal of Econometrics*, 161(1), 6–23.
- Barnett, W. A., Offenbacher, E. K., & Spindt, P. A. (1984). The New Divisia Monetary Aggregates. *Journal of Political Economy*, 92(6), 1049–85.
- Barnett, W. A. & Serletis, A. (2000). *The Theory of Monetary Aggregation*. Contributions to Economic Analysis. Elsevier.

- Barnett, W. A. & Singleton, K. J. (1987). *The microeconomic theory of monetary aggregation*. Cambridge University Press.
- Belongia, M. T. (1996). Measurement matters: Recent results from monetary economics reexamined. *Journal of Political Economy*, 104(5), 1065–83.
- Belongia, M. T. & Binner, J. M. (2000). *Divisia Monetary Aggregates: Theory and Practice*. Palgrave.
- Belongia, M. T. & Ireland, P. N. (2012). *Quantitative Easing: Interest Rates and Money in the Measurement of Monetary Policy*. Boston College Working Papers in Economics 801, Boston College Department of Economics.
- Belongia, M. T. & Ireland, P. N. (2014). The barnett critique after three decades: A new keynesian analysis. *Journal of Econometrics*, forthcoming.
- Canova, F. & Menz, T. (2011). Does money matter in shaping domestic business cycles? an international investigation. *Journal of Money, Credit and Banking*, 43(4), 577–607.
- Carlstrom, C. T. & Fuerst, T. S. (1995). Interest rate rules vs. money growth rules a welfare comparison in a cash-in-advance economy. *Journal of Monetary Economics*, 36(2), 247–267.
- Carlstrom, C. T. & Fuerst, T. S. (2003). Money Growth Rules and Price Level Determinacy. *Review of Economic Dynamics*, 6(2), 263–275.
- Diewert, W. E. (1976). Exact and superlative index numbers. *Journal of Econometrics*, 4(2), 115–145.
- Divisia, F. (1926). L’indice monetaire et la theorie de la monnaie. *Revue d’econ polit*, LX(01), 49–81.
- Evans, G. W. & Honkapohja, S. (2003). Friedman’s money supply rule vs. optimal interest rate policy. *Scottish Journal of Political Economy*, 50(5), 550–566.

- Fahr, S., Motto, R., Rostagno, M., Smets, F., & Tristani, O. (2013). A monetary policy strategy in good and bad times: lessons from the recent past. *Economic Policy*, 28(74), 243–288.
- Friedman, M. (1960). *A Program for Monetary Stability*. Fordham University Press.
- Friedman, M. & Schwartz, A. J. (1971). *A Monetary History of the United States, 1867-1960*. Princeton University Press.
- Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.
- Ireland, P. N. (1996). The Role of Countercyclical Monetary Policy. *Journal of Political Economy*, 104(4), 704–23.
- Ireland, P. N. (2009). On the welfare cost of inflation and the recent behavior of money demand. *American Economic Review*, 99(3), 1040–52.
- Keating, J. & Smith, A. L. (2013). *Price Versus Financial Stability: A role for money in Taylor rules?* WORKING PAPERS SERIES IN THEORETICAL AND APPLIED ECONOMICS 201307, University of Kansas, Department of Economics.
- Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics*, 27(1), 129–147.
- McCallum, B. T. (1998). Solutions to linear rational expectations models: a compact exposition. *Economics Letters*, 61(2), 143–147.
- McCallum, B. T. & Nelson, E. (2011). Money and inflation: Some critical issues. In B. M. Friedman & M. Woodford (Eds.), *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics* chapter 3, (pp. 97–153). Elsevier.
- Newey, W. K. & West, K. D. (1987). A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, 55(3), 703–08.

Sims, C. A. & Zha, T. (2006). Were there regime switches in u.s. monetary policy? *American Economic Review*, 96(1), 54–81.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

2.A Appendix: Proofs

In this section we present the proofs to our results stated in the paper. The equilibrium model we use in this paper is based on the model described in Belongia & Ireland (2014), however we assume additively separable preferences between consumption and monetary services allowing us to obtain analytic results for the determinacy regions. Also, to arrive at the standard dynamic IS and NKPC, we slightly alter the timing of the transactions for the representative household.

Proposition 2.1

Consider the dynamic system defined by equations (2.1), (2.2), (2.8) and (2.1),

$$AE_t[\tilde{w}_{t+1}] = B\tilde{w}_t + Cs_t.$$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \eta_r & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \kappa & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

where $\tilde{w}_t = \begin{bmatrix} \tilde{y}_t^e, & \tilde{\pi}_t, & \tilde{r}_{t-1}, & \tilde{l}_{t-1} \end{bmatrix}^T$ and where $\tilde{l}_t = m_t - p_t$ is real money balances. The system has two non-predetermined variables and two predetermined variables. Therefore, the system will have a unique rational expectations equilibrium (REE) if, and only if, two roots λ which satisfy $|B - \lambda A| = 0$ lie outside the unit circle and remaining two roots lie

inside the unit circle McCallum (1998). The generalized eigenvalues of the matrix pencil $B - \lambda A$ are given by

$$\Lambda = \begin{bmatrix} 0 \\ \frac{1}{2} \frac{\beta+1+\kappa-\sqrt{(\beta-1)^2+2\beta\kappa+2\kappa+\kappa^2}}{\beta} \\ \frac{1+\eta_r}{\eta_r} \\ \frac{1}{2} \frac{\beta+1+\kappa+\sqrt{(\beta-1)^2+2\beta\kappa+2\kappa+\kappa^2}}{\beta} \end{bmatrix}. \quad (2.A.1)$$

The proof to Proposition 1 follows from (A.1) since $|\lambda_i| < 1$ for $i \in \{1, 2\}$, $|\lambda_4| > 1$ and finally $|\lambda_3| > 1$ since $\eta_r \geq 0$ by assumption.

Lemma 2.1

From Definition 1, we have the growth rate of the Divisia monetary aggregate is given by

$$\ln(g_t^{Divisia}) = \left(\frac{s_t^n + s_{t-1}^n}{2} \right) \ln \left(\frac{n_t}{n_{t-1}} \right) + \left(\frac{s_t^d + s_{t-1}^d}{2} \right) \ln \left(\frac{d_t}{d_{t-1}} \right).$$

Now derive the factor demands for n_t and d_t given the CES monetary aggregate defined in (2.4). These factor demands can be derived from a two-stage budgeting problem where in the first stage the household chooses how much of their budget to allocate towards monetary services m_t at price u_t and in the second stage the household chooses how to allocate their monetary expenditures between n_t and d_t . Suppose the household chooses to allocate $m_t u_t = Z$ towards monetary services. Then the second stage problem is to:

$$\max_{n_t, d_t} m_t \quad \text{subject to} \quad u_t^n n_t + u_t^d d_t = Z \text{ and (2.4).}$$

The resulting factor demands are given by:

$$n_t = \nu m_t \left(\frac{u_t}{u_t^n} \right)^\omega \quad d_t = (1 - \nu) m_t \left(\frac{u_t}{u_t^d} \right)^\omega. \quad (2.A.2)$$

which satisfy the constraint $u_t^n n_t + u_t^d d_t = u_t m_t$ for

$$u_t = \left[\nu \left(\frac{r_t - 1}{r_t} \right)^{1-\omega} + (1-\nu) \left(\frac{r_t - r_t^d}{r_t} \right)^{1-\omega} \right]^{\frac{1}{1-\omega}}.$$

Substituting these factor demands into the definition of the Divisia monetary aggregate, we obtain the following useful expression for the difference between the growth rates of the Divisia monetary aggregate and the true monetary aggregate:

$$\begin{aligned} & \ln(g_t^{divisia}) - \Delta \ln(m_t) \\ &= \omega \left[\Delta \ln(u_t(u_t^n, u_t^d)) - \frac{1}{2} (s_t^n + s_{t-1}^n) \Delta \ln(u_t^n) - \frac{1}{2} (s_t^d + s_{t-1}^d) \Delta \ln(u_t^d) \right] \\ &\equiv E^{Div}(\ln(u_t^n), \ln(u_t^d), \ln(u_{t-1}^n), \ln(u_{t-1}^d)), \end{aligned}$$

where E^{Div} is the error-term which quantifies the difference between the Divisia monetary aggregate and the true monetary aggregate.

To verify the claim, take a first-order Taylor approximation of E^{Div} around the steady-state, hence all derivatives below are evaluated at the steady-state.

$$\begin{aligned} E^{Div} &= \left[\frac{\partial E^{Div}}{\partial \ln(u_t^n)} \right] \tilde{u}_t^n + \left[\frac{\partial E^{Div}}{\partial \ln(u_t^d)} \right] \tilde{u}_t^d + \left[\frac{\partial E^{Div}}{\partial \ln(u_{t-1}^n)} \right] \tilde{u}_{t-1}^n + \left[\frac{\partial E^{Div}}{\partial \ln(u_{t-1}^d)} \right] \tilde{u}_{t-1}^d + \mathcal{O}(\|\xi\|^2) \\ &= \left[\frac{\partial E^{Div}}{\partial \ln(u_t^n)} \right] \tilde{u}_t^n + \left[\frac{\partial E^{Div}}{\partial \ln(u_t^d)} \right] \tilde{u}_t^d - \left[\frac{\partial E^{Div}}{\partial \ln(u_t^n)} \right] \tilde{u}_{t-1}^n - \left[\frac{\partial E^{Div}}{\partial \ln(u_t^d)} \right] \tilde{u}_{t-1}^d + \mathcal{O}(\|\xi\|^2) \\ &= \left[\frac{\partial E^{Div}}{\partial \ln(u_t^n)} \right] \Delta \tilde{u}_t^n + \left[\frac{\partial E^{Div}}{\partial \ln(u_t^d)} \right] \Delta \tilde{u}_t^d + \mathcal{O}(\|\xi\|^2) \end{aligned}$$

Finally, notice that

$$\begin{aligned}
\left[\frac{\partial E^{Div}}{\partial \ln(u_t^n)} \right] &= \left[\frac{\partial \ln(u_t)}{\partial \ln(u_t^n)} - \frac{1}{2}(s_t^n + s_{t-1}^n) \right] \\
&= \frac{\nu (\bar{u}^n)^{1-\omega}}{\nu (\bar{u}^n)^{1-\omega} + (1-\nu) (\bar{u}^d)^{1-\omega}} - \frac{\nu (\bar{u}^n)^{1-\omega}}{(\bar{u})^{1-\omega}} = 0
\end{aligned}$$

and

$$\begin{aligned}
\left[\frac{\partial E^{Div}}{\partial \ln(u_t^d)} \right] &= \left[\frac{\partial \ln(u_t)}{\partial \ln(u_t^d)} - \frac{1}{2}(s_t^d + s_{t-1}^d) \right] \\
&= \frac{(1-\nu) (\bar{u}^d)^{1-\omega}}{\nu (\bar{u}^n)^{1-\omega} + (1-\nu) (\bar{u}^d)^{1-\omega}} - \frac{(1-\nu) (\bar{u}^d)^{1-\omega}}{(\bar{u})^{1-\omega}} = 0
\end{aligned}$$

which verifies our claim since we have $\ln(g_t^{divisia}) - \Delta \ln(m_t) = \mathcal{O}(\|\xi\|^2)$.

Lemma 2.2

Much like the proof of Lemma 1, substitute the factor demands for currency and deposits defined in equation (2.A.2) into the definition of the simple-sum aggregate in definition 2 to arrive at

$$\begin{aligned}
&\ln(g_t^{simple-sum}) - \Delta \ln(m_t) \\
&= \omega \Delta \ln(u_t(u_t^n, u_t^d)) + \Delta \ln(\nu (u_t^n)^{-\omega} + (1-\nu) (u_t^d)^{-\omega}) \\
&\equiv E^{SS}(\ln(u_t^n), \ln(u_t^d), \ln(u_{t-1}^n), \ln(u_{t-1}^d)),
\end{aligned}$$

where E^{SS} is the error-term which quantifies the difference between the simple-sum monetary aggregate and the true monetary aggregate.

To verify the claim, take a first-order Taylor approximation of E^{SS} around the steady-state,

hence all derivatives below are evaluated at the steady-state.

$$\begin{aligned}
E^{SS} &= \left[\frac{\partial E^{SS}}{\partial \ln(u_t^n)} \right] \tilde{u}_t^n + \left[\frac{\partial E^{SS}}{\partial \ln(u_t^d)} \right] \tilde{u}_t^d + \left[\frac{\partial E^{SS}}{\partial \ln(u_{t-1}^n)} \right] \tilde{u}_{t-1}^n + \left[\frac{\partial E^{SS}}{\partial \ln(u_{t-1}^d)} \right] \tilde{u}_{t-1}^d + \mathcal{O}(\|\xi\|^2) \\
&= \left[\frac{\partial E^{SS}}{\partial \ln(u_t^n)} \right] \tilde{u}_t^n + \left[\frac{\partial E^{SS}}{\partial \ln(u_t^d)} \right] \tilde{u}_t^d - \left[\frac{\partial E^{SS}}{\partial \ln(u_t^n)} \right] \tilde{u}_{t-1}^n - \left[\frac{\partial E^{SS}}{\partial \ln(u_t^d)} \right] \tilde{u}_{t-1}^d + \mathcal{O}(\|\xi\|^2) \\
&= \left[\frac{\partial E^{SS}}{\partial \ln(u_t^n)} \right] \Delta \tilde{u}_t^n + \left[\frac{\partial E^{SS}}{\partial \ln(u_t^d)} \right] \Delta \tilde{u}_t^d + \mathcal{O}(\|\xi\|^2) \\
&= \omega \left[\frac{\partial \ln(u_t)}{\partial \ln(u_t^n)} - \frac{\partial \ln(\nu(u_t^n)^{-\omega} + (1-\nu)(u_t^d)^{-\omega})}{\partial \ln(u_t^n)} \right] \Delta \tilde{u}_t^n \\
&+ \omega \left[\frac{\partial \ln(u_t)}{\partial \ln(u_t^d)} - \frac{\partial \ln(\nu(u_t^n)^{-\omega} + (1-\nu)(u_t^d)^{-\omega})}{\partial \ln(u_t^d)} \right] \Delta \tilde{u}_t^d + \mathcal{O}(\|\xi\|^2) \\
&= \omega \left[\bar{s}^n - \frac{\nu(\bar{u}^n)^{-\omega}}{\nu(\bar{u}^n)^{-\omega} + (1-\nu)(\bar{u}^d)^{-\omega}} \right] \Delta \tilde{u}_t^n \\
&+ \omega \left[\bar{s}^d - \frac{(1-\nu)(\bar{u}^d)^{-\omega}}{\nu(\bar{u}^n)^{-\omega} + (1-\nu)(\bar{u}^d)^{-\omega}} \right] \Delta \tilde{u}_t^d + \mathcal{O}(\|\xi\|^2) \\
&= \omega \left[\frac{\nu(\bar{u}^n)^{1-\omega}}{\nu(\bar{u}^n)^{1-\omega} + (1-\nu)(\bar{u}^d)^{1-\omega}} - \frac{\nu(\bar{u}^n)^{-\omega}}{\nu(\bar{u}^n)^{-\omega} + (1-\nu)(\bar{u}^d)^{-\omega}} \right] \Delta \tilde{u}_t^n \\
&+ \omega \left[\frac{(1-\nu)(\bar{u}^d)^{1-\omega}}{\nu(\bar{u}^n)^{1-\omega} + (1-\nu)(\bar{u}^d)^{1-\omega}} - \frac{(1-\nu)(\bar{u}^d)^{-\omega}}{\nu(\bar{u}^n)^{-\omega} + (1-\nu)(\bar{u}^d)^{-\omega}} \right] \Delta \tilde{u}_t^d + \mathcal{O}(\|\xi\|^2)
\end{aligned}$$

Let $\alpha = \bar{u}^d / \bar{u}^n \in (0, 1)$, then we have

$$\begin{aligned}
E^{SS} &= \omega \left[\frac{\nu(\alpha)^{\omega-1}}{\nu(\alpha)^{\omega-1} + (1-\nu)} - \frac{\nu(\alpha)^\omega}{\nu(\alpha)^\omega + (1-\nu)} \right] \Delta \tilde{u}_t^n \\
&+ \omega \left[\frac{(1-\nu)}{\nu(\alpha)^{\omega-1} + (1-\nu)} - \frac{(1-\nu)}{\nu(\alpha)^\omega + (1-\nu)} \right] \Delta \tilde{u}_t^d + \mathcal{O}(\|\xi\|^2) \\
&= \omega \left[\frac{\nu(\alpha)^{\omega-1}(\nu(\alpha)^\omega + (1-\nu)) - \nu(\alpha)^\omega(\nu(\alpha)^{\omega-1} + (1-\nu))}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \Delta \tilde{u}_t^n \\
&+ \omega \left[\frac{(1-\nu)(\nu(\alpha)^\omega + (1-\nu)) - (1-\nu)(\nu(\alpha)^{\omega-1} + (1-\nu))}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \Delta \tilde{u}_t^d + \mathcal{O}(\|\xi\|^2).
\end{aligned}$$

Further simplifying this expression, we have:

$$\begin{aligned}
E^{SS} &= \omega \left[\frac{\alpha^\omega [(\alpha)^{-1} \nu(1-\nu) - \nu(1-\nu)]}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \left[\frac{1}{\bar{r}-1} - \left(\frac{\bar{\tau} - \bar{x}}{(\bar{r}-1)(\bar{\tau}(\bar{r}-1) + \bar{x})} \right) \right] \Delta \tilde{r}_t \\
&\quad - \omega \left[\frac{\alpha^\omega [(\alpha)^{-1} \nu(1-\nu) - \nu(1-\nu)]}{(\nu(\alpha)^\omega + (1-\nu))(\nu(\alpha)^{\omega-1} + (1-\nu))} \right] \left[\left(\frac{\bar{\tau}}{\alpha} \right) \Delta \tilde{r}_t + \left(\frac{\bar{x}}{(\bar{r}-1)\bar{\tau} + \bar{x}} \right) \Delta \tilde{x}_t \right] + \mathcal{O}(\|\xi\|^2) \\
&\equiv \psi(\omega) \left[\frac{\bar{r}\bar{x}}{(\bar{r}-1)(\bar{\tau}(\bar{r}-1) + \bar{x})} \right] \Delta \tilde{r}_t - \psi(\omega) \left(\frac{\bar{\tau}}{\alpha} \right) \Delta \tilde{r}_t - \psi(\omega) \left(\frac{\bar{x}}{(\bar{r}-1)\bar{\tau} + \bar{x}} \right) \Delta \tilde{x}_t + \mathcal{O}(\|\xi\|^2) \\
&\equiv \psi_r(\omega) \Delta \tilde{r}_t - \psi_\tau(\omega) \Delta \tilde{r}_t - \psi_x(\omega) \Delta \tilde{x}_t + \mathcal{O}(\|\xi\|^2).
\end{aligned}$$

To complete the proof, first notice that since $(\bar{\tau}, \bar{x}) > 0$ we have that $\psi_r(\omega) > 0$ and moreover since $\psi(\omega) > 0$ we have that $(\psi_\tau(\omega), \psi_x(\omega)) \gg (0, 0)$. Finally, to obtain the limiting result apply L'Hospital's rule to the function

$$h(\omega) = \omega e^{\omega \ln(\alpha)} = \frac{\omega}{e^{-\omega \ln(\alpha)}}$$

to find that $\lim_{\omega \rightarrow \infty} h(\omega) = 0$ which implies $\lim_{\omega \rightarrow \infty} \psi(\omega) = 0$ verifying the claim.

Corollary 2.2

By Lemma 2, the constant simple-sum growth rule implies a policy rule of the form

$$0 = \Delta \tilde{g}_t^{\text{simple-sum}} = \Delta \tilde{m}_t + \psi_r(\omega) \Delta \tilde{r}_t - \psi_\tau(\omega) \Delta \tilde{r}_t - \psi_x(\omega) \Delta \tilde{x}_t.$$

We can rearrange this expression as an interest rate feedback rule.

$$\tilde{r}_t = \tilde{r}_{t-1} - \frac{1}{\psi_r(\omega)} \Delta \tilde{m}_t - \frac{\psi_\tau(\omega)}{\psi_r(\omega)} \Delta \tilde{r}_t - \frac{\psi_x(\omega)}{\psi_r(\omega)} \Delta \tilde{x}_t \quad (2.A.3)$$

This rule is part of a more general class of rules studied in section 4. More generally, consider rules of the form

$$\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_m \Delta \tilde{m}_t$$

which is equation (2.1) where we drop the stochastic terms from (2.A.3) since they have no impact on the determinacy properties. The dynamic system consisting of (2.1), (2.2), (2.8) and (2.A.3) can be expressed as

$$AE_t[\tilde{w}_{t+1}] = B\tilde{w}_t + Cs_t.$$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -\beta & 0 & 0 \\ 0 & 0 & 1 & -\phi_m \\ 0 & 0 & \eta_r & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \kappa & -1 & 0 & 0 \\ 0 & \phi_m & \phi_r & -\phi_m \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

where $\tilde{w}_t = \begin{bmatrix} \tilde{y}_t^e, & \tilde{\pi}_t, & \tilde{r}_{t-1}, & \tilde{l}_{t-1} \end{bmatrix}^T$ and where $\tilde{l}_t = m_t - p_t$ is real money balances. The system has two non-predetermined variables and two predetermined variables. Therefore, the system will have a unique rational expectations equilibrium (REE) if, and only if, two roots λ which satisfy $|B - \lambda A| = 0$ lie outside the unit circle and remaining two roots lie inside the unit circle McCallum (1998). The generalized eigenvalues of the matrix pencil $B - \lambda A$ are given by

$$\Lambda = \begin{bmatrix} 0 \\ \frac{1}{2} \frac{\beta+1+\kappa-\sqrt{(\beta-1)^2+2\beta\kappa+2\kappa+\kappa^2}}{\beta} \\ \frac{\phi_r+\phi_m+\eta_r\phi_m}{1+\eta_r\phi_m} \\ \frac{1}{2} \frac{\beta+1+\kappa+\sqrt{(\beta-1)^2+2\beta\kappa+2\kappa+\kappa^2}}{\beta} \end{bmatrix}. \quad (2.A.4)$$

The proof for Corollary 2 follows by letting $\phi_r = 1$ and $\phi_m = -\frac{1}{\psi_r(\omega)}$. In which case we have that $|\lambda_i| < 1$ for $i \in \{1, 2\}$, $|\lambda_4| > 1$ and finally $|\lambda_3| > 1$ if and only if

$$\left| \frac{\psi_r(\omega) - \eta_r - 1}{\psi_r(\omega) - \eta_r} \right| > 1.$$

This condition will be satisfied if and only if $\psi_r(\omega) < \eta_r + \frac{1}{2}$ allowing for a possibly infinite generalized eigenvalue for $\psi_r(\omega) = \eta_r$.

Proposition 2.2

This result follows from examining the eigenvalues presented in (3.B.5) above which considers the dynamic model consisting of (2.1), (2.2), (2.8) and (2.1). We have that $|\lambda_i| < 1$ for $i \in \{1, 2\}$, $|\lambda_4| > 1$ and finally $|\lambda_3| > 1$ if and only if

$$\left| \frac{\phi_r + \phi_m + \eta_r \phi_m}{1 + \eta_r \phi_m} \right| > 1.$$

Since we are assuming all terms are non-negative this is equivalent to having that $|\lambda_3| > 1$ if and only if $\phi_r + \phi_m > 1$.

Corollary 2.4

Applying Lemma 2 to the interest rate rule in (2.3) we have the policy rule

$$\tilde{r}_t = \phi_r \tilde{r}_{t-1} + \phi_{ss} [\Delta \tilde{m}_t + \psi_r(\omega) \Delta \tilde{r}_t - \psi_\tau(\omega) \Delta \tilde{\tau}_t - \psi_x(\omega) \Delta \tilde{x}_t].$$

Rearranging this expression this expression we have a policy rule of the form

$$\tilde{r}_t = \left[\frac{\phi_r - \phi_{ss} \psi_r(\omega)}{1 - \phi_{ss} \psi_r(\omega)} \right] \tilde{r}_{t-1} + \left[\frac{\phi_{ss}}{1 - \phi_{ss} \psi_r(\omega)} \right] \Delta \tilde{m}_t \quad (2.A.5)$$

where we drop the exogenous shock terms since they have no affect on the determinacy properties of the policy rule. Since the rule in (2.A.5) is of the same general form as (2.1), we can analyze the eigenvalues in (3.B.5) above. We then have that $|\lambda_i| < 1$ for $i \in \{1, 2\}$,

$|\lambda_4| > 1$ and finally $|\lambda_3| > 1$ if and only if

$$\left| \frac{\phi_r + \phi_{ss} - \phi_{ss}[\psi_r(\omega) - \eta_r]}{1 - \phi_{ss}[\psi_r(\omega) - \eta_r]} \right| > 1$$

or equivalently

$$|\phi_r + \phi_{ss} - \phi_{ss}[\psi_r(\omega) - \eta_r]| > |1 - \phi_{ss}[\psi_r(\omega) - \eta_r]|.$$

Chapter 3

A Working Solution to Working Capital Indeterminacy

3.1 Introduction

The advice to central banks that a well designed interest rate reaction function mechanically adjusts the policy rate more than one for one to deviations of inflation from target (c.f. Taylor (1993)) is one of the most robust policy prescriptions in monetary theory. So much so that leading undergraduate textbooks introduce students to the so called *Taylor principle*, “The principle that the monetary authorities should raise nominal interest rates by more than the increase in the inflation rate has been named the Taylor principle, and it is critical to the success of monetary policy” (Mishkin, 2013).

However, the Taylor principle as described above is not without its caveats. Importantly, Christiano et al. (2010) show numerically that an upper bound may need to be placed on the central bank’s reaction to inflation in order to ensure expectations remain well anchored.¹ This upper bound arises when there is a timing mismatch between when the firm produces its product and when it gets paid for the product. When this occurs, firms will typically have to

¹Another apparent failure of the Taylor principle involves the interaction of trend-inflation and determinacy. For example, Coibion & Gorodnichenko (2011) show that under the widely used Calvo style pricing assumption and trend inflation, the Taylor principle breaks down.

finance inputs with short-term loans called working capital. This environment introduces a working-capital channel of monetary policy which works counter to the typical transmission mechanism of monetary policy in New-Keynesian models.²

To illustrate both of these transmission mechanisms, consider for the moment a prototypical dynamic AS-AD model where expectations are intentionally simplified to allow for graphical intuition. This model consists of 3 log-linear equations:

$$\begin{aligned}\hat{y}_t &= -\hat{R}_t + \varepsilon_t^{ad} \\ \hat{\pi}_t &= \begin{cases} \kappa\hat{y}_t + \beta\hat{\pi}_{t-1} \\ \kappa\hat{y}_t + \kappa_r\hat{R}_t + \beta\hat{\pi}_{t-1} \end{cases} \\ \hat{R}_t &= \phi_\pi \hat{\pi}_t\end{aligned}$$

where the first equation is an IS equation relating cyclical output to the real interest rate and an exogenous white noise shock, ε_t^{ad} , the second equation is a dynamic Phillip's curve relating inflation to cyclical output and possibly the real policy rate if working capital is required for production and finally the last equation is a simple real interest rate reaction function which is assumed to describe the behavior of the central bank.

3.1.1 The Taylor Principle: A lower bound on ϕ_π

Assume the typical Phillips curve governs firms' price setting behavior so that: $\hat{\pi}_t = \kappa\hat{y}_t + \beta\hat{\pi}_{t-1}$. The Taylor principle is represented in the above system as the requirement on the magnitude of ϕ_π . Since \hat{R}_t is the *real* interest rate the, the appropriate condition for a stable dynamic system $\phi_\pi > 0$. To see why this is so, eliminate \hat{R}_t from the above system and plot the resulting aggregate demand and supply curves. Notice that in Figure (3.1a), the aggregate demand embodies the typical negative relationship between inflation and cyclical output. This negative relationship emerges from the assumption that $\phi_\pi > 0$ so that when

²Throughout the paper, the terms working-capital channel and cost channel can be used interchangeably. For consistency, I will refer to this mechanism as the working-capital channel.

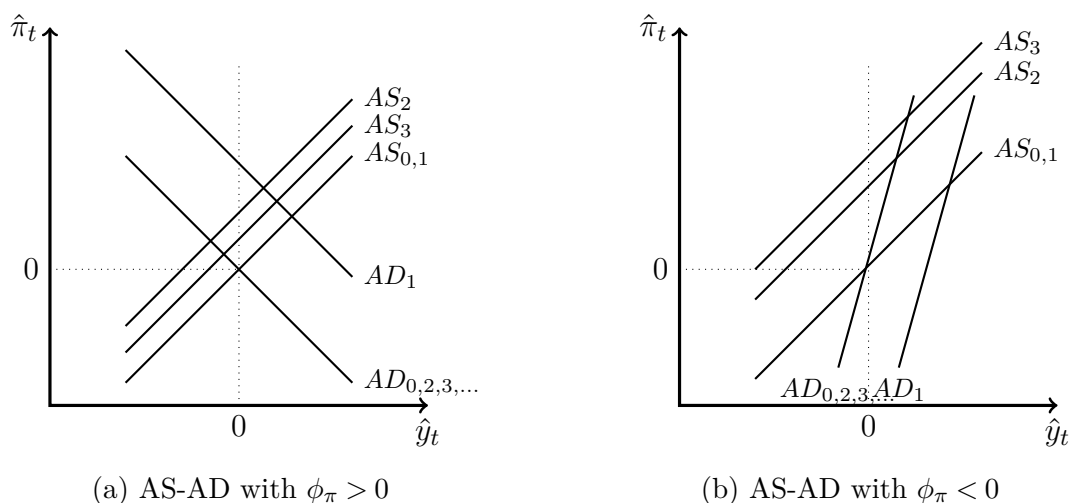


Figure 3.1: The Taylor Principle

inflation increases the central bank generates an increase in the real interest rate, causing cyclical output to decline. If the central bank were to violate this condition and set $\phi_\pi < 0$, the resulting system would consist of two upward sloping curves. To see why this creates dynamic instability in this simple model, consider the behavior of both models following a one time increase in ε_1^{ad} that returns to zero in periods 2,3 and so on.

The dynamics in the model under the Taylor principle are stable with output and inflation returning to their steady state values. The increase in inflation that results from the aggregate demand disturbance is met with a systematic increase in the real interest rate from the central bank, which dampens output and therefore inflation. In fact, an excessively aggressive central bank can set ϕ_π arbitrarily large and eliminate any increase in inflation from the aggregate demand disturbance. This policy prescription emerges from several papers including Schmitt-Grohe & Uribe (2007).

On the other hand, when the central bank is passive and sets $\phi_\pi < 0$, the aggregate demand disturbance causes inflation to initially increase, which the central bank amplifies by lowering the real interest rate, causing output to increase further. The increase in output resulting from passive monetary policy causes firms to set even higher prices resulting in even more inflation and evermore policy accommodation. Clearly the dynamic system in Figure

(3.1b) is not stable as inflation and output expand without bounds. To the extent that the central bank cares about the volatility of inflation and output, this outcome is clearly undesirable. Thus, the widespread recommendation that central banks should aggressively combat inflation seems a natural conclusion. In the following section, I will motivate why this prescription should not be given without pause.

3.1.2 The Taylor principle isn't sufficient: An upper bound on ϕ_π

Now suppose the working-capital channel of monetary policy is active so that inflation dynamics are governed by $\hat{\pi}_t = \kappa \hat{y}_t + \kappa_r \hat{R}_t + \beta \hat{\pi}_{t-1}$. Notice now changes in the real interest rate directly impact both aggregate demand and aggregate supply. The aggregate demand channel posits that an increase in the real interest rate will lower aggregate demand, which indirectly lowers inflation through the Phillips curve. The working-capital channel appends to this mechanism an aggregate supply mechanism which posits that when the real interest rate increases, so too do firms' marginal cost and therefore there is upward pressure on inflation.

Given these opposing channels, it is not surprising that a well intentioned central bank which aggressively adjusts the policy rate in response to changes in inflation may unintentionally induce instability through the working-capital channel. To illustrate this possibility, substitute the monetary policy rule into the IS and Phillips curves to eliminate \hat{R}_t from the above system and plot the resulting AS and AD curves. There are two possibilities, both of which are illustrated in figure 2, $\kappa_r \phi_\pi < 1$ or $\kappa_r \phi_\pi > 1$. If we perform the same thought experiment as above and consider the dynamic response to an aggregate demand disturbance then it becomes clear why the latter condition is to be avoided.

First suppose that $1 > \kappa_r \phi_\pi$. Then the Phillips curve is upward sloping as usual; however, the slope is larger than would be in the case without the working-capital channel. None the less, when aggregate demand increases inflation and cyclical output raise above their steady state level. This rise in inflation results in a mechanical increase in the real policy rate by the central bank. Over time, the counter-cyclical monetary policy decreases output which

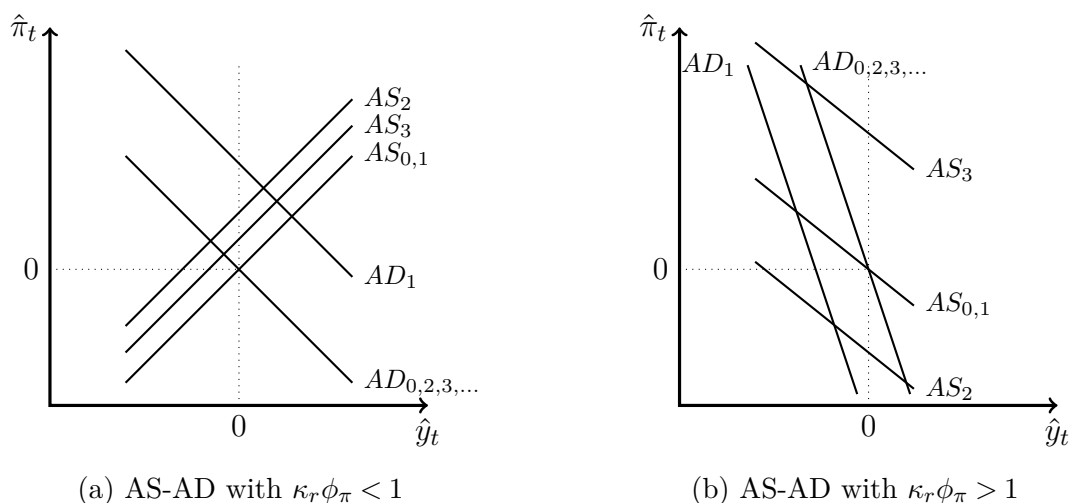


Figure 3.2: Working Capital Indeterminacy

puts pressure on inflation to fall back to its target.

Now suppose that $1 < \kappa_r \phi_\pi$ as may be the case for a central bank which seeks to aggressively target inflation. Then the Phillips curve posits a negative relationship between inflation and cyclical output, and therefore both AS and AD curves slope downwards. When inflation increases from the aggregate demand disturbance the central bank mechanically raises the policy rate. This dampens output through the IS equation; however, it also increases inflation through the working-capital channel. This additional increase in inflation induces the central bank to further raise the policy rate which ultimately further increases inflation through the working-capital channel. The cumulative effect of this feedback loop is higher inflation and a fall in output. Dynamically, this same mechanism causes inflation to become unstable and further wander from target while output falls further below the economy's potential level.

3.1.3 Roadmap

The combination of the above results suggests the Taylor principle alone is not sufficient to guarantee existence of a unique stable equilibrium. Instead, the central bank's inflation response must be bounded below (the Taylor principle) *and* above when there is an active working-capital channel. In what follows, I will formalize these bounds and the sense in which violating them produces instability using the full discipline of a structural New-Keynesian

model and rational expectations. I also show that Friedman’s k -percent money growth rule and an interest rate rule reacting to money growth with a coefficient satisfying the lower bound condition $\phi_m > 1$ are sufficient for determinacy. In other words, the working capital induced indeterminacy is a potential pitfall exclusive to inflation targeting regimes. I also show the above determinacy results are robust to weakening the assumption that firms borrow at the policy rate. Finally, I augment the standard model with real wage rigidities. This additional friction further restricts the central bank’s reaction to inflation but has no impact on the determinacy regions of money growth targeting rules.

3.2 DSGE Model

This section describes the DSGE model used in the paper in detail. The model motivates the working capital channel through a timing mismatch between when firms pay their wage bill and receive payment for their output. To align the model as much as possible with the textbook New-Keynesian model, I assume that all financial transactions are intratemporal and therefore all interest rates are known at the beginning of the period. This allows firms to adjust their prices before actually paying back their working capital loans at the end of the period.³

3.2.1 The Household

The representative household enters any period $t = 0, 1, 2, \dots$ with a portfolio consisting of 3 assets. The household holds maturing bonds b_{t-1} , shares of monopolistically competitive firm $i \in [0, 1]$ $s_{t-1}(i)$, and base money totaling mb_{t-1} . The household faces a sequence of budget constraints in any given period. This budgeting can be described by dividing period t into 2 separate periods: first a securities trading session and then bank settlement period. In the securities trading session the household can buy and sell stocks, bonds, receives wages w_t for hours worked h_t during the period, purchases consumption goods c_t and obtains any loans l_t^{hh} needed to facilitate these transactions. Any government transfers are also made

³Christiano et al. (2005) provide an alternative timing assumption in which the policy rate (which equals the loan rate in their model) is not known when period t prices are set.

at this time, denoted by t_t . Any remaining funds can be allocated between currency n_t and deposits d_t . This is summarized in the constraint below.

$$n_t + d_t = \frac{mb_{t-1}}{\pi_t} + \frac{b_{t-1}}{\pi_t} - \frac{b_t}{r_t} - \int_0^1 q_t(s_t(i) - s_{t-1}(i)) di + w_t h_t + l_t - c_t + t_t \quad (3.1)$$

At the end of the period, the household receives dividends $f_t(i)$ on shares of stock owned in period t , $s_t(i)$, and settles all interest payments with the bank. In particular, the household is owed interest on deposits made at the beginning of the period, $r_t^d d_t$ and owes the bank interest on loans taken out, $r_t^l l_t^{hh}$. I assume as in Curdia & Woodford (2010) that an exogenous fraction ξ_t of all loans to the household and firm will be defaulted upon in which case the bank is not paid back but the proceeds are instead transferred to the household. Any remaining funds can be carried over in the form of base money into period $t+1$, mb_t .

$$mb_t = n_t + \int_0^1 f_t(i) s_t(i) di + r_t^d d_t - r_t^l l_t^{hh} + \xi_t [l_t^{hh} + l_t^f] + f_t^b. \quad (3.2)$$

I assume the default rate follows an AR(1) (in logs),

$$\ln(\xi_t) = (1 - \rho_\xi) \bar{\xi} + \rho_\xi \ln(\xi_{t-1}) + \varepsilon_t^\xi \quad \varepsilon_t^\xi \sim \mathcal{N}(0, \sigma_\xi). \quad (3.3)$$

The household seeks to maximize their lifetime utility, discounted at rate β , subject to these constraints. The period flow utility of the household takes the following form.

$$u_t = a_t [\ln(c_t) + v_t \ln(m_t) + \eta(1 - h_t)]$$

The time-varying preference parameter a_t enters the linearized Euler equation as an IS shock and similarly, v_t enters the linearized money demand equation as a money demand shock.

Both of these processes are assumed to follow an AR(1) (in logs).

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a \quad \varepsilon_t^a \sim \mathcal{N}(0, \sigma_a) \quad (3.4)$$

$$\ln(v_t) = (1 - \rho_v) \ln(\bar{v}) + \rho_v \ln(v_{t-1}) + \varepsilon_t^v \quad \varepsilon_t^v \sim \mathcal{N}(0, \sigma_v) \quad (3.5)$$

The true monetary aggregate, m_t , which enters the period utility function takes a rather general CES form,

$$m_t = \left[\nu^{\frac{1}{\omega}} (n_t)^{\frac{\omega-1}{\omega}} + (1 - \nu)^{\frac{1}{\omega}} (d_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (3.6)$$

where ν calibrates the relative expenditure shares on currency and deposits and ω calibrates the elasticity of substitution between the two monetary assets.

The representative household faces the problem of maximizing its lifetime utility subject to its budget constraints. Letting $\zeta_t = [c_t, h_t, m_t, n_t, d_t, l_t^{hh}, b_t, mb_t, s_t(i)]$ denote the vector of choice variables, the household's optimization problem can be recursively defined using Bellman's method.

$$\begin{aligned} V_t(b_{t-1}, mb_{t-1}, s_{t-1}(i)) = \max_{\zeta_t} & \left\{ a_t [\ln(c_t) + v_t \ln(m_t) + \eta(1 - h_t)] \right. \\ & - \lambda_t^1 \left(d_t + n_t + c_t - w_t h_t - l_t^{hh} - mb_{t-1}/\pi_t - t_t - b_{t-1} + \int_0^1 q_t(i)(s_t(i) - s_{t-1}(i)) di + b_t/r_t \right) \\ & - \lambda_t^2 \left(m_t - \left[\nu^{\frac{1}{\omega}} (n_t)^{\frac{\omega-1}{\omega}} + (1 - \nu)^{\frac{1}{\omega}} (d_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right) \\ & \left. - \lambda_t^3 \left(mb_t - n_t - \int_0^1 f_t(i) s_t(i) di - r_t^d d_t + r_t^l l_t^{hh} - \xi_t [l_t^{hh} + l_t^f] - f_t^b \right) + \beta \mathbb{E}_t [V_{t+1}(b_t, mb_t, s_t(i))] \right\} \end{aligned}$$

The first order necessary conditions are given by the following equations:

$$\frac{a_t}{c_t} = \beta \mathbb{E}_t \left[\frac{a_{t+1}}{c_{t+1}} \frac{r_t}{\pi_{t+1}} \right] \quad (3.7)$$

$$mrs_t = \eta c_t \quad (3.8)$$

$$w_t = w_{t-1}^{\rho_w} mrs_t^{(1-\rho_w)} \quad (3.9)$$

$$m_t = c_t \left(\frac{\lambda_t^2}{\lambda_t^1} \right)^{-1} v_t \quad (3.10)$$

$$n_t = \nu m_t \left[\frac{\lambda_t^2 / \lambda_t^1}{(r_t - 1) / r_t} \right]^\omega \quad (3.11)$$

$$d_t = (1 - \nu) m_t \left[\frac{\lambda_t^2 / \lambda_t^1}{(r_t - r_t^d) / r_t} \right]^\omega \quad (3.12)$$

$$r_t^l = r_t + \xi_t \quad (3.13)$$

$$\lambda_t^1 q_t(i) = \lambda_t^3 f_t(i) + \beta \mathbb{E}_t [\lambda_{t+1}^1 q_{t+1}(i)]. \quad (3.14)$$

The above equations are quite standard with a few exceptions. Notice that equations (5.7) and (5.8) can be combined to yield the typical condition that the real wage equals the marginal rate of substitution when $\rho_w = 0$. However, when $1 > \rho_w > 0$ this condition only holds in steady state and there may be short-run deviations from this optimality condition due to real wage rigidity as in Blanchard & Gali (2007).

Additionally, the demand for the monetary aggregate and the various component quantities shows that when money is modeled as an aggregate of imperfectly substitutable assets, the demand for the aggregate and each component asset doesn't depend exclusively on the short term policy rate as is typically the case in the standard New-Keynesian economies (see for e.g. Woodford (1999) and Gali (2008)). Instead, these demand functions depend on the relative user costs of currency, deposits and the monetary aggregate: $u_t^n = (r_t - 1)/r_t$, $u_t^d = (r_t - r_t^d)/r_t$ and $u_t = \lambda_t^2 / \lambda_t^1$. Finally, notice the loan rate deviates from the rate on 1-period bonds by the default rate, giving rise to a time varying finance premium driven default rates.

3.2.2 The Goods Producing Sector

The goods producing sector features a final goods firm and an intermediate goods firm. There are a unit measure of intermediate goods producing firms indexed by $i \in [0,1]$ who produce a differentiated product. The final goods firm produces y_t combining inputs $y_t(i)$ using the constant returns to scale technology,

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\theta_t-1}{\theta_t}} di \right)^{\frac{\theta_t}{\theta_t-1}}$$

in which $\theta_t > 1$ governs the elasticity of substitution between inputs, $y_t(i)$. As in Smets & Wouters (2007), I assume this elasticity is time varying:

$$\ln(\theta_t) = (1 - \rho_\theta)\bar{\theta} + \rho_\theta \ln(\theta_{t-1}) + \varepsilon_t^\theta \quad \varepsilon_t^\theta \sim \mathcal{N}(0, \sigma_\theta), \quad (3.15)$$

which results in pure cost-push shocks in the New-Keynesian Phillips curve. The final goods producing firm sells its product in a perfectly competitive market, hence solving the profit maximization problem,

$$\max_{y_t(i)_{i \in [0,1]}} p_t \left(\int_0^1 y_t(i)^{\frac{\theta_t-1}{\theta_t}} di \right)^{\frac{\theta_t}{\theta_t-1}} - \int_0^1 p_t(i) y_t(i) di.$$

The resulting first order condition defines the demand curve for each intermediate goods producing firm's product.

$$y_t(i) = \left(\frac{p_t(i)}{p_t} \right)^{-\theta_t} y_t \quad (3.16)$$

Intermediate Goods Producing Firm

Given the downward sloping demand for its product in (3.16), the intermediate goods producing firm has the ability to set the price of its product above marginal cost. Unlike the

final goods market, the intermediate goods market is not purely competitive as evident by the downward sloping demand for its product in equation (3.16). To permit aggregation and allow for the consideration of a representative intermediate goods producing firm i , I assume all such firms have the same constant returns to scale technology which implies linearity in the single input labor $h_t(i)$,

$$y_t(i) = z_t h_t(i). \quad (3.17)$$

The z_t term in (3.17) is an aggregate technology shock that follows an AR(1) in log differences,

$$\ln(z_t/z_{t-1}) = (1 - \rho_z)\bar{z} + \rho_z \ln(z_{t-1}/z_{t-2}) + \varepsilon_t^z \quad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z). \quad (3.18)$$

The price setting ability of each firm is constrained in two ways. First, each intermediate goods producing firm faces a demand for its product from the representative final goods producing firm defined in (3.16). Second, each intermediate goods producing firm faces a convex cost of price adjustment proportional one unit of the final good defined by Rotemberg (1982) to take the form,

$$\Phi(p_t(i), p_{t-1}(i), p_t, y_t) = \frac{\phi}{2} \left[\frac{p_t(i)}{\bar{\pi} p_{t-1}(i)} - 1 \right]^2 y_t.$$

The intermediate goods producing firm maximizes its period t real stock price, $q_t(i)$. Using the representative household's demand for firm i 's stock (3.14) and iterating forward defines the real (no-bubbles) share price as the discounted sum of future dividend payments,

$$q_t(i) = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}^3}{\lambda_t^1} f_{t+j}(i) \right].$$

Though the firm maximizes period t share price, the costly price adjustment constraint makes the intermediate goods producing firm's problem dynamic. Mathematically summarizing,

each intermediate goods producing firm solves to following dynamic problem.

$$\max_{\{y_t(i), h_t(i), p_t(i)\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}^3}{\lambda_t^1} \left[\frac{p_t(i)}{p_t} y_t(i) - r_t^l w_t h_t(i) - \frac{\phi}{2} \left[\frac{p_t(i)}{\bar{\pi} p_{t-1}(i)} - 1 \right]^2 y_t \right]$$

subject to

$$y_t(i) = \left[\frac{p_t(i)}{p_t} \right]^{-\theta_t} y_t$$

$$y_t(i) = z_t h_t(i)$$

The presence of the gross nominal loan rate in the firm's profit function is the result of the timing assumption which stipulates that firms must finance inputs with working capital loans before being paid for their output. In this case, the firm borrows their entire wage bill each period:

$$l_t^f(i) = w_t h_t(i). \quad (3.19)$$

After paying the loan back with interest the firms total cost of obtaining inputs is given by $r_t^l l_t^f(i) - l_t^f(i) + w_t h_t(i) = r_t^l w_t h_t(i)$ where the equality follows from (3.19).

The problem can be simplified by substituting the inverse of the technology constraint for $h_t(i)$ and then substituting the factor demand into the resulting expression for $y_t(i)$ so that now the representative intermediate goods producing firm solves the following recursive problem defined by Bellman's equation.

$$V_t(p_{t-1}(i)) = \max_{p_t(i)} \left\{ \frac{\lambda_t^3 y_t}{\lambda_t^1} \left[\left[\frac{p_t(i)}{p_t} \right]^{1-\theta_t} - r_t^l \frac{w_t}{z_t} \left[\frac{p_t(i)}{p_t} \right]^{-\theta_t} - \frac{\phi}{2} \left[\frac{p_t(i)}{\bar{\pi} p_{t-1}(i)} - 1 \right]^2 \right] + \beta \mathbb{E}_t [V_{t+1}(p_t(i))] \right\}$$

The first order condition for the problem is given by

$$(1 - \theta_t) \left[\frac{p_t(i)}{p_t} \right]^{-\theta_t} + \theta_t r_t^l \frac{w_t}{z_t} \left[\frac{p_t(i)}{p_t} \right]^{-1-\theta_t} - \phi \left[\frac{p_t(i)}{\bar{\pi} p_{t-1}(i)} - 1 \right] \frac{p_t}{\bar{\pi} p_{t-1}(i)} + \beta \phi \mathbb{E}_t \left[\frac{\lambda_{t+1}^3 y_{t+1}}{\lambda_t^3 y_t} \left[\frac{p_{t+1}(i)}{\bar{\pi} p_t(i)} - 1 \right] \frac{p_t p_{t+1}(i)}{\bar{\pi} (p_t(i))^2} \right] = 0. \quad (3.20)$$

In a symmetric equilibrium where $p_t(i) = p_t \forall i \in [0, 1]$, (3.20) can be log-linearized in which case it takes the form of a working-capital augmented New-Keynesian Phillips Curve relating current inflation to the real marginal cost (which depends on real wages, technology and loan rates) and expected future inflation.

3.2.3 The Financial Firm

The financial firm produces deposits d_t and loans l_t for its clients, the household and the firm. Following Belongia & Ireland (2014), I assume that producing d_t deposits requires $\chi_t d_t$ units of non-labor inputs. In this case, χ_t is the marginal cost of producing deposits and varies according to the AR(1) process (in logs),

$$\ln(\chi_t) = (1 - \rho_\chi) \ln(\bar{\chi}) + \rho_\chi \ln(\chi_{t-1}) + \varepsilon_t^\chi \quad \varepsilon_t^\chi \sim \mathcal{N}(0, \sigma_\chi). \quad (3.21)$$

Therefore an increase in χ_t can be interpreted as an adverse financial productivity shock. To keep the model as close as possible to the canonical New-Keynesian, I assume these resources only rented by the firm and none are destroyed in the production process. For this reason, at the end of period t , they are transferred back to the household in the form of dividends. This assumption is made without losing any generality regarding the determinacy results. However, it does have a meaningful change on optimal monetary policy. For example, if the resources are destroyed in the production process, then these resource costs should be minimized by an optimizing central bank.

In addition to these non-labor inputs, the financial firm must satisfy the accounting identity

which specifies assets (loans to households and firms plus reserves) equal liabilities (deposits),

$$l_t + \tau_t d_t = d_t. \quad (3.22)$$

Although changes in banking regulation have effectively eliminated reserve requirements, banks may often choose to hold reserves in lieu of making loans - a flight to quality of sorts. Therefore, instead of assuming the central bank controls the reserve ratio τ_t , assume it varies exogenously according to the AR(1) process (in logs),

$$\ln(\tau_t) = (1 - \rho_\tau)\ln(\bar{\tau}) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_t^\tau \quad \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau). \quad (3.23)$$

An increase in τ_t can therefore be interpreted as a reserves demand shock, as opposed to a change in policy.

The financial firm chooses l_t and d_t in order to maximize period profits

$$\max_{l_t, d_t} (1 - \xi_t)r_t^l l_t - r_t^d d_t - l_t + d_t - \chi_t d_t$$

subject to the balance sheet constraint (3.22). Since the loan and deposits markets are perfectly competitive, substituting the balance-sheet constraint into the profit function and imposing zero profits due to the perfect competition in the banking sector results in the loan-deposit spread,

$$r_t^l - r_t^d = r_t^l \xi_t + [r_t^l(1 - \xi_t) - 1]\tau_t + \chi_t. \quad (3.24)$$

To understand this expression, notice the first term captures the revenues lost to non-performing loans while the second term captures the foregone revenue of making loans (net of defaults) when deposits are held as reserves instead of being loaned out. The last term is the cost of renting $\chi_t d_t$ units of the final good to produce d_t deposits.

3.2.4 Central Bank Policy

As is standard in New-Keynesian economies, the system of equations is under determined without a specification of monetary policy. The goal of this paper is to understand how the working-capital channel impacts the determinacy regions of various policy rules. One of the most widely prescribed policy rules (see for e.g. Schmitt-Grohe & Uribe (2007)) is given by the simple feedback rule:

$$\left(\frac{r_t}{\bar{r}}\right) = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} e^{\varepsilon_t^r}.$$

For values of ϕ_π sufficiently large, this rule can deliver welfare performance arbitrarily close to optimal Ramsey policies in New-Keynesian models (see for e.g. Faia & Monacelli (2007)). After showing rules these inflation targeting interest rate rules are subject to both lower and upper bounds on ϕ_π , I provide analogous results which show that rules reacting the growth rate of money can achieve determinacy under less stringent conditions. In particular, the money growth targeting rule is defined by:

$$\left(\frac{r_t}{\bar{r}}\right) = \left(\frac{r_t}{\bar{r}}\right)^{\phi_r} \left(\frac{m_t}{m_{t-1}} \frac{\pi_t}{\bar{\pi}}\right)^{\phi_m} e^{\varepsilon_t^r}.$$

This rule is only subject to a lower bound condition $\phi_r + \phi_m > 1$ regardless of the strength of the working-capital channel.

Given that Friedman (1960) alluded a similar rule to implement his famous k-percent rule:

$$\frac{m_t}{m_{t-1}} \frac{\pi_t}{\bar{\pi}} = (1 + k) e^{\varepsilon_t^r} \quad (3.25)$$

it is perhaps not surprising that Friedman's k-percent rule is also always determinate, regardless of the strength of the working-capital channel.

Although these rules are of interest for understanding the design of policy rules, it is more widely believed actual Federal reserve policy is best described by a Taylor (1993) rule since

the late 1980's (Thornton (2012) provides narrative evidence to this point):

$$\left(\frac{r_t}{\bar{r}}\right) = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{x_t}{\bar{x}}\right)^{\phi_x} e^{\varepsilon_t^r},$$

where x_t is the output gap, the difference between output and the potential level of output. In this economy, this level of output can be characterized by the efficient level output. In other words, the level of output arising from the frictionless problem:

$$\max_{c_t, h_t(i)} a_t [\ln(c_t) + \eta(1 - \int_0^1 h_t(i) di)] \quad \text{subject to} \quad c_t = z_t \left(h_t(i)^{\frac{\theta_t-1}{\theta_t}} \right)^{\frac{\theta_t}{\theta_t-1}}.$$

The resulting first order condition yields the efficient level of output: $y_t^* = c_t^* = z_t/\eta$, which yields the follow expression for the efficient output gap, deonted by x_t :

$$x_t = \frac{y_t}{y_t^*} = \eta \frac{y_t}{z_t}. \quad (3.26)$$

These various interest rate rules can be concisely embedded in the generalized interest rate rule:

$$\left(\frac{r_t}{\bar{r}}\right) = \left(\frac{r_t}{\bar{r}}\right)^{\rho_r} \left(\frac{m_t}{m_{t-1}} \frac{\pi_t}{\bar{\pi}}\right)^{\phi_m} \left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi} \left(\frac{x_t}{\bar{x}}\right)^{\phi_x} e^{\varepsilon_t^r}. \quad (3.27)$$

3.2.5 Market Clearing

It is now possible to define the equilibrium conditions which close the model. Equilibrium in the final goods market requires that the accounting identity

$$y_t = c_t + \frac{\phi}{2} \left[\frac{\pi_t}{\bar{\pi}} - 1 \right]^2 y_t \quad (3.28)$$

holds. Equilibrium in the money market, equity market, bond market and loan market requires that at all times

$$\begin{aligned}
mb_t &= \frac{mb_{t-1}}{\pi_t} + t_t \\
s_t(i) &= s_{t-1}(i) = 1 \\
b_t &= b_{t-1} = 0 \\
l_t &= l_t^{hh} + \int_0^1 l_t^f(i) di
\end{aligned}$$

respectively. Finally, imposing the symmetry among the intermediate goods producing firms requires that in equilibrium $y_t(i) = y_t$, $h_t(i) = h_t$, $l_t^f(i) = l_t^f$, $p_t(i) = p_t$, $f_t(i) = f_t$ and $q_t(i) = q_t$.

3.3 Working Capital Indeterminacy

The equilibrium model presented above has the following log-linear structure:

$$\tilde{x}_t = \mathbb{E}_t[\tilde{x}_{t+1}] - (\tilde{r}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]) + (1 - \rho_a)\tilde{a}_t + \rho_z\tilde{z}_t \quad (3.1)$$

$$\tilde{\pi}_t = \kappa\tilde{x}_t + \kappa_r\tilde{r}_t + \kappa_r\frac{\tilde{\xi}}{\bar{r}}\tilde{\xi}_t + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] - \tilde{\theta}_t \quad (3.2)$$

$$\tilde{m}_t = \tilde{x}_t - \eta_r\tilde{r}_t - \eta_\tau\tilde{\tau}_t - \eta_\xi\tilde{\xi}_t - \eta_\chi\tilde{\chi}_t + \tilde{v}_t + \tilde{z}_t \quad (3.3)$$

The model is fairly standard with one notable exception, the inclusion of the policy rate in the New-Keynesian Phillips Curve. In what follows, I will show this working-capital channel alters the determinacy conditions on inflation targeting interest rules so that the Taylor principle is no longer sufficient to guarantee the existence of a unique rational expectations equilibrium. An additional condition bounding the inflation response from above emerges. On the other hand, money growth rules and interest rate rules targeting the growth rate of money are not subject to this additional restriction.

3.3.1 Inflation targeting when wages are flexible

In this section, I analyze the conditions for equilibrium existence and uniqueness when the central bank targets inflation. Inflation targeting is a particularly interesting policy regime to analyze since it has been implemented to some degree by numerous central banks.⁴ *Strict Inflation Targeting* of the sorts advocated by Svensson (1999) can be defined by the targeting rule:

$$\tilde{\pi}_t = 0 \tag{3.4}$$

The practice of this sort of policy is well supported by the findings in many New-Keynesian models. For example, Woodford (1999) (pg. 290) shows that such a criterion replicates many desirable aspects of optimal monetary policy, including determinacy. However, the following proposition shows this result is not robust to the inclusion of a working-capital channel.

Proposition 3.1. *For $\rho_w = 0$, for any $0 < \beta < 1$, for any $\kappa > 0$ and for any $\kappa \geq \kappa_r \geq 0$ if the central bank follows the policy rule $\tilde{\pi}_t = 0$ then there exists a unique REE if and only if $\kappa_r < \frac{1}{2}\kappa$.*

Unless noted otherwise, all proofs are in the appendix. Proposition (3.1) shows that if a central bank is committed in the most extreme sense to achieving their inflation target then equilibrium determinacy is unlikely, unless the working-capital channel is sufficiently weak. One interesting case, which has received the bulk of attention in the cost-channel literature is $\kappa_r = \kappa$. This emerges as a special case in this model when $\xi_t = 0$ which then implies $r_t = r_t^l$. The interpretation of this assumption is that firms borrow at the central bank's policy rate. In this restricted case, strict inflation targeting leads to indeterminacy. However, Proposition (3.1) shows that in a more general case where the loan rate differs from the policy rate, the condition for equilibrium determinacy depends on the degree to which the loan rate varies with the policy rate. In Section (3.4), I will show that assuming $\kappa_r = \kappa$ is not far from

⁴The clearest example is New Zealand which officially adopted inflation targeting in 1990. See Roger (2010) for a full list of known inflation targeting central banks.

reality; thus, strict inflation targeting is unlikely to pin down a unique rational expectations equilibrium.

In practice, strict inflation targeting seems difficult at best. The primary difficulty is that the central bank must set its policy rate in such a way to achieve the target, before the current period inflation rate is observed (or at least, determined). For this reason, it may be more reasonable to describe an inflation targeting regime by way of an interest rate rule reacting to inflation:

$$\tilde{r}_t = \phi_\pi \tilde{\pi}_t. \quad (3.5)$$

This type of policy not only embodies an implementable approach to inflation targeting, but it also serves as the basis for exemplifying the Taylor principle; the principle that ϕ_π should be greater than one. The following proposition shows that this principle is no longer sufficient for equilibrium uniqueness when the working-capital channel is sufficiently strong.

Proposition 3.2. *For $\rho_w = 0$, for any $0 < \beta < 1$, for any $\kappa > 0$ and for any $\kappa \geq \kappa_r \geq 0$ if the central bank follows the policy rule $\tilde{r}_t = \phi_\pi \tilde{\pi}_t$ then there exists a unique REE if and only if*

$$1 < \phi_\pi \leq \begin{cases} \infty & \text{if } \kappa_r < \frac{1}{2}\kappa \\ \frac{2(1+\beta)+\kappa}{\kappa(2\alpha-1)} & \text{if } \kappa_r > \frac{1}{2}\kappa. \end{cases}$$

Proposition (3.2) shows that if the central bank is too aggressive in adjusting its policy rate to movements in inflation, unwanted volatility may emerge due to a multiplicity of equilibria. This result confirms the intuition from the AD-AS model presented in the introduction carries over to a fully specified, micro-founded equilibrium model with rational expectations. When the working-capital channel is sufficiently strong, an over-aggressive central bank may actually cause inflation to rise when they increase their policy rate in response to inflation rising above their desired target. This induces higher inflation, thus calling for a further

mechanical increase in the policy rate leading to ultimately higher inflation and higher policy rates. Once again, the additional constraint on the policy maker's reaction to inflation only emerges when $\kappa_r > \frac{1}{2}\kappa$. As I show later, this is the empirically relevant case.

3.3.2 Money growth targeting when wages are flexible

This section shows inflation targeting rules are particularly susceptible to this working-capital feedback loop. I reach this conclusion by showing that rules which target the growth rate of the monetary aggregate have stable determinacy regions independent of the strength of the working-capital channel. Although money growth oriented rules have fallen from favor amongst central bankers, they have a rich history in monetary theory. Arguably, all policy rules trace their roots to Friedman's k-percent money growth rule which called for the central bank to fix the growth rate of a broad monetary aggregate (such as M2) at k-percent. Friedman argued that doing so shielded the economy from potential mistakes by central bankers, such as passively allowing the money supply to collapse, a chief explanation for the severity of the Great Depression according to Milton Friedman and Anna Schwartz (1971).

Instead of preventing a central bank from becoming too passive, in what follows I show that money growth rules shield the economy from an overly active central banker. In particular, if inflation rises above target, a central bank which attempts to stabilize the growth rate of nominal money will react by increasing interest rates (since nominal money growth implicitly includes inflation). However, the increase in nominal rates decreases the demand for money. This opposing effect tempers the central bank's contraction, preventing the possibility of sunspot equilibria emerging regardless of the strength of the working-capital channel.

Proceeding in a similar fashion as above, first consider a strict money growth target as proposed by Friedman (1960) whereby each period the central bank sets the growth rate of money to the desired rate of k-percent:

$$\Delta\tilde{m}_t + \tilde{\pi}_t = 0, \tag{3.6}$$

where k is absorbed into the steady-state growth rate of nominal money. The following proposition establishes the determinacy properties of Friedman's k -percent rule.

Proposition 3.3. *For $\rho_w = 0$, for any $\eta_r \geq 0$, for any $0 < \beta < 1$, for any $\kappa > 0$ and for any $\kappa \geq \kappa_r \geq 0$ if the central bank follows the policy rule $\Delta \tilde{m}_t + \tilde{\pi}_t = 0$ then there exists a unique REE.*

Unlike the strict inflation targeting rule examined in Proposition (3.1), Friedman's strict money targeting rule delivers a unique rational expectations equilibrium, regardless of the strength of the working capital. One immediate corollary to this result is proven in Keating & Smith (2013), Friedman's k -percent rule is determinate when there is no working-capital channel. However, what is more surprising is the stability of this determinacy condition across models with and without working capital channels. As Proposition (3.1) shows, strict inflation targeting rules don't share this stability across these two models.

Similar to the implementation issues that make strict inflation targeting rules infeasible, implementing Friedman's k -percent rule seems difficult at best. Friedman immediately recognized the primary problem. When he first suggested his k -percent growth rule in 1960, he also recommended implementing 100% reserve requirements. In this case, the money multiplier collapses to one and broad monetary aggregates, such as M2 (Friedman's preferred measure of money) become perfectly controllable by the central bank. That being said, Friedman (1960) still believed broad money growth could be stabilized under a fractional reserve banking system:

This [constant money growth rule] is not, under our present System, an easy thing to do. It involves a great many technical difficulties and there will be some deviations from it. If the other changes I suggested [including 100% reserve requirements] were made in the System, it would make the task easier; but even without those changes, it could be done under the present System.

In fact, Friedman believed the stability in the U.S. economy that preceded the financial crisis was due in large part to the Federal Reserve's ability to stabilize the growth rate of money by using the short-term interest rate. This is evident from the following exchange between Milton Friedman and Russ Roberts in an EconTalk interview in September, 2006.⁵

Russ Roberts: [Shortly after *A Monetary History of the United States, 1867-1960* was published the central bank's role was focused on the money supply.] Something changed in the last 25 or 30 years. That's not what Alan Greenspan or Ben Bernanke talk about. They talk about other things and they play with that short-term interest rate, not the so-called stock of money that you focused on so intensely in the book.

Friedman: That's what they talk about but that's not what they do.

Russ Roberts: What do they do?

Friedman: They use the short-term interest rate as a way of controlling the quantity of money.

Clearly this interpretation of monetary policy over the great moderation is not consistent with FOMC transcripts or most of the monetary policy research.⁶ None the less, it provides an interesting point to consider the monetary policy rule:

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_m (\Delta \tilde{m}_t + \tilde{\pi}_t) \quad (3.7)$$

as it should be feasible for the central bank to adjust the short term policy rate to achieve its desired nominal money growth rate. Although money growth targeting rules are typically considered inferior to inflation targeting rules from a normative perspective, the following proposition shows this conclusion may be premature.

⁵Friedman passed away just two months later, on November 16th.

⁶See for example Kahn & Benolkin (2007) and the quote within from Lawrence Meyer, former Federal Reserve Governor, "money plays no role in today's consensus macro model ... and virtually no role in the conduct of monetary policy, at least in the United States." Moreover, Woodford's (1999) "cashless" economy models of monetary policy are pervasively used by academic and central bank researchers.

Proposition 3.4. *For $\rho_w = 0$, for any $\eta_r \geq 0$, for any $0 < \beta < 1$, for any $\kappa > 0$ and for any $\kappa \geq \kappa_r \geq 0$ if the central bank follows the policy rule $\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_m(\Delta \tilde{m}_t + \tilde{\pi}_t)$ then there exists a unique REE if and only if $\rho_r + \phi_m > 1$.*

Proposition (3.4) shows that money growth targeting rules satisfy a Taylor principle of their own sorts. In particular, adjusting the policy rate more than one percentage point for each one percentage point deviation of nominal money growth from the desired target. This simple sufficient condition is robust to models with and without the working-capital channel. Furthermore, in the numerical analysis that follows, I show this condition is sufficient to anchor expectations on a unique rational expectations equilibrium even when a strong response to inflation is added. Hence, aggressive inflation targeting can be implemented once the central bank is sufficiently committed to stabilizing money growth.

Implementing money growth targeting does face a potential shortcoming relative to inflation targeting. In particular, the policy rules in Propositions (3.3) and (3.4) require the central bank to control a monetary aggregate which contains unknown structural parameters such as ν and ω . A similar issue emerges in small open New-Keynesian models where the CPI price index is typically modeled as a CES aggregate of domestic and foreign goods. Just as chain weighted index numbers are used in practice to calculate CPI figures, index number theory can be combined with monetary aggregation theory to construct a monetary index.

Barnett (1978, 1980) made this possible by deriving the expression for the user-costs of monetary assets which can then be combined with data on the quantity of each assets to create a chain-weighted index number. The result is the Divisia monetary aggregate (index) which is defined as follow in this model economy.

Definition 3.1. The growth rate of the nominal Divisia monetary aggregate is defined by

$$\ln(\mu_t) = \left(\frac{s_t^n + s_{t-1}^n}{2} \right) \ln \left(\frac{n_t}{n_{t-1}} \right) + \left(\frac{s_t^d + s_{t-1}^d}{2} \right) \ln \left(\frac{d_t}{d_{t-1}} \right) + \ln(\pi_t), \quad (3.8)$$

where s_t^n and s_t^d are the expenditure shares of currency and interest bearing deposits respectively defined by:

$$s_t^n = \frac{u_t^n n_t}{u_t^n n_t + u_t^d d_t} = \frac{[(r_t - 1)/r_t]n_t}{[(r_t - 1)/r_t]n_t + [(r_t - r_t^d)/r_t]d_t} \quad (3.9)$$

$$s_t^d = \frac{u_t^d d_t}{u_t^n n_t + u_t^d d_t} = \frac{[(r_t - r_t^d)/r_t]d_t}{[(r_t - 1)/r_t]n_t + [(r_t - r_t^d)/r_t]d_t}. \quad (3.10)$$

By weighting the growth rate of the individual assets (with time-varying weights) the Divisia monetary aggregate is able to successfully account for changes in the composition of the aggregate which may have no impact on the overall aggregate. This superior accuracy places the Divisia index amongst Diewart's (1976) class of *superlative* index numbers, meaning the Divisia index has the ability track any linearly homogenous function (with continuous second-derivatives) up to second-order accuracy. In Lemma (3.1), I show this accuracy is not lost in this linearized model. In particular, I show a result analogous to Keating & Smith (2013): A log linear approximation to the growth rate of the Divisia monetary aggregate tracks the true monetary aggregate to second-order accuracy.

Lemma 3.1. *For a given bound on the amplitude of the exogenous structural shocks $\|\delta\|$, for any $0 < \nu < 1$, and for any $\omega > 0$, the difference between the Divisia monetary aggregate and the true monetary aggregate (each in growth rates) is given by*

$$\ln(\mu_t) - (\ln(m_t/m_{t-1}) + \ln(\pi_t/\bar{\pi})) = \mathcal{O}(\|\delta\|^2).$$

Combing Lemma(3.1) with Propositions (3.3) and (3.4) results in two immediate corollaries: Friedman's k-percent rule and flexible money growth targeting rules can be implemented with the Divisia monetary aggregate instead of the true monetary aggregate without any loss of generality regarding the determinacy results in Propositions (3.3) and (3.4). These results follow because determinacy is a local property and locally the Divisia monetary aggregate

perfectly tracks the true monetary aggregate. Thus, implementing money growth targeting rules is feasible in practice but requires constructing the appropriate index, just as inflation targeting requires the Bureau of Labor Statistics or the Bureau of Economic Analysis to construct the appropriate price index. Currently, there are two sources of chain weighted index monetary aggregates for the United States. One is produced by the Federal Reserve Bank of St.Louis and the other is produced by the Center for Financial Stability. Both are constructed using the Divisia index defined above.⁷

3.4 Adding Real Wage Rigidities

In this section, I relax the assumption that $\rho_w = 0$ and allow for real wages to adjust slowly to the output gap (which is an affine combination of the marginal rate of substitution and exogenous shocks). Wage rigidities have proven to be an important source of propagating business cycles and are largely considered a ‘stock’ feature of DSGE models (see: Blanchard & Gali (2007); Smets & Wouters (2007); Christiano et al. (2005); Taylor (1999)). The equilibrium model presented above has the following log-linear structure when $0 \leq \rho_w < 1$ is generally different from zero:

$$\tilde{x}_t = \mathbb{E}_t[\tilde{x}_{t+1}] - (\tilde{r}_t - \mathbb{E}_t[\tilde{\pi}_{t+1}]) + (1 - \rho_a)\tilde{a}_t + \rho_z\tilde{z}_t \quad (3.1)$$

$$\tilde{\pi}_t = \kappa[\tilde{w}_t - \tilde{z}_t] + \kappa_r\tilde{r}_t + \kappa_r\frac{\bar{\xi}}{\bar{r}}\tilde{\xi}_t + \beta\mathbb{E}_t[\tilde{\pi}_{t+1}] - \tilde{\theta}_t \quad (3.2)$$

$$\tilde{w}_t = \rho_w\tilde{w}_{t-1} + (1 - \rho_w)\tilde{x}_t - \rho_w\tilde{z}_{t-1} + (1 - \rho_w)\tilde{z}_t \quad (3.3)$$

$$\tilde{m}_t = \tilde{x}_t - \eta_r\tilde{r}_t - \eta_\tau\tilde{\tau}_t - \eta_\xi\tilde{\xi}_t - \eta_\chi\tilde{\chi}_t + \tilde{v}_t + \tilde{z}_t. \quad (3.4)$$

What is especially interesting about wage rigidities in the presence of a working capital channel is the ability to generate the so called “price puzzle.” This puzzle is said to be

⁷The Monetary Services Index is produced by the Federal Reserve Bank of St.Louis for M1, M2, MZM and MSI-ALL aggregates where the details of the assets included in these aggregates can be found in Anderson & Jones (2011). The Center for Financial Stability produces M1 - M4 aggregates, providing broader measures of money. Details of these aggregates can be found in Barnett et al. (2013). Both sources produce the aggregates at a monthly frequency.

present when a monetary contraction leads to increases in the price level/inflation. Sims (1992) first noted the price puzzle as a prevalent feature of monetary vector autoregressions across multiple countries. Christiano et al. (2005) go on to show that this feature can be captured by DSGE models with (1) a working capital channel and (2) wage rigidities. Intuitively, from the Phillips Curve in equation (3.2), if wages adjust slowly to a monetary contraction then inflation will increase when interest rates rise. To better understand this interaction, it is useful to analyze the model's reaction to a monetary contraction under various assumption on ρ_w .

The majority of the parameters are quite standard in the DSGE literature; however, this does not mean there is a consensus of values. I set $\beta = 0.9975$ which together with $\bar{\pi} = 1.005$ and $\bar{z} = 1.0025$ implies an annualized nominal bond rate equal to 400 basis points. The choice of κ is more ambiguous. Schorfheide (2008) highlights a range of estimates from $\kappa \in [0.005, 4]$. For the purpose of analyzing the impulse response I choose a moderate value of $\kappa = 0.25$. Following Ireland (2009), I set $\eta_r = 2$. As for κ_r , the model implies that $\kappa_r = \kappa \frac{\bar{r}}{\bar{r}^I}$. From the data, the average ratio of the gross federal funds rate to the gross C&I loan rate is 0.997. Hence, I simply set $\kappa_r = \kappa$.⁸

The following impulse response functions to a monetary contraction verify that a model with a working capital channel and real wage rigidities are capable of generating a price puzzle. Given the large body of evidence pointing to wage rigidities and the prevalent price puzzle from the VAR literature, a natural robustness check of the above determinacy results can be performed by allowing for wage rigidities in this working capital environment.

3.4.1 Inflation targeting with sticky wages

The initial rise in prices following the monetary contraction is evidence that the transmission mechanism of monetary policy is further altered by the inclusion of wage rigidities. One clear

⁸One alternative way to interpret a value of $\kappa_r < \kappa$ would be to assume that firms only have to finance a fraction, α , of their wage bill. If however the household must wait until the end of the period to receive the remaining fraction of their paycheck then they will be forced to borrow $1 - \alpha$ of their paycheck at the same loan rate. Hence, the cumulative effect of the working capital/working paycheck channels is equivalent to assuming $\alpha = 1$.

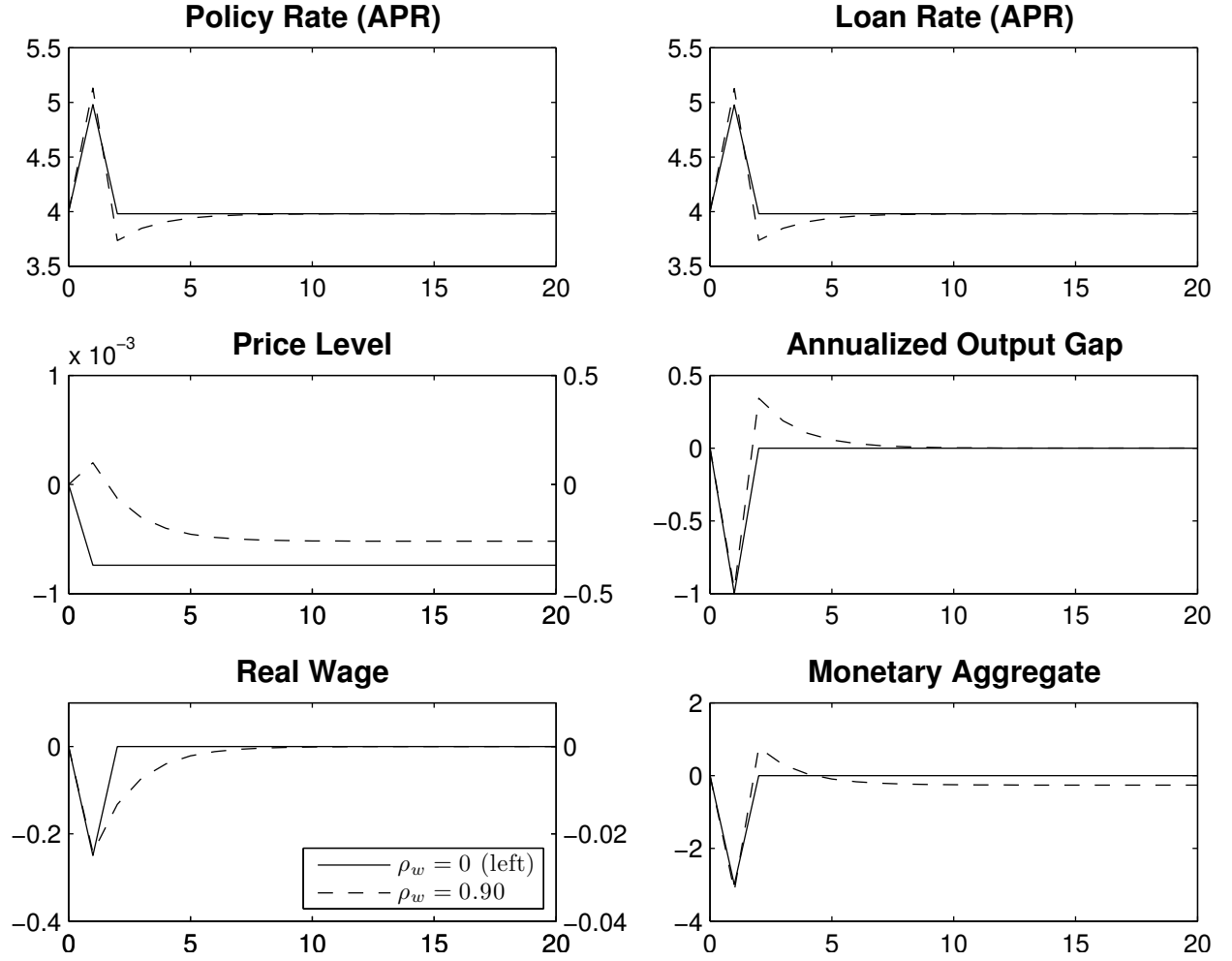


Figure 3.3: Impulse response functions to an exogenous monetary contraction when the central bank follows the inflation targeting rule: $\tilde{r}_t = \phi_\pi \tilde{\pi}_t$. The responses above are graphed for $\phi_\pi = 1.5$ with the other parameters fixed at the values presented in Table 2.3.

conclusion from Figure (3.3) is that the strength of the working capital channel (relative to the Euler channel) of monetary policy is amplified by the sluggish response of wages. From the perspective of this study, this suggests the determinacy regions for inflation targeting policy rules will be further reduced by the interaction of real wage rigidities and working capital. Indeed, the following numerical analysis verifies this is indeed the case.

Figure (3.4) shows determinacy regions under four different cases including the extremes of flexible wages and relatively rigid wages. Notice, in the first case of flexible wages $\rho_w = 0$, the bold black line denotes the upper bound defined in Proposition (3.1). Although

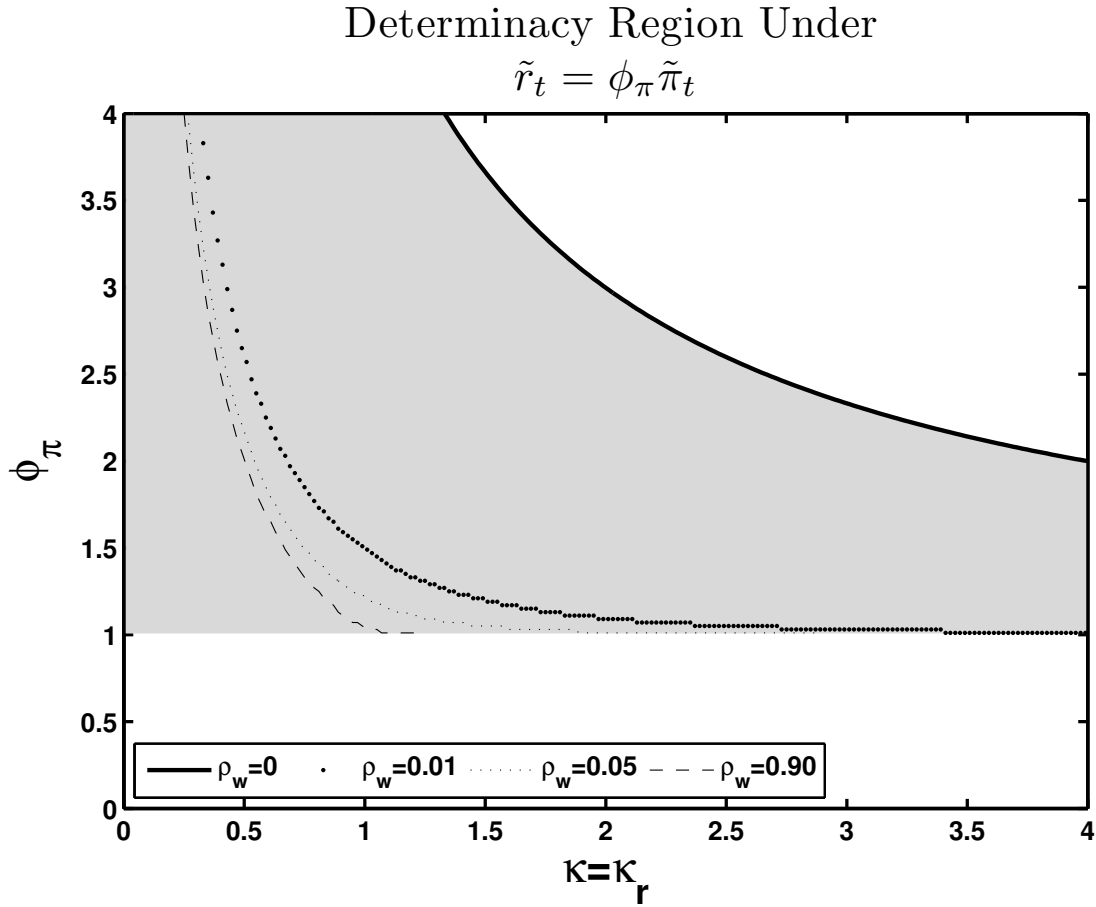


Figure 3.4: Determinacy regions with a working capital channel and various degrees of real wage rigidity when the central bank follows the inflation targeting rule: $\tilde{r}_t = \phi_\pi \tilde{\pi}_t$. The gray area to the southwest of the respective curves are determinate while the white areas are indeterminate.

analytic results are out of reach when wages are rigid, the figure clearly illustrates that the upper bound on the inflation response is a decreasing function of ρ_w . Thus, even if the central bank has a prior belief that κ is relatively small - approximately 0.5, if real wages adjust slowly then a modest inflation reaction around 2 would produce dynamic instability. Interestingly, this value is well below those recommended in a host of New-Keynesian models which call for aggressive inflation targeting (Schmitt-Grohe & Uribe, 2007; Faia & Monacelli, 2007). Regardless of the upper bound on the inflation reaction term, the lower bound of one continues to be sufficient for determinacy, but not necessary.

This may be a relatively minor concern if interest rate smoothing can substantially widen

the determinacy region. For example, it is well known that interest rate smoothing decreases the lower bound on the inflation response needed to produce a unique rational expectations equilibrium. This modified Taylor principle states that $\rho_r + \phi_\pi > 1$ is a sufficient condition to produce determinacy. Thus, it clearly widens the determinacy region in the sense that a smaller inflation response may still be determinate. Does it also increase the upper bound on the inflation response consistent with a unique equilibrium when the working capital channel is active? The figure below shows the answer is a tepid yes.

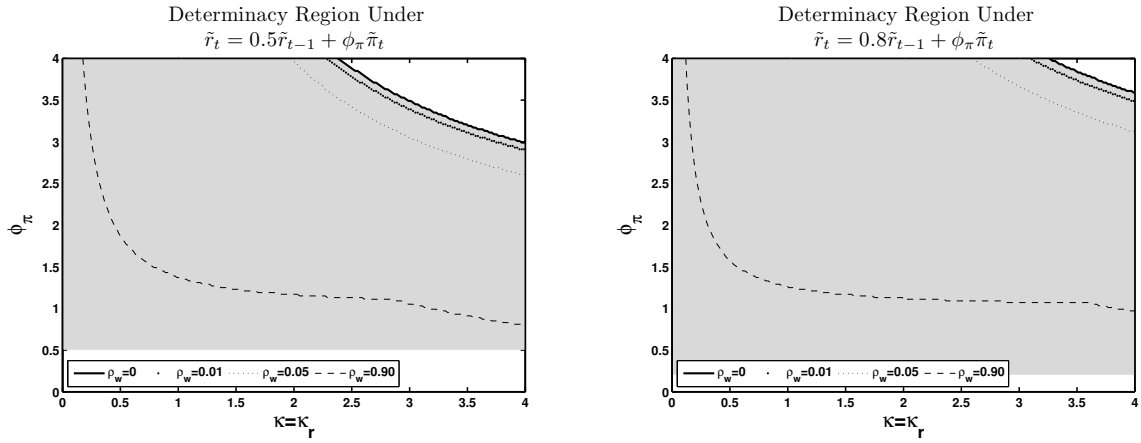


Figure 3.5: Determinacy regions with a working capital channel and various degrees of real wage rigidity when the central bank follows the inflation targeting rule: $\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_\pi \tilde{\pi}_t$. The gray area to the southwest of the respective curves are determinate while the white areas are indeterminate.

Figure (3.5) shows that interest rate smoothing does, in fact, enlarge the determinacy regions of inflation targeting rules. However, the smoothing is far more effective at systematically decreasing the lower bound of one than it is at increasing the upper bound imposed by working capital and rigid real wages. Specifically, the lower bound is changed to $\rho_r + \phi_\pi > 1$, but the impact of the smoothing parameter on the upper bound of the inflation response depends on the degree of real wage rigidities. When wages are perfectly flexible or only mildly rigid increasing the smoothing parameter can moderately increase in the upper bound. However, when $\rho_w = 0.9$ (the empirically relevant case (Blanchard & Gali, 2007)) the upper bound condition is only mildly affected.

3.4.2 Money growth targeting with sticky wages

Section(3.4.1) shows that adding real wage rigidities shrinks the determinacy regions of inflation targeting rules. This results from a decrease in the upper bound on the inflation response due to the interaction of the working capital channel and real wage rigidities. Since money growth targeting rules are only subject to a lower bound restriction that $\rho_r + \phi_m > 1$, the inclusion of real wage rigidities should have no impact on the determinacy regions of such rules. Figure (3.6) verifies this is true. The lines which denote the different determinacy regions for various degrees of wage rigidities all lie on top of one another. In other words, the monetarists Taylor principle (which states that for the money growth targeting rule: $\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_m (\Delta \tilde{m}_t + \tilde{\pi}_t) = \rho_r \tilde{r}_{t-1} + \phi_m \tilde{\mu}_t$, $\rho_r + \phi_m > 1$ is both necessary and sufficient for determinacy) holds with and without the working capital channel and slowly adjusting real wages. The robustness of this result combined with the sensitivity of the inflation targeting rule's determinacy region suggests that inflation targeting rules achieve (or fail to achieve) determinacy in a fundamentally different way than money growth targeting rules.

3.5 Conclusion: A Working Solution to Working Capital Indeterminacy

When firms have to pay for production inputs prior to receiving the revenue for their output, the transmission mechanism of monetary policy is altered. Instead of the usual storyline regarding how monetary policy works: the central bank increases their policy rate which, because of sticky prices, increases the real interest rate inducing consumers to shift consumption to the future, decreasing demand and inflation; there is an additional transmission mechanism - the working capital channel. The above story is augmented to include the following: the central bank increases their policy rate which increases firms' borrowing costs resulting in higher inflation.

This paper shows these competing effects have a significant impact on the determinacy

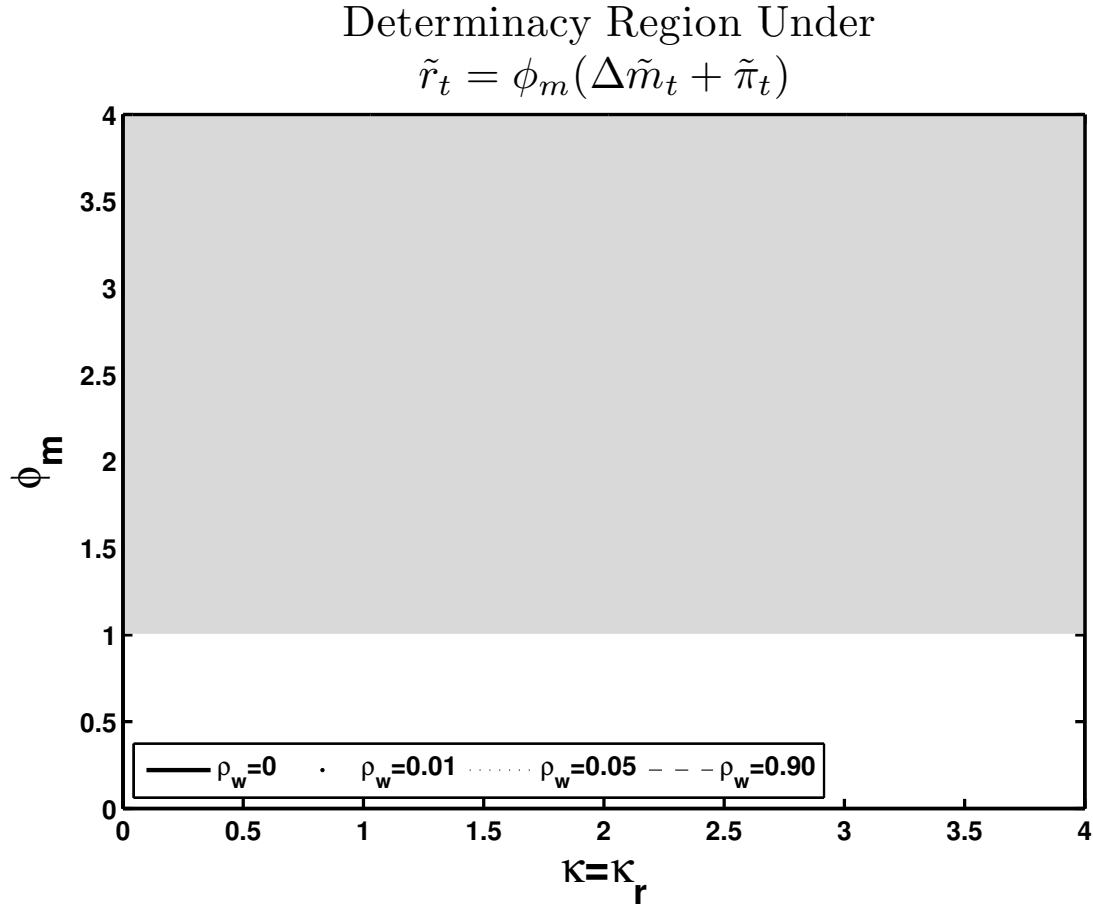


Figure 3.6: Determinacy regions with a working capital channel and various degrees of real wage rigidity when the central bank follows the inflation targeting rule: $\tilde{r}_t = \phi_m(\Delta\tilde{m}_t + \tilde{\pi}_t)$. The gray area to the southwest of the respective curves are determinate while the white areas are indeterminate.

regions of inflation targeting rules. Most notably, strict inflation targeting is especially susceptible to indeterminacy. The more common form of inflation targeting whereby the central bank adjusts its policy rate to achieve their inflation objective is also vulnerable to working capital indeterminacy. Typically, such rules are only subject to the well known lower bound of one on the inflation objective known as the Taylor principle. However, when the working capital channel is active, the inflation response is also subject to an upper bound. The exact value depends on the slope of the Phillips curve and the degree of real wage rigidity and lesser so on the degree of inertia in the policy rule.

What is perhaps more surprising than the sensitivity of the inflation targeting rule's deter-

minacy region is the robustness of the determinacy region of money growth targeting rules. Whether the central bank chooses to fully stabilize the growth rate of nominal money such as Friedman (1960) suggested when proposing his k-percent rule, or they simply adjust their policy rate to achieve their money growth objective the determinacy regions are large and stable. Specifically, for flexible money growth targeting rules $\tilde{r}_t = \rho_r \tilde{r}_{t-1} + \phi_m (\Delta \tilde{m}_t + \pi_t)$ the simple condition that $\rho_r + \phi_m > 1$ is both necessary and sufficient for the existence of a unique rational expectations equilibrium across models with and without the working capital channel and sticky real wages.

Given the fact that aggressive inflation targeting is typically considered optimal from a normative standpoint (Schmitt-Grohe & Uribe, 2007; Faia & Monacelli, 2007), the inability of this rule to bring about a unique rational expectations equilibrium poses a serious hurdle to implementing inflation targeting. The ability of money growth targeting to anchor expectations on a unique equilibrium offers one solution to this problem: Augment the typical inflation targeting interest rate rule with a money growth response. Figure (3.7) shows that a relatively modest focus on money growth can greatly widen the determinacy regions of inflation targeting rules. Unlike interest rate smoothing, the money growth response enlarges the determinacy region even for large degrees of real wage rigidity.

Although the money growth response stabilizes the determinacy region of inflation targeting rules across working capital and sticky wage models, the question of whether this response is optimal still remains. In particular, there is a long standing literature which argues that reacting to money exposes the economy to the additional volatility of money demand shocks (Poole, 1970). Thus, money growth targeting is typically thought to be inferior to inflation targeting (Gali, 2008). However, the inability of inflation targeting rules to pin down a unique equilibrium may pose a trade-off for policy makers.

Future work should explore the details of this trade-off. By anchoring expectations, some degree of money growth targeting may be optimal since it enables the central bank to more aggressively target inflation. However, this is likely an empirical question, as the degree

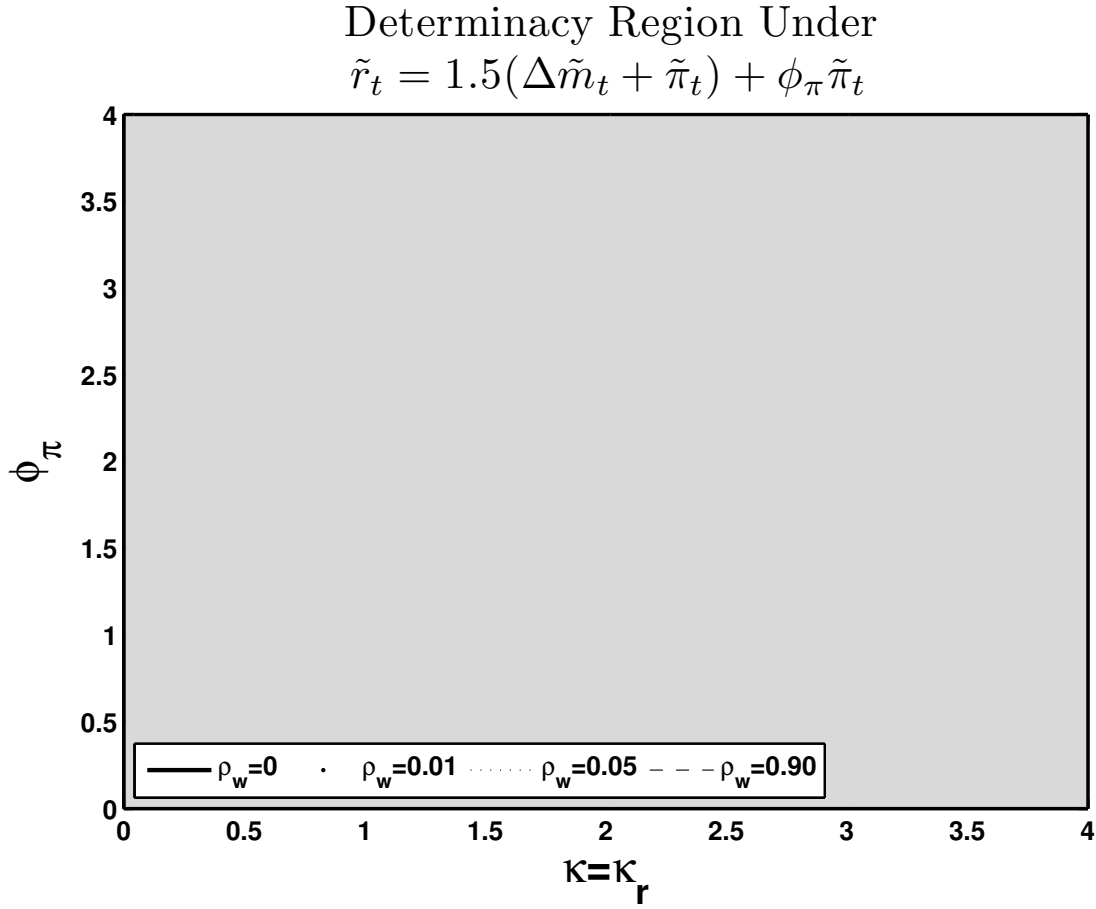


Figure 3.7: Determinacy regions with a working capital channel and various degrees of real wage rigidity when the central bank follows the inflation targeting rule: $\tilde{r}_t = 1.5(\Delta\tilde{m}_t + \tilde{\pi}_t) + \phi_\pi \tilde{\pi}_t$. The gray area is determinate.

of money growth targeting that is optimal depends on the slope of the Phillips curve, the degree of real wage rigidities and the relative sizes of the standard deviation of money demand disturbances to drivers of the business cycle. The survey by Schorfheide (2008) suggests there may be a wide range of estimates of the slope parameter. Given this parameter uncertainty, a useful exercise would be to analyze robustly optimal monetary policy in this working capital augmented New-Keynesian model as in Giannoni (2002).

References

- Anderson, R. G. & Jones, B. (2011). A comprehensive revision of the U.S. monetary services (divisia) indexes. *Review*, (Sep), 325–360.
- Barnett, W., Liu, J., Mattson, R., & Noort, J. (2013). The New CFS Divisia Monetary Aggregates: Design, Construction, and Data Sources. *Open Economies Review*, 24(1), 101–124.
- Barnett, W. A. (1978). The user cost of money. *Economics Letters*, 1(2), 145–149.
- Barnett, W. A. (1980). Economic monetary aggregates an application of index number and aggregation theory. *Journal of Econometrics*, 14(1), 11–48.
- Belongia, M. T. & Ireland, P. N. (2014). The barnett critique after three decades: A new keynesian analysis. *Journal of Econometrics*, forthcoming.
- Blanchard, O. & Gali, J. (2007). Real Wage Rigidities and the New Keynesian Model. *Journal of Money, Credit and Banking*, 39(s1), 35–65.
- Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1), 1–45.
- Christiano, L. J., Trabandt, M., & Walentin, K. (2010). DSGE Models for Monetary Policy Analysis. In B. M. Friedman & M. Woodford (Eds.), *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics* chapter 7, (pp. 285–367). Elsevier.

- Coibion, O. & Gorodnichenko, Y. (2011). Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation. *American Economic Review*, 101(1), 341–70.
- Curdia, V. & Woodford, M. (2010). Credit spreads and monetary policy. *Journal of Money, Credit and Banking*, 42(s1), 3–35.
- Diewert, W. E. (1976). Exact and superlative index numbers. *Journal of Econometrics*, 4(2), 115–145.
- Faia, E. & Monacelli, T. (2007). Optimal interest rate rules, asset prices, and credit frictions. *Journal of Economic Dynamics and Control*, 31(10), 3228–3254.
- Friedman, M. (1960). *A Program for Monetary Stability*. Fordham University Press.
- Friedman, M. & Schwartz, A. J. (1971). *A Monetary History of the United States, 1867-1960*. Princeton University Press.
- Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.
- Giannoni, M. P. (2002). Does model uncertainty justify caution? robust optimal monetary policy in a forward-looking model. *Macroeconomic Dynamics*, 6(01), 111–144.
- Ireland, P. N. (2009). On the welfare cost of inflation and the recent behavior of money demand. *American Economic Review*, 99(3), 1040–52.
- Kahn, G. A. & Benolkin, S. (2007). The role of money in monetary policy: why do the Fed and ECB see it so differently? *Economic Review*, (Q III), 5–36.
- Keating, J. W. & Smith, A. L. (2013). *Determinacy and Indeterminacy in Monetary Policy Rules with Money*. WORKING PAPERS SERIES IN THEORETICAL AND APPLIED ECONOMICS 201310, University of Kansas, Department of Economics.

- McCallum, B. T. (1998). Solutions to linear rational expectations models: a compact exposition. *Economics Letters*, 61(2), 143–147.
- Mishkin, F. (2013). *The Economics Of Money, Banking And Financial Markets*. Pearson.
- Poole, W. (1970). Optimal choice of monetary policy instruments in a simple stochastic macro model. *The Quarterly Journal of Economics*, 84(2), 197–216.
- Roger, S. (2010). Inflation targeting turns 20. *Finance & Development*, 47(1), 46–49.
- Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90(6), 1187–1211.
- Schmitt-Grohe, S. & Uribe, M. (2007). Optimal simple and implementable monetary and fiscal rules. *Journal of Monetary Economics*, 54(6), 1702–1725.
- Schorfheide, F. (2008). Dsge model-based estimation of the new keynesian phillips curve. *FRB Richmond Economic Quarterly*, 94(4), 397–433.
- Sims, C. A. (1992). Interpreting the macroeconomic time series facts : The effects of monetary policy. *European Economic Review*, 36(5), 975–1000.
- Smets, F. & Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3), 586–606.
- Svensson, L. E. O. (1999). Inflation targeting as a monetary policy rule. *Journal of Monetary Economics*, 43(3), 607–654.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.
- Taylor, J. B. (1999). Staggered price and wage setting in macroeconomics. In J. B. Taylor & M. Woodford (Eds.), *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics* chapter 15, (pp. 1009–1050). Elsevier.

Thornton, D. L. (2012). *Greenspan's conundrum and the Fed's ability to affect long-term yields*. Federal reserve bank of st. louis, working papers, Federal Reserve Bank of St. Louis.

Woodford, M. (1999). Commentary : how should monetary policy be conducted in an era of price stability? *Proceedings - Economic Policy Symposium - Jackson Hole*, (pp. 277–316).

3.A Appendix: The Stationary, Non-Linear Model

Since the technology shock is modeled as an $I(1)$ process, most variables will inherit a unit root. Thus, before I can apply standard perturbation procedures and solve the linearized model, I must define a new system of stationary variables as follows: $\hat{c}_t = c_t/z_{t-1}$, $\hat{m}rs_t = mrs_t/z_{t-1}$, $\hat{w}_t = w_t/z_{t-1}$, $\hat{m}_t = m_t/z_{t-1}$, $\hat{n}_t = n_t/z_{t-1}$, $\hat{d}_t = d_t/z_{t-1}$, $\hat{l}_t = l_t/z_{t-1}$, $\hat{m}b_t = mb_t/z_{t-1}$, $\hat{y}_t = y_t/z_{t-1}$, $\hat{\lambda}_t^1 = \lambda_t^1/z_{t-1}$, $\hat{\lambda}_t^2 = \lambda_t^2/z_{t-1}$, $\hat{\lambda}_t^3 = \lambda_t^3/z_{t-1}$, r_t^l , r_t^d , r_t , π_t , x_t , a_t , v_t , ξ_t , $\hat{z}_t = z_t/z_{t-1}$, θ_t , τ_t . Of course, the system is an equation short which is the description of monetary policy, the focus of the paper.

$$\frac{a_t}{\hat{c}_t} = \beta \mathbb{E}_t \left[\frac{a_{t+1}}{\hat{c}_{t+1}} \frac{1}{\hat{z}_t} \frac{r_t}{\pi_{t+1}} \right] \quad (3.A.1)$$

$$\hat{m}rs_t = \eta \hat{c}_t \quad (3.A.2)$$

$$\hat{w}_t = (\hat{w}_{t-1}/\hat{z}_{t-1})^{\rho_w} \hat{m}rs_t^{(1-\rho_w)} \quad (3.A.3)$$

$$\hat{m}_t = \hat{c}_t u_t^{-1} v_t \quad (3.A.4)$$

$$\hat{n}_t = \nu \hat{m}_t \left[\frac{u_t}{(r_t - 1)/r_t} \right]^\omega \quad (3.A.5)$$

$$\hat{d}_t = (1 - \nu) \hat{m}_t \left[\frac{u_t}{(r_t - r_t^d)/r_t} \right]^\omega \quad (3.A.6)$$

$$\hat{m}_t = \left[\nu^{\frac{1}{\omega}} (\hat{n}_t)^{\frac{\omega-1}{\omega}} + (1 - \nu)^{\frac{1}{\omega}} (\hat{d}_t)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (3.A.7)$$

$$\hat{\lambda}_t^1 = \frac{a_t}{\hat{c}_t} \quad (3.A.8)$$

$$u_t = \hat{\lambda}_t^2 / \hat{\lambda}_t^1 \quad (3.A.9)$$

$$\hat{\lambda}_t^1 = \hat{\lambda}_t^3 [r_t^l + \xi_t] \quad (3.A.10)$$

$$r_t^l = r_t + \xi_t \quad (3.A.11)$$

$$\hat{y}_t = \hat{z}_t h_t \quad (3.A.12)$$

$$\begin{aligned} \phi \left[\frac{\pi_t}{\bar{\pi}} - 1 \right] \frac{\pi_t}{\bar{\pi}} &= (1 - \theta_t) + \theta_t r_t^l \frac{\hat{w}_t}{\hat{z}_t} \\ &+ \beta \phi \mathbb{E}_t \left[\frac{\hat{\lambda}_{t+1}^3 \hat{y}_{t+1}}{\hat{\lambda}_t^3 \hat{y}_t} \left[\frac{\pi_{t+1}}{\bar{\pi}} - 1 \right] \frac{\pi_{t+1}}{\bar{\pi}} \right] \end{aligned} \quad (3.A.13)$$

$$\hat{l}_t = \hat{d}_t(1 - \tau_t) \quad (3.A.14)$$

$$r_t^l - r_t^d = r_t^l \xi_t + [r_t^l(1 - \xi_t) - 1]\tau_t + \chi_t \quad (3.A.15)$$

$$x_t = \eta \frac{\hat{y}_t}{\hat{z}_t} \quad (3.A.16)$$

$$\hat{y}_t = \hat{c}_t + \frac{\phi}{2} \left[\frac{\pi_t}{\bar{\pi}} - 1 \right] \hat{y}_t \quad (3.A.17)$$

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_t^a \quad (3.A.18)$$

$$\ln(v_t) = (1 - \rho_v) \ln(\bar{v}) + \rho_v \ln(v_{t-1}) + \varepsilon_t^v \quad (3.A.19)$$

$$\ln(\xi_t) = (1 - \rho_\xi) \bar{\xi} + \rho_\xi \ln(\xi_{t-1}) + \varepsilon_t^\xi \quad (3.A.20)$$

$$\ln(\hat{z}_t) = (1 - \rho_z) \bar{z} + \rho_z \ln(\hat{z}_{t-1}) + \varepsilon_t^z \quad (3.A.21)$$

$$\ln(\theta_t) = (1 - \rho_{\theta_t}) \ln(\bar{\theta}) + \rho_{\theta_t} \ln(\theta_{t-1}) + \varepsilon_t^\theta \quad (3.A.22)$$

$$\ln(\tau_t) = (1 - \rho_\tau) \ln(\bar{\tau}) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_t^\tau \quad (3.A.23)$$

$$\ln(\chi_t) = (1 - \rho_\chi) \ln(\bar{\chi}) + \rho_\chi \ln(\chi_{t-1}) + \varepsilon_t^\chi \quad (3.A.24)$$

3.B Appendix: Proofs

In this section I present the proofs to our results stated in the paper. All proofs for determinacy omit the possibility that an eigenvalue is exactly equal to one. In such a case, a log-linear approximation to the non-linear model can not pin down the question of local existence and uniqueness.

Proposition 3.1

Consider the dynamic system defined by equations (3.1), (3.2) and (3.4),

$$AE_t[\tilde{w}_{t+1}] = B\tilde{w}_t + C\tilde{s}_t.$$

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & \beta & \kappa_r \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

where $\tilde{w}_t = \begin{bmatrix} \tilde{x}_t & \tilde{\pi}_t & \tilde{r}_{t-1} \end{bmatrix}^T$ and C is a conformable matrix for the vector $\tilde{s}_t = \begin{bmatrix} \tilde{a}_t & \tilde{z}_t & \tilde{\xi}_t & \tilde{\theta}_t \end{bmatrix}^T$. The system has two non-predetermined variables and one predetermined variable. Therefore, the system will have a unique rational expectations equilibrium (REE) if, and only if, two roots λ which satisfy $|B - \lambda A| = 0$ lie outside the unit circle and remaining two roots lie inside the unit circle McCallum (1998). The generalized eigenvalues

of the matrix pencil $B - \lambda A$ are given by

$$\Lambda = \begin{bmatrix} 0 \\ \frac{\kappa_r - \kappa}{\kappa_r} \\ \infty \end{bmatrix}. \quad (3.B.1)$$

The proof to Proposition (3.1) follows from (3.B.1) since $|\lambda_1| < 1$, $|\lambda_3| > 1$ and finally $|\lambda_2| > 1$ if and only if $\kappa_r < \frac{1}{2}\kappa$.

Proposition 3.2

Consider the dynamic system defined by equations (3.1), (3.2) and (3.5),

$$AE_t[\tilde{w}_{t+1}] = B\tilde{w}_t + C\tilde{s}_t.$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & \phi_\pi \\ -\kappa & 1 - \kappa_r \phi_\pi \end{bmatrix}$$

where $\tilde{w}_t = \begin{bmatrix} \tilde{x}_t & \tilde{\pi}_t \end{bmatrix}^T$ and C is a conformable matrix for the vector $\tilde{s}_t = \begin{bmatrix} \tilde{a}_t & \tilde{z}_t & \tilde{\xi}_t & \tilde{\theta}_t \end{bmatrix}^T$.

Since A is invertible define $F = A^{-1}B$. The system has two non-predetermined variables.

Therefore, from Proposition C.1 in Woodford (1999), the system will have a unique rational expectations equilibrium (REE) if, and only if all of the following are true:

$$|F| > 1 \quad (3.B.2)$$

$$|F| - \text{tr}(F) > -1 \quad (3.B.3)$$

$$|F| + \text{tr}(F) > -1. \quad (3.B.4)$$

Since $\kappa \geq \kappa_r$, let $\kappa_r = \alpha\kappa$ where $\alpha \in [0, 1]$. First notice that (3.B.2) is satisfied, $|F| = \frac{1+\kappa\phi_\pi(1-\alpha)}{\beta} > 1$. Moreover, since $tr(F) = \frac{1+\kappa(1-\alpha\phi_\pi)\beta}{\beta}$, it follows that

$$|F| - tr(F) = \frac{\kappa(\phi_\pi - 1) - \beta}{\beta} > -1 \iff \kappa(\phi_\pi - 1) > 0 \iff \phi_\pi > 1.$$

Finally, for the condition in (3.B.4) to be satisfied, we must have:

$$|F| + tr(F) = \frac{2 + \kappa + \beta + \phi_\pi\kappa(1 - 2\alpha)}{\beta} > -1 \iff 2(1 + \beta) + \kappa + \phi_\pi\kappa(1 - 2\alpha) > 0.$$

Rearranging the last inequality, the condition from Proposition (3.2) emerges:

$$\phi_\pi < \begin{cases} \infty & \text{if } \alpha \leq \frac{1}{2} \\ \frac{2(1+\beta)+\kappa}{\kappa(2\alpha-1)} & \text{if } \alpha > \frac{1}{2}. \end{cases}$$

Proposition 3.3

The dynamic system consisting of (3.1), (3.2), (3.3) and (3.7) can be expressed as

$$AE_t[\tilde{w}_{t+1}] = B\tilde{w}_t + Cs_t.$$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & \beta & \kappa_r & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \eta_r & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\kappa & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

where $\tilde{w}_t = \begin{bmatrix} \tilde{x}_t, & \tilde{\pi}_t, & \tilde{r}_{t-1}, & \tilde{m}_{t-1} \end{bmatrix}^T$ and C is a conformable matrix for the vector $\tilde{s}_t = \begin{bmatrix} \tilde{a}_t, & \tilde{z}_t, & \tilde{\xi}_t, & \tilde{\theta}_t, & \tilde{\tau}_t, & \tilde{\chi}_t, & \tilde{v}_t \end{bmatrix}^T$. The system has two non-predetermined variables and two predetermined variables. Therefore, the system will have a unique rational expectations equilibrium (REE) if, and only if, two roots λ which satisfy $|B - \lambda A| = 0$ lie outside the unit circle and the remaining two roots lie inside the unit circle McCallum (1998). The generalized eigenvalues of the matrix pencil $B - \lambda A$ are given by

$$\Lambda = \begin{bmatrix} 0 \\ \frac{\beta+1+\kappa-\sqrt{(\beta-1)^2+2\beta\kappa+2\kappa+\kappa^2}}{2\beta} \\ \frac{1+\eta_r}{\eta_r} \\ \frac{\beta+1+\kappa+\sqrt{(\beta-1)^2+2\beta\kappa+2\kappa+\kappa^2}}{2\beta} \end{bmatrix}. \quad (3.B.5)$$

The proof to Proposition (3.3) follows from (3.B.5) since $|\lambda_1| < 1$, $|\lambda_2| < 1$, $|\lambda_4| > 1$ and finally $|\lambda_3| > 1$, allowing for a possibly infinite generalized eigenvalue at $\eta_r = 0$.

Proposition 3.4

The dynamic system consisting of (3.1), (3.2), (3.3) and (3.6) can be expressed as

$$AE_t[\tilde{w}_{t+1}] = B\tilde{w}_t + Cs_t.$$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & \beta & \kappa_r & 0 \\ 0 & 0 & 1 & -\phi_m \\ 0 & 0 & \eta_r & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\kappa & 1 & 0 & 0 \\ 0 & \phi_m & \rho_r & -\phi_m \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

where $\tilde{w}_t = \begin{bmatrix} \tilde{x}_t, & \tilde{\pi}_t, & \tilde{r}_{t-1}, & \tilde{m}_{t-1} \end{bmatrix}^T$ and C is a conformable matrix for the vector $\tilde{s}_t = \begin{bmatrix} \tilde{a}_t, & \tilde{z}_t, & \tilde{\xi}_t, & \tilde{\theta}_t, & \tilde{\tau}_t, & \tilde{\chi}_t, & \tilde{v}_t \end{bmatrix}^T$. The system has two non-predetermined variables and two predetermined variables. Therefore, the system will have a unique rational expectations equilibrium (REE) if, and only if, two roots λ which satisfy $|B - \lambda A| = 0$ lie outside the unit circle and the remaining two roots lie inside the unit circle McCallum (1998). The generalized eigenvalues of the matrix pencil $B - \lambda A$ are given by

$$\Lambda = \begin{bmatrix} 0 \\ \frac{\beta+1+\kappa-\sqrt{(\beta-1)^2+2\beta\kappa+2\kappa+\kappa^2}}{2\beta} \\ \frac{\rho_r+\phi_m+\eta_r\phi_m}{1+\eta_r\phi_m} \\ \frac{\beta+1+\kappa+\sqrt{(\beta-1)^2+2\beta\kappa+2\kappa+\kappa^2}}{2\beta} \end{bmatrix}. \quad (3.B.6)$$

The proof to Proposition (3.4) follows from (3.B.6) since $|\lambda_1| < 1$, $|\lambda_2| < 1$, $|\lambda_4| > 1$ and finally $|\lambda_3| > 1$ if, and only if, $\rho_r + \phi_m > 1$.

Lemma 3.1

Before deriving the final a result, a few useful observations will allow for a simpler proof. First, define the user costs of n_t and d_t respectively by:

$$u_t^n = \frac{r_t - 1}{r_t} \quad (3.B.7)$$

$$u_t^d = \frac{r_t - r_t^d}{r_t}. \quad (3.B.8)$$

These expressions appear from the first order conditions for currency and deposits; in particular,

$$\frac{\frac{\partial u_t}{\partial n_t}}{\frac{\partial u_t}{\partial d_t}} = \frac{\frac{\partial u_t}{\partial m_t} \frac{\partial m_t}{\partial n_t}}{\frac{\partial u_t}{\partial m_t} \frac{\partial m_t}{\partial d_t}} = \frac{\frac{r_t - 1}{r_t}}{\frac{r_t - r_t^d}{r_t}} \equiv \frac{u_t^n}{u_t^d},$$

where the last equality uses the representative household's first order conditions (5.10) and (3.12). Using these definitions of the user-cost of monetary assets, combine (5.10) and (3.12) to eliminate n_t and d_t from (3.6). Solving the resulting expression for λ_t^2/λ_t^1 and define the resulting expression as:

$$\frac{\lambda_t^2}{\lambda_t^1} = u_t = \left[\nu (u_t^n)^{1-\omega} + (1-\nu) (u_t^d)^{1-\omega} \right]^{\frac{1}{1-\omega}}, \quad (3.B.9)$$

the user-cost of the entire monetary aggregate. Finally, notice that given the definitions of (3.B.7), (3.B.8) and (3.B.9), u_t satisfies Irving Fisher's factor reversal test which mathematically states that total expenditures on monetary assets must equal the product of the quantity and price aggregate:

$$u_t^n n_t + u_t^d d_t = m_t u_t^\omega \left[\nu (u_t^n)^{1-\omega} + (1-\nu) (u_t^d)^{1-\omega} \right] = m_t u_t. \quad (3.B.10)$$

This expression will prove very useful in the following proof. From Definition (3.1), we have

the growth rate of the Divisia monetary aggregate is given by

$$\ln(\mu_t) = \left(\frac{s_t^n + s_{t-1}^n}{2} \right) \ln \left(\frac{n_t}{n_{t-1}} \right) + \left(\frac{s_t^d + s_{t-1}^d}{2} \right) \ln \left(\frac{d_t}{d_{t-1}} \right) + \ln(\pi_t).$$

Now substituting the factor demands ((5.10) and (3.12)) into the definition of the Divisia monetary aggregate to eliminate n_t and d_t , we obtain the following useful expression for the difference between the growth rates of the Divisia monetary aggregate and the true monetary aggregate:

$$\begin{aligned} \ln(\mu_t) - \Delta \ln(m_t) - \ln(\pi_t) &= \omega \left[\Delta \ln(u_t(u_t^n, u_t^d)) \right] \\ &\quad - \omega \left[\frac{1}{2} (s_t^n + s_{t-1}^n) \Delta \ln(u_t^n) - \frac{1}{2} (s_t^d + s_{t-1}^d) \Delta \ln(u_t^d) \right] \\ &\equiv E(\ln(u_t^n), \ln(u_t^d), \ln(u_{t-1}^n), \ln(u_{t-1}^d)), \end{aligned}$$

where E is the error-term which quantifies the difference between the Divisia monetary aggregate and the true monetary aggregate. This expression is a function only of user-costs and completely independent of the quantities of monetary assets. This can easily be seen since the denominator of the expenditure share weights s_t^n and s_t^d is total expenditures which depends only on u_t and m_t from (3.B.10):

$$\begin{aligned} s_t^n &= \frac{u_t^n n_t}{u_t^n n_t + u_t^d n_t} = \frac{m_t u_t^\omega \nu (u_t^n)^{1-\omega}}{u_t m_t} = \nu \left(\frac{u_t^n}{u_t} \right)^{1-\omega} \\ s_t^d &= \frac{u_t^d d_t}{u_t^n n_t + u_t^d n_t} = \frac{m_t u_t^\omega (1-\nu) (u_t^d)^{1-\omega}}{u_t m_t} = (1-\nu) \left(\frac{u_t^d}{u_t} \right)^{1-\omega}. \end{aligned}$$

To verify the claim, take a first-order Taylor approximation of E around the steady state, hence all derivatives below are evaluated at the steady state. For this reason, all derivatives with respect to the share weights $s^n(u_t^n, u_t^d)$ and $s^d(u_t^n, u_t^d)$ evaluate to zero since in steady

state, $\Delta \ln(\bar{u}^n) = \Delta \ln(\bar{u}^d) = 0$.

$$\begin{aligned}
E &= \left[\frac{\partial E}{\partial \ln(u_t^n)} \right] \tilde{u}_t^n + \left[\frac{\partial E}{\partial \ln(u_t^d)} \right] \tilde{u}_t^d + \left[\frac{\partial E}{\partial \ln(u_{t-1}^n)} \right] \tilde{u}_{t-1}^n + \left[\frac{\partial E}{\partial \ln(u_{t-1}^d)} \right] \tilde{u}_{t-1}^d + \mathcal{O}(\|\delta\|^2) \\
&= \left[\frac{\partial E}{\partial \ln(u_t^n)} \right] \tilde{u}_t^n + \left[\frac{\partial E}{\partial \ln(u_t^d)} \right] \tilde{u}_t^d - \left[\frac{\partial E}{\partial \ln(u_t^n)} \right] \tilde{u}_{t-1}^n - \left[\frac{\partial E}{\partial \ln(u_t^d)} \right] \tilde{u}_{t-1}^d + \mathcal{O}(\|\delta\|^2) \\
&= \left[\frac{\partial E}{\partial \ln(u_t^n)} \right] \Delta \tilde{u}_t^n + \left[\frac{\partial E}{\partial \ln(u_t^d)} \right] \Delta \tilde{u}_t^d + \mathcal{O}(\|\delta\|^2)
\end{aligned}$$

where the second equality makes use of the fact that $\partial E / \partial \ln(u_t^n) = -\partial E / \partial \ln(u_{t-1}^n)$ and $\partial E / \partial \ln(u_t^d) = -\partial E / \partial \ln(u_{t-1}^d)$.

Finally, notice that

$$\begin{aligned}
\left[\frac{\partial E}{\partial \ln(u_t^n)} \right] &= \left[\frac{\partial \ln(u_t)}{\partial \ln(u_t^n)} - \frac{1}{2}(s_t^n + s_{t-1}^n) \right] \\
&= \frac{\nu(\bar{u}^n)^{1-\omega}}{\nu(\bar{u}^n)^{1-\omega} + (1-\nu)(\bar{u}^d)^{1-\omega}} - \frac{\nu(\bar{u}^n)^{1-\omega}}{(\bar{u})^{1-\omega}} = 0
\end{aligned}$$

and

$$\begin{aligned}
\left[\frac{\partial E}{\partial \ln(u_t^d)} \right] &= \left[\frac{\partial \ln(u_t)}{\partial \ln(u_t^d)} - \frac{1}{2}(s_t^d + s_{t-1}^d) \right] \\
&= \frac{(1-\nu)(\bar{u}^d)^{1-\omega}}{\nu(\bar{u}^n)^{1-\omega} + (1-\nu)(\bar{u}^d)^{1-\omega}} - \frac{(1-\nu)(\bar{u}^d)^{1-\omega}}{(\bar{u})^{1-\omega}} = 0
\end{aligned}$$

which verifies our claim since we have $\ln(\mu_t) - \Delta \ln(m_t) - \ln(\pi_t) = E = \mathcal{O}(\|\delta\|^2)$.

Chapter 4

House Prices, Heterogeneous Banks and Unconventional Monetary Policy Options

4.1 Introduction

The first drop in U.S. home prices since the Great Depression resulted in the 2008 Financial Crisis, forcing policy makers to ‘Fly Blind,’ and take the exceptional actions of injecting equity into the largest U.S. financial firms and making a myriad of asset purchases without a playbook from economists. Little was known regarding the exact financial mechanisms linking home prices to the rest of the economy, and to date, many questions remain. Why did a drop in home prices set off a financial panic that forced the largest commercial banks to shed relatively more assets than their smaller counter-parts? What is the transmission mechanism of equity injections into these big banks and large-scale asset purchases amidst a housing-centered credit crunch?

In this paper, I propose an empirically motivated financial mechanism which provides an answer to both of these questions. I embed this financial structure into an otherwise stan-

dard Real Business Cycle model to illustrate its ability to generate salient features of the U.S. economy. Quantitatively, the model matches empirical correlations that the traditional financial accelerator mechanism (Bernanke et al., 1999) (BGG) fails to capture, including the correlation of finance premiums with home prices, investment and output. I then test the model’s qualitative predictions against an estimated VAR. The results of these empirical comparisons support the model’s financial structure.

I am not alone in my efforts to adapt existing financial accelerator frameworks to simulate the effects of unconventional monetary policy in light of the financial crisis. However, these models typically rely on ‘Financial’ shocks to generate a crisis scenario and quadratic investment adjustment costs to propagate the downturn (Gertler & Karadi, 2011). In such models, no distinction is made between housing secured debt and other assets.

I take the alternative viewpoint that housing, and housing secured debt instruments, played a particularly important role in the crisis. In particular, the housing and financial sector in the model I develop are highly integrated – a feature that allows for a financial crunch to originate from a change in primitives such as preferences over housing. Moreover, the use of housing as an ultimate source of collateral for financial assets provides a natural source to propagate downturns as opposed to quadratic investment adjustment costs. Upon default, the liquidation of housing – a durable – increases the stock of homes and therefore decreases prices into the future. Furthermore, the time lag between default and liquidation leads to hump-shaped impulse response functions of output, an empirical feature used to justify quadratic adjustment costs (cf Christiano et al. (2005)).

Besides treating the housing and the financial sector as independent, other models that have been used to analyze unconventional monetary policy fail to provide an answer regarding the effects, and desirability, of equity injections into “Too Big to Fail” banks. These papers assume either: (i) All banks are the same in terms of size and efficiency (Gertler & Kiyotaki, 2010) or (ii) Banks differ in terms of size and efficiency, but banks face no agency problems (Hafstead & Smith, 2012).

However, both of these assumption are at odds with the empirical evidence. Recent estimates of scale efficiency in the financial sector, including off-balance sheet activities, have found that “Bigger is Better” (Wheelock & Wilson, 2012; Bos & Kolari, 2013). As for the later assumption, Demirguc-Kunt et al. (2013) find that bank capital is an important determinant for the performance of banks, most markedly for big banks – empirical support for overturning the Modigliani-Miller theorem.¹

With this empirical evidence in mind, I incorporate bank heterogeneity into the financial sector. Big banks naturally arise in equilibrium due to their superior efficiency intermediating housing-secured assets, relative to small banks. The only question is: Why do small banks operate at all in this market if they are technologically dis-advantaged? This is where eschewing the assumption that banks face no agency problems is key. The big/productive banks in the model face a moral-hazard problem. Therefore, the amount of housing-secured assets the productive bank can hold is determined by its capital. In this sense, bank-capital is key for the operation of big/productive banks, consistent with the empirical evidence.

By capturing the rich interaction between the housing market and the heterogeneous financial sector, the model is able to offer an explanation as to why the concentration of assets in the biggest commercial banks decreased with home prices, as illustrated below.

The financial mechanism I propose is capable of endogenously generating the above co-movement. Beyond simply capturing a correlation, the model provides the insight that the above redistribution increased the severity and length of the Great Recession. However, the same feedback mechanism provides traction for equity injections into big banks and large-scale asset purchase programs.

¹The Modigliani-Miller theorem states that how a firm chooses to fund itself (debt vs. equity) is irrelevant.

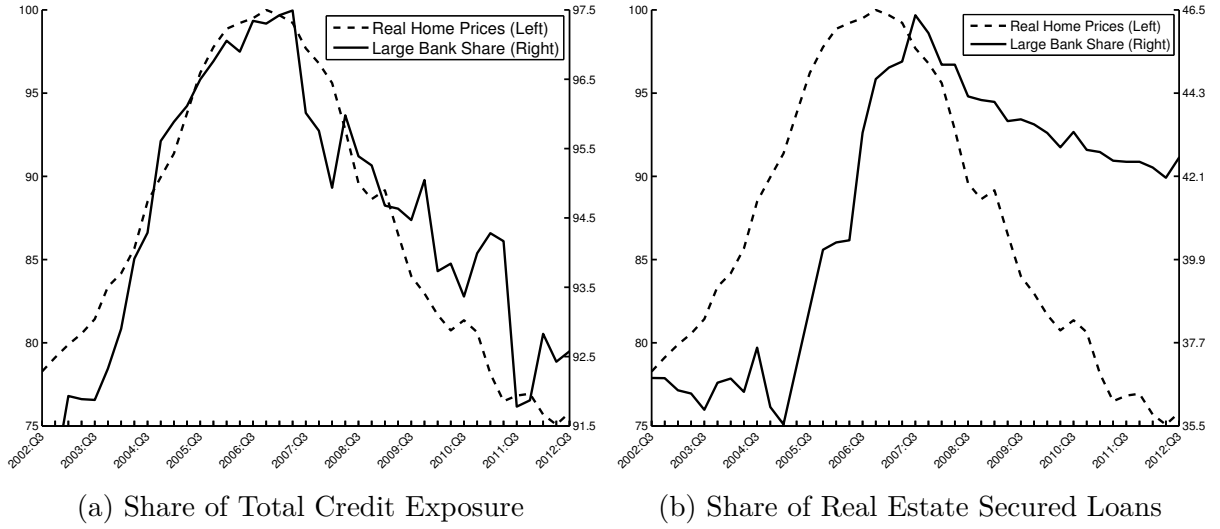


Figure 4.1: Real home prices (normalized to pre-recession peak) and the share of various assets held by the largest commercial banks.

4.2 A DSGE Model with Integrated Housing and Financial Markets

In this section I develop a general equilibrium model capable of capturing the asset redistribution between big and small banks that occurs when home prices change. The model implies that bank heterogeneity amplifies typical business cycles when large banks are leverage constrained. To the extent these large financial firms can more efficiently intermediate secured-debt, the asset concentration that occurs during an upswing moves the economy closer to a Pareto outcome. The other side of this coin however implies that slumps can be made more severe due to this redistribution.

4.2.1 Related Models and How this Model Differs

The notion that heterogeneity between agents can lead to amplification effects is not new. Kiyotaki & Moore (1997) (hereafter KM) show that an asset price spiral may occur if productive agents are borrowing constrained and the tightness of this constraint depends on asset prices. If an adverse shock results in decreased output for the productive agent, they have fewer assets to borrow against which results in unproductive agents absorbing these

assets, which they are willing to do so only at reduced prices. However, since the productive agents binding constraint depends on the price of the asset, the decrease in price only further tightens the collateral constraint - starting the process over.

The model I present here builds on the keen insight of this amplification effect, however the KM model is silent with regards to default rates since agents in the model never default. This aspect was critical during the recent crisis where home prices and MBS fell in value in large part due to rising defaults on risky sub-prime loans. However, Bernanke et al. (1999) (hereafter BGG) develop a financial friction where risky projects are financed and each period some share of financed projects will fail resulting in default. However, BGG features no illiquid collateral, instead all assets are already monetary. This feature of the model misses the role of market liquidity (or illiquidity) which is elegantly captured by KM. Moreover, BGG and KM don't explicitly include banks.² To the extent that deposits and bank capital are perfect substitutes, the financial sector can be left in the background as household's can then directly fund projects with deposits. On the other hand, if bank capital serves a special role in mitigating financial frictions faced by the bank (on the supply side), then these agents must be explicitly modeled.

Gertler & Kiyotaki (2010) present a model with financial intermediaries where bank capital facilitates bank's ability to obtain funds. Banks in their model face frictions in raising loanable funds however, there are no demand side frictions, so that all loans face zero default risk. Additionally, there is no clear distinction between big banks and small banks. Banks in their model differ in terms of their investment opportunities but not their efficiency in intermediating loans. Therefore, building on Hafstead & Smith (2012), I include heterogeneous banks with market power - creating bank capital in the model. Banks in this environment differ in size due to differences with regards to their efficiency. Along this dimension, my

²In KM the agents can simply be re-interpreted as banks with investment opportunities and this would capture supply side financial frictions. However, to the extent that there are also demand side financial frictions this single banker/investor model would understate the role of collateral values in mitigating this friction.

work resembles Adrian & Shin (2010) who model supply side financial heterogeneity in terms of financial firms ability to value assets.

Finally, in contrasts to these previous contributions, I follow Iacoviello (2005) and Iacoviello & Neri (2010) who explicitly model a housing market. In particular, I do not interpret asset prices generally, but instead I model assets whose underlying value as collateral is tied to home prices. Although my inclusion of demand side and supply side financial frictions in terms of large productive banks and small unproductive banks is novel, this is one of the clearest contributions of this paper. By focusing exclusively on assets whose values are tied to house prices, I am able to generate financial market deterioration from changes in primitives such as technology and preferences.

4.2.2 Model Description

The model consists of a household, a housing producer, a continuum of entrepreneurs who produce the final consumption good, a banking sector with 2 types of banks (productive and unproductive), both of whom finance investment for goods producers and lastly a central bank is modeled. In this section I will describe the behavior of each agent in turn. Many of the details including the full set of model equations are in the appendix as to not distract from the basic mechanism at work in the model. I follow the convention throughout the model that lower case variables are nominal and uppercase variables are in real terms - including interest rates.

4.2.2.1 Household

The household earns wages by renting labor to goods producers L_t and home builders L_t^H . Additionally, they earn non-labor income from banks in the form of dividends $p_t Div_t$, transfer payments $p_t Trans_t$ and principal plus interest payments $p_{t-1} D_{t-1} r_{t-1}^D$ on last periods deposits. The monetary authority may transfer any revenue back to the household in lump-sum form via $p_t T_t$. This income can be saved in the form of bank deposits $p_t D_t$ or spent on consumption $p_t C_t$ and housing $p_t^H H_t$. Also, any non-depreciated housing stock, $(1 - \delta) H_{t-1}$, can

be resold at the market price p_t^H . The resulting budget constraint in any period $t = 0, 1, 2, \dots$ is given by,

$$p_t C_t + p_t^H [H_t - (1 - \delta^H) H_{t-1}] + p_t D_t \leq w_t L_t + w_t^H L_t^H + p_{t-1} D_{t-1} r_t^D + p_t Div_t + p_t Trans_t + p_t T_t.$$

Although I follow Iacoviello (2005) and Iacoviello & Neri (2010) by modeling a housing market, I do not include home equity borrowing constraints on the household side. Where they focus on the wealth effects of housing price changes in terms of financing consumption, I focus more so on real-estate as collateral for investment and the interaction of house prices and housing-secured assets. That being said, in general equilibrium changes in home prices can impact consumption via traditional income and substitution effects, but not through the home equity channel highlighted by Iacoviello (2005).

The household maximizes their lifetime expected utility subject to the flow budget constraint above. The household's lifetime expected utility is specified by

$$U = \sum_{i=0}^{\infty} \mathbb{E}_t \left\{ \ln(C_{t+i}) + \eta_{t+i}^H \ln(H_{t+i}) + \eta^L \ln(1 - L_{t+i}) + \eta^{L^H} \ln(1 - L_{t+i}^H) \right\},$$

where η_t^H represents a shift in the elasticity of demand for housing. I specify this as an exogenous process following a first order auto-regressive process, in line with Iacoviello & Neri (2010).

$$\ln(\eta_t^H) = (1 - \rho_{\eta^H}) \bar{\eta}^H + \rho_{\eta^H} \ln(\eta_{t-1}^H) + \varepsilon_t^{\eta^H} \quad \varepsilon_t^{\eta^H} \sim \mathcal{N}(0, \sigma_{\eta^H}) \quad (4.1)$$

4.2.2.2 Goods Production

The goods producing sector is comprised of a continuum of entrepreneurs. Entrepreneurs have limited resources to finance capital required to produce the final good so they must borrow from banks. A financial friction arises whereby entrepreneurs borrow funds from banks this period to purchase capital used in production next period. Their output next

period is subject to idiosyncratic productivity disturbances only observable by banks after paying a monitoring cost.

Entrepreneur's Debt Contract: The Demand Side Financial Friction

A continuum of entrepreneurs $j \in \mathbb{R}_+$ supply wholesale goods to retailers using capital and labor. Entrepreneurs only live for 2 periods and only care about their second period utility. In the first period they have no endowment and no technology but they have a unit of labor supply. In the second period of their lives they are endowed with 1 unit of an asset which can be narrowly thought of as land, \bar{N} , which can be transformed into housing only by entrepreneurs and banks. However, capital must be purchased this period to be useful tomorrow. Denote the quantity of capital purchased in period t by entrepreneur j by K_t^j and denote the period t price of capital by q_t . To purchase this capital the entrepreneur will receive financing from the banking sector. More specifically, the entrepreneur uses last period's wages $p_t W_t^E$ and pledges tomorrow's endowment \bar{N}^j as collateral for a secured loan in the amount $p_t^N \bar{N}^j$ and the remaining portion of the capital purchase is financed with an unsecured loan in the amount $p_t B_t^j$. More concretely,

$$q_t K_t^j = p_t^N \bar{N}^j + p_t B_t^j + p_t W_t^E \quad (4.2)$$

is entrepreneur j 's budget constraint.

Without default, distinguishing between secured and unsecured loans is trivial. However, in the second period of their life, entrepreneurs are subjected to an idiosyncratic productivity shock ω_{t+1}^j which is *i.i.d.* across entrepreneurs and time. I assume throughout the analysis in this paper, $\omega_{t+1}^j \sim \ln \mathcal{N}\left(\frac{-(\sigma_t^\omega)^2}{2}, \sigma_t^\omega\right)$ with CDF at time t denoted by $F_t(\omega_{t+1}^j)$. The choice of parameters implies $\mathbb{E}\{\omega_{t+1}^j\} = 1$ so that in the aggregate this idiosyncratic shock has no direct impact on production, but the existence of uncertainty at the firm level impacts aggregate output through financial imperfections (BGG). To capture exogenous increases in the cross-sectional dispersion of idiosyncratic productivity shocks I allow σ_t^ω to vary over

time. I posit the simple auto-regressive process,

$$\ln(\sigma_t^\omega) = (1 - \rho_{\sigma^\omega}) \overline{\sigma^\omega} + \rho_{\sigma^\omega} \ln(\sigma_{t-1}^\omega) + \varepsilon_t^{\sigma^\omega} \quad \varepsilon_t^{\sigma^\omega} \sim \mathcal{N}(0, \sigma_{\sigma^\omega}) \quad (4.3)$$

for this demand-side *risk* shock. Christiano et al. (2013) show that such shocks have played a significant role in shaping the U.S. business cycle. Moreover, these shocks prove useful in the empirical analysis of the paper as they provide a structural interpretation for exogenous increase in the external finance premium.

Since projects are financed before the idiosyncratic productivity shock can be observed by either the entrepreneur or the bank, entrepreneurs who receive a low productivity value will default upon their loan. Denote the real gross interest rate on unsecured loans by $R_t^{L,j}$ and denote the gross real return on capital common to all entrepreneurs by R_{t+1}^K . Then for any entrepreneur j , we can define the cut-off value of $\bar{\omega}_{t+1}^j$ by the equation

$$\bar{\omega}_{t+1}^j R_{t+1}^K K_t^j q_t = p_t B_t^j R_t^{L,j}. \quad (4.4)$$

This equation defines the minimum level of productivity needed to pay back the unsecured loan. For entrepreneur j , the loan will be repaid if $\omega_{t+1}^j \geq \bar{\omega}_{t+1}^j$ and will otherwise be defaulted upon. However, the bank can not observe the level of productivity without paying an auditing cost in proportion $\mu \in (0, 1)$ to the entrepreneur's revenue.³ Banks who do not pay for auditing never find out if the entrepreneur actually received a low productivity draw or if they simply chose to renege on their loan. Given this arrangement, the optimal debt-contract dictates that banks will audit only defaulting entrepreneurs and only entrepreneurs who receive a bad-draw will default on their loans.

To make matters more explicit I define the expected revenue to the bank for loaning $p_t B_t^j$ to entrepreneur j in (5.17). This expected revenue is comprised of 2 terms, the first of

³This follows from Townsend (1979), but has been popularized in this context by Carlstrom & Fuerst (1997) and BGG.

which is non-defaulting loan revenue and the second is revenue net of auditing costs on non-performing loans.

$$\underbrace{\left(1 - F_t(\bar{\omega}_{t+1}^j)\right) p_t B_t^j R_t^L}_{\text{Non-Defaulting Payoff}} + \underbrace{(1 - \mu) \int_0^{\bar{\omega}_{t+1}^j} \omega_{t+1}^j dF_t(\omega_{t+1}^j) R_{t+1}^K K_t^j q_t}_{\text{Defaulting Payoff}} \quad (4.5)$$

For the bank to be willing to make this loan, this expected pay-off must be at least equal to bank's cost of making the loan. In BGG the cost of making the loan is simply the cost of obtaining the funds via deposits - $p_t B_t^j R_t^D$. However since the banking sector in this model has market power this is no longer the case. For the time being simply define the real cost per dollar loaned by R_t^E .⁴ Then the incentive compatibility constraint can be expressed as,

$$p_t B_t^j R_t^E = \left(1 - F_t(\bar{\omega}_{t+1}^j)\right) p_t B_t^j R_t^L + (1 - \mu) \int_0^{\bar{\omega}_{t+1}^j} \omega_{t+1}^j dF_t(\omega_{t+1}^j) R_{t+1}^K K_t^j q_t. \quad (4.6)$$

We can simplify this expression (and the resulting entrepreneur's optimization problem) by defining the following terms. First let $G_t(\bar{\omega}_{t+1}^j)$ be defined as the expected productivity value for defaulting entrepreneurs.

$$G_t(\bar{\omega}_{t+1}^j) = \int_0^{\bar{\omega}_{t+1}^j} \omega_{t+1}^j dF_t(\omega_{t+1}^j) \quad (4.7)$$

Also let $\Gamma_t(\bar{\omega}_{t+1}^j)$ be defined as the expected share of entrepreneurial profits going to the bank gross of auditing costs.

$$\Gamma_t(\bar{\omega}_{t+1}^j) = \left(1 - F_t(\bar{\omega}_{t+1}^j)\right) \bar{\omega}_{t+1}^j + G_t(\bar{\omega}_{t+1}^j) \quad (4.8)$$

Now I can combine (4.6) with (5.16), (5.19) and (5.20) to rewrite the bank's incentive

⁴ R_t^E is explicitly defined in the description of the banking sector in the appendix using the approach laid out in Hafstead & Smith (2012)

compatibility as,

$$p_t B_t^j R_t^E = \left(\Gamma_t(\bar{\omega}_{t+1}^j) - \mu G_t(\bar{\omega}_{t+1}^j) \right) R_{t+1}^K K_t^j q_t. \quad (4.9)$$

We can now formally state the problem faced by entrepreneur j . To keep the debt-contract tractable, I assume the entrepreneur is risk-neutral with regards to aggregate consumption. In particular, I assume they seek to maximize their income and then allocate that income between consumption and housing services. Entrepreneur j therefore seeks to maximize total income⁵ subject to the bank's IC constraint.

$$\begin{aligned} \max_{K_t^j, \bar{\omega}_{t+1}^j} \quad & \left(1 - \Gamma_t(\bar{\omega}_{t+1}^j) \right) R_{t+1}^K K_t^j q_t \\ \text{subject to} \quad & \left[q_t K_t^j - p_t^N \bar{N}^j - p_t W_t^E \right] R_t^E = \left(\Gamma_t(\bar{\omega}_{t+1}^j) - \mu G_t(\bar{\omega}_{t+1}^j) \right) R_{t+1}^K K_t^j q_t \end{aligned}$$

The solution to this optimization problem pins down the cut-off value $\bar{\omega}_{t+1}^j$ and the entrepreneur's demand for capital K_t^j .⁶ The problem is identical in nature to the problem entrepreneurs face in BGG who show the optimal debt contract has the property that the default rate and external finance premium move inversely with net-worth. In this model, the net-worth component is replaced with the collateral value, implying that $\partial \bar{\omega}_{t+1}^j / \partial p_t^N < 0$ - finance premiums and default rates will move in the opposite direction of collateral prices.

Aggregate Goods Production

The previous section describes the firm-level behavior in the goods producing sector, specifically it describes the debt-contract problem faced by each producer. In this section, I describe the industry wide behavior. Each entrepreneur (in the second period of their life) has access to the production technology, $Y_t^j = \omega_t^j Z_t^G \left(K_{t-1}^j \right)^{\alpha_G} \left(L_t^{G,j} \right)^{1-\alpha_G}$, which can be aggregated over due to constant returns to scale. The aggregate goods production technology

⁵Notice the entrepreneur's income can be re-written as $\int_{\bar{\omega}_{t+1}^j}^{\infty} \omega_{t+1}^j dF(\omega_{t+1}^j) R_{t+1}^K K_t^j q_t - \left(1 - F_t(\bar{\omega}_{t+1}^j) \right) R_t^L B_t^j = \left(1 - \Gamma_t(\bar{\omega}_{t+1}^j) \right) R_{t+1}^K K_t^j q_t$ where I use (5.16) and (5.20).

⁶The first order conditions for this problem are in the appendix.

in any given period t is specified as

$$Y_t = Z_t^G (K_{t-1})^{\alpha_G} (L_t^G)^{1-\alpha_G} \quad (4.10)$$

where Z_t^G is an exogenous technology process which affects all entrepreneurs equally. I assume this technology follows a first-order autoregressive process.

$$\ln(Z_t^G) = (1 - \rho_{Z^G}) \overline{Z^G} + \rho_{Z^G} \ln(Z_{t-1}^G) + \varepsilon_t^{Z^G} \quad \varepsilon_t^{Z^G} \sim \mathcal{N}(0, \sigma_{Z^G}) \quad (4.11)$$

The gross aggregate real return on holding a unit of capital from period $t-1$ to period t is defined by

$$R_t^K = \frac{\alpha_G \frac{Y_t}{K_{t-1}} + (1 - \delta^K) Q_t}{Q_{t-1}}, \quad (4.12)$$

where I utilize the aggregate marginal product of capital from the Cobb-Douglas specification above - $MP_K = \alpha_G \frac{Y_t}{K_{t-1}}$.

The labor aggregate in the production function is a composite of labor supplied by the household, L_t , and labor supplied by this periods young entrepreneurs, L_t^E ,

$$L_t^G = (L_t^E)^{\alpha_E} (L_t)^{1-\alpha_E}. \quad (4.13)$$

This implies the wage paid to the household's labor and the wage paid to entrepreneurial labor are given by,

$$W_t = (1 - \alpha_G)(1 - \alpha_E) \frac{Y_t}{L_t} \quad (4.14)$$

$$W_t^E = (1 - \alpha_G) \alpha_E \frac{Y_t}{L_t^E} \quad (4.15)$$

I calibrate $\alpha_E = .01$ so that in equilibrium the household receives the majority of wages and variations in collateral values are the primary sources of movement in entrepreneur's balance sheets. The aggregate income of entrepreneurs in period t is $(1 - \Gamma_{t-1}(\bar{\omega}_t)) R_t^K K_{t-1} q_{t-1}$.

I assume entrepreneurs, like the household, receive utility from consuming both the consumption good and housing services. Unlike households, entrepreneurs have the ability to transform their endowment of ‘land’ - \bar{N}^j - into non-tradable housing. Recall however, this endowment was leveraged last period to secure a loan in the amount $p_{t-1}^N \bar{N}^j$. Hence, entrepreneurs who are able, choose to payback the secured loan with interest $p_{t-1}^N \bar{N}^j r_{t-1}^D$ and then convert \bar{N}^j into housing services one for one. If they don’t payback the secured loan then they default on this contract and the bank takes possession of the collateral \bar{N}^j .

I assume in the aggregate, all the entrepreneurs who did not default on their unsecured loan, payback their secured loan and use the rest of their income on the consumption good. Those who defaulted on the unsecured loan have lost all income and hence do not consume anything. More specifically, the aggregate real consumption of entrepreneurs is given by

$$C_t^E = (1 - \Gamma_{t-1}(\bar{\omega}_t)) R_t^K K_{t-1} Q_{t-1} - (1 - F_{t-1}(\bar{\omega}_t)) P_{t-1}^N \bar{N}^j R_{t-1}^D. \quad (4.16)$$

“What micro-level preferences would give rise to this aggregate consumption behavior?” is an interesting question. In the appendix I describe one possible micro-structure that would lead to this aggregate consumption behavior. An appealing aspect of this description is the existence of a single default rate in the economy.⁷

4.2.2.3 New Housing Production

I assume new housing is produced in a purely competitive market and free from financial frictions. In particular, housing producers combine labor L_t^H with housing specific technology, Z_t^H in the production technology,

$$H_t^{New} = Z_t^H (L_t^H)^{1-\alpha_H} \quad (4.17)$$

⁷That is to say, the default rate on unsecured loans is the same as the default rate on secured loans. I choose this as a starting point although this assumption can be relaxed.

where

$$\ln(Z_t^H) = (1 - \rho_{ZH}) \overline{Z^H} + \rho_{ZH} \ln(Z_{t-1}^H) + \varepsilon_t^{ZH} \quad \varepsilon_t^{ZH} \sim \mathcal{N}(0, \sigma_{ZH}). \quad (4.18)$$

I model housing specific technology independent of technology in the goods producing sector since much of the economic growth over the last two decades has been IT-driven and housing production is a non IT-intensive industry. Moreover, this specification allows for goods technology process, Z_t^G , to play a significant role in determining output without implying a counterfactual negative correlation between home prices and GDP (see for example Davis & Heathcote (2005)). The resulting demand for labor from the housing sector takes the form

$$W_t^H = (1 - \alpha_H) P_t^H \frac{H_t^{New}}{L_t^H}. \quad (4.19)$$

4.2.2.4 Banking Sector: The Supply Side Financial Friction

There are a unit measure of banks in the model each belonging to one of two types. I distinguish bank types by a superscript “P” for productive banks and a superscript “U” for unproductive banks (who make up ν share of the population). Productive banks represent the *Large* commercial banks in the data. These banks are more productive with repossessed collateral pledged by entrepreneurs to secure loans and hence value these housing-secured assets more than their unproductive counterparts. However, this efficiency creates a moral hazard problem for borrowers due to the possibility of productive banks wrongfully repossessing collateral and absconding with the profits. If this occurs, the entrepreneur’s only recourse is to take the productive banks accumulated capital. To this extent, bank-capital mitigates the moral hazard concerns and allows the productive banks to hold more of these collateralized assets. An amplification effect emerges from the endogenous tightening and loosening of this moral hazard constraint which forces productive banks to adjust their holding of collateralized assets in response to movements in home prices.

For ease of exposition I describe factors common to both types of bank before describing each type's optimizing behavior. In particular, all banks have some degree of market power, face a balance sheet constraint and remit a fraction of profits each period back to the household in the form of dividends and various transfers. In what follows I generically refer to bank i to reference one of the infinitely many identical banks within either type.

Each bank possesses a degree of market power which is captured by assuming a Dixit-Stiglitz type aggregator function. As Hafstead & Smith (2012) point out, this has the simplifying feature that all banks serve all entrepreneurs and therefore face the same ex-ante and ex-post default rates. More specifically, aggregate loans are a CES index

$$B_t = \left(\int_0^1 B_t(i)^{\frac{\theta_B-1}{\theta_B}} \right)^{\frac{\theta_B}{\theta_B-1}} \quad (4.20)$$

where θ_B is the elasticity of substitution between different bank loans and is calibrated to match aggregate lending rates. The corresponding price index which is dual⁸ to this quantity index is given by

$$r_t^B = \left(\int_0^1 r_t^B(i)^{1-\theta_B} \right)^{\frac{1}{1-\theta_B}}. \quad (4.21)$$

This specification of the aggregate indexes implies each bank i of type $T \in \{P, U\}$ faces the downward sloping demand for loans,

$$B_t^T(i) = \left(\frac{r_t^{B,T}(i)}{r_t^B} \right)^{-\theta_B} B_t. \quad (4.22)$$

Each bank i must not only satisfy their demand for loans, but they must also abide to the balance sheet constraint,

$$p_t B_t^T(i) + p_t^N N_t^T(i) = p_t D_t^T(i) + p_{t-1} B K_{t-1}^T(i) \quad (4.23)$$

⁸Duality here refers to the price and quantity indexes which satisfy Fisher's factor reversal test, $\int_0^1 r_t^B(i) B_t(i) di = r_t^B B_t$.

which simply states that assets (bank loans) must equal liabilities (bank deposits) plus bank capital, respectively.

Since banks use share holder's retained earnings to fund risky loans, I assume shareholders (households) require compensation. More specifically, each period the bank is allowed (by the central bank) to expose a fraction ψ_t of bank capital to cover *expected* losses on unsecured defaulted loans. Each period t , the fraction of loans that actually default is given by $F_{t-1}(\bar{\omega}_t)$. Hence, each period the bank transfers the nominal payment

$$p_t Trans_t^T(i) = F_{t-1}(\bar{\omega}_t) p_{t-1} \psi_{t-1} BK_{t-1}^T(i) \quad (4.24)$$

to shareholders in order to compensate them for the capital that was exposed to covering losses on loans originated in period $t-1$. In this sense, the transfer to shareholders occurs only on realized, or ex-post, loan losses. Combining this transfer payment with dividend payments, bank capital evolves according to the following law of motion.

$$p_t BK_t^T(i) = \gamma_t^T p_t \Pi_t^T(i) - p_t Trans_t^T(i) + (1 - \delta^{BK}) p_{t-1} BK_{t-1}^T(i) \quad (4.25)$$

To summarize this, banks of type T pay out a time varying fraction γ_t^T of period t profits as dividends and invest the remaining fraction in bank capital. Additionally, banks compensate shareholders for exposing retained earnings to potential losses via $p_t Trans_t^T$ and lose a fraction of bank capital to depreciation. Following Gerali et al. (2010), I assume bank investment decisions are made independently from bank profit maximization, however, I assume this fraction is time varying. In particular, I assume that $\gamma_t^T = \frac{\bar{\gamma}^T}{p_t \Pi_t^T}$ so that each period a constant amount of new equity is injected into the banks from shareholders. This implies realistically that banks respond to falling profits by paying out a smaller share of profits in dividends. Before proceeding to a specific description of each type of bank's problem, it useful to summarize these transfers by defining net investment in the banking

sector.

$$I_t^{BK} = \sum_{T \in \{P, U\}} s_T \left(\gamma_t^T p_t \Pi_t^T - p_t \text{Trans}_t^T \right) \quad \text{where } s_T = \begin{cases} \nu & \text{if } T = U \\ 1 - \nu & \text{if } T = P \end{cases} \quad (4.26)$$

Productive Bank

The productive bank enters each period t with inflows consisting of maturing unsecured loans $r_{t-1}^{B,P}(i) p_t B_{t-1}^P(i)$ and maturing secured loans $p_{t-1}^N N_{t-1}^P r_{t-1}^D$, of which, $(1 - F_{t-1}(\bar{\omega}_t))$ will be repaid in full. Denote the real income of all borrowers who are unable to repay last periods loan by $\Phi_{t-1}(\bar{\omega}_t)$. The productive bank will receives the fraction $\frac{B_{t-1}^P(i)}{B_{t-1}}$ of $\phi_{t-1}(\bar{\omega}_t)$ net of auditing costs μ for the unsecured loan defaults. Additionally, the productive bank repossesses $F_{t-1}(\bar{\omega}_t) N_{t-1}^P(i)$, which is the collateral posted on the secured loans who defaulted. This repossessed collateral is transformed in to housing using the technology common to all banks, $H_t^R(i) = Z^R N_{t-1}^P(i)$. Finally, the productive bank also has incoming deposits totaling $p_t D_t^P(i)$. At the same time, the productive bank has outflows of newly originated unsecured and secured loans totaling $p_t B_t^P(i) + p_t^N N_t^P(i)$ and maturing deposits from period $t-1$ totaling $p_{t-1} D_{t-1}^P(i) r_{t-1}^D$. This is stated more concisely below in (4.30) which defines the productive bank's period t nominal profits.

$$\begin{aligned} p_t \Pi_t^P(i) = & (1 - F_{t-1}(\bar{\omega}_t)) r_{t-1}^{B,P}(i) p_t B_{t-1}^P(i) + (1 - \mu) \frac{B_{t-1}^P(i)}{B_{t-1}} p_t \Phi_{t-1}(\bar{\omega}_t) \\ & + (1 - F_{t-1}(\bar{\omega}_t)) p_{t-1}^N N_{t-1}^P(i) r_{t-1}^D + F_{t-1}(\bar{\omega}_t) p_t^H Z^R N_{t-1}^P(i) \\ & - p_t B_t^P(i) - p_t^N N_t^P(i) - p_{t-1} D_{t-1}^P(i) r_{t-1}^D + p_t D_t(i) \end{aligned} \quad (4.27)$$

The productive bank's ability to liquidate repossessed collateral - N_t^P at zero marginal cost raises a moral hazard. In particular, if the productive bank were to claim default on all the secured loans originated in period t and repossess the collateral the following period, they would earn a gross real return totaling $N_t^P \mathbb{E}_t \left\{ \frac{Z^R P_{t+1}^H - (1 - F_t(\bar{\omega}_{t+1}) R_t^D P_t^N)}{P_t^N} \right\}$. The first term

represents income from the selling the unlawfully repossessed collateral and the second term subtracts the foregone income that would have been received from entrepreneurs paying back their loans. I assume a fraction of this return will be lost when taken so that productive banks only net a fraction $\psi_t^{N,P}$ of this return. This varies stochastically according to the AR(1) process.

$$\ln(\psi_t^{N,P}) = (1 - \rho_{\psi^{N,P}}) \overline{\psi^{N,P}} + \rho_{\psi^{N,P}} \ln(\psi_{t-1}^{N,P}) + \varepsilon_t^{\psi^{N,P}} \quad \varepsilon_t^{\psi^{N,P}} \sim \mathcal{N}(0, \sigma_{\psi^{N,P}}) \quad (4.28)$$

These disturbances provide a model equivalent to the large bank share shock from that will be analyzed in the VAR.

If the productive bank chooses to abscond with the assets, entrepreneurs are entitled to the remaining equity of the bank after preferred shareholders (households) receive their risk premium. Hence, entrepreneurs would be entitled to a claim of $(1 - \psi_t F_t(\bar{\omega}_{t+1})) BK_t^P(i)$. Thus, the incentive for productive banks to claim default and abscond with these housing-secured assets is eliminated when the equity claims of exploited entrepreneurs exceeds the gross return on unlawful liquidations.

$$\underbrace{(1 - \psi_t F_t(\bar{\omega}_{t+1})) BK_t^P(i)}_{\text{Equity Claim of Exploited Entrepreneurs}} \geq \underbrace{N_t^P \mathbb{E}_t \left\{ \frac{Z^R P_{t+1}^H - (1 - F_t(\bar{\omega}_{t+1})) R_t^D P_t^N}{P_t^N} \right\} \psi_t^{N,P}}_{\text{Gross Return on Unlawful Repossessions}} \quad (4.29)$$

When (4.29) holds with equality, the productive bank will be limited in how many housing-secured assets it can hold. Moreover, this constraint will endogenously loosen and tighten in response to various macroeconomic shocks which affect home prices or default rates. Let Λ_t denote the household's stochastic discount factor used for valuing future real payments. The problem faced by the productive bank is then defined below.

$$\max_{\{r_{t+j}^{B,P}(i), B_{t+j}^P(i), N_{t+j}^P(i), D_{t+j}^P(i)\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \mathbb{E}_t \{ \Lambda_{t+j} \Pi_{t+j}^P(i) \} \quad \text{subject to} \quad (4.22), (4.23), (4.29)$$

Due to the complications that arise from solving a model with an occasionally binding constraint, I calibrate the model so that the productive bank's moral hazard constraint binds in the non-stochastic steady-state.

Unproductive Bank

The unproductive bank is identical to the productive bank with one noticeable exception - they are less productive. To make matters more concrete, when a secured loan defaults the unproductive bank repossesses collateral $F_{t-1}(\bar{\omega}_t)N_{t-1}^U(i)$. Unlike their productive counterparts, the unproductive bank liquidates this collateral while bearing an increasing marginal cost. On defaulted secured loans, the unproductive bank transforms repossessed collateral into housing yielding revenue $p_t^H Z^R N_{t-1}^U(i)$ at a resource cost of $p_t \mu^{R,U} \left(N_{t-1}^U(i)\right)^{\chi^{R,U}}$. This captures the heterogeneity between commercial banks (empirically found by (Wheelock & Wilson, 2012; Bos & Kolari, 2013)) with regards to their ability to evaluate and trade off-balance sheet assets. Most notably, as unproductive banks increase their holding of these assets, the value of the assets will fall due to the increasing marginal cost. Hence, the market liquidity of such assets depends on the distribution of these assets (i.e. it depends on who is holding the assets). A point first made by KM and applied to here to housing backed securities within this model.

With this exception, the unproductive bank's profit function is very similar to the productive bank's stated below.

$$\begin{aligned}
p_t \Pi_t^U(i) = & (1 - F_{t-1}(\bar{\omega}_t)) r_{t-1}^{B,U}(i) p_t B_{t-1}^U(i) + (1 - \mu) \frac{B_{t-1}^U(i)}{B_{t-1}} p_t \Phi_{t-1}(\bar{\omega}_t) \\
& + (1 - F_{t-1}(\bar{\omega}_t)) p_{t-1}^N N_{t-1}^U(i) r_{t-1}^{D,U} \\
& + F_{t-1}(\bar{\omega}_t) \left[p_t^H Z^R N_{t-1}^U(i) - p_t \mu^{R,U} \left(N_{t-1}^U(i)\right)^{\chi^{R,U}} \right] \\
& - p_t B_t^U(i) - \chi^{B,U} B_t^U(i) - p_t^N N_t^U(i) - p_{t-1} D_{t-1}^U(i) r_{t-1}^D + p_t D_t^U(i)
\end{aligned} \tag{4.30}$$

Notice the lack of productivity spills over to unsecured loans. The parameter $\chi^{B,U}$ is cal-

ibrated to match the average share of resources allocated to financial intermediation. The increasing resource cost of repossessing collateral implies the unproductive bank is *not* subject to a moral hazard constraint. In particular, if any single unproductive bank i attempted to purchase a large amount of these assets at a given market price p_t^N and falsely claim default, their cost of liquidating the assets would exceed what they paid for them. Therefore the existence of these increasing marginal cost eliminates any incentive to steal away these assets.

Let Λ_t denote the household's stochastic discount factor used for valuing future real payments. The problem faced by the unproductive bank is then defined below.

$$\max_{\{r_{t+j}^{B,U}(i), B_{t+j}^U(i), N_{t+j}^U(i), D_{t+j}^U(i)\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \mathbb{E}_t \left\{ \Lambda_{t+j} \Pi_{t+j}^U(i) \right\} \quad \text{subject to} \quad (4.22), (4.23)$$

4.2.2.5 Central Bank

The central bank is charged with setting a *macroprudential* policy rule and a monetary policy rule. The macroprudential policy instrument is the regulatory maximum share of capital that can be allocated to loan losses. This essentially controls the amount of owners equity the bank can allocate to cover loan losses. Here, I assume the central bank simply sets this to a constant level,

$$\psi_t = \bar{\psi}. \quad (4.31)$$

As for the monetary policy instrument I assume the central bank follows the simple interest rate rule whereby the rate on one-period deposits adjusts to itself lagged, the inflation rate, and the growth rate of real GDP:

$$\left(\frac{r_t^D}{r^D} \right) = \left(\frac{r_{t-1}^D}{r^D} \right)^{\rho_r} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\psi_\pi} \left(\frac{GDP_t}{GDP_{t-1}} \right)^{\psi_{GDP}}. \quad (4.32)$$

4.2.2.6 Market Clearing

Sections 5.2.1.1 - 5.2.1.8 describe the optimal behavior of all agents in the economy. A symmetric competitive equilibrium is defined as a sequence of quantities, prices and Lagrange multipliers (shadow prices) which satisfy all optimality conditions, policy rules and market clearing conditions. In particular, the demand for housing must equate the supply of housing on the market which consists of newly built homes, repossessed collateral being liquidated on the housing market and non-depreciated housing from last period. Put more simply,

$$H_t = H_t^{New} + F_{t-1}(\bar{\omega}_t)Z^R(\nu N_{t-1}^U + (1-\nu)N_{t-1}^P) + (1-\delta^H)H_{t-1}. \quad (4.33)$$

The above expression can be further simplified by noting that the market for secured lending (or more narrowly, land) clears when

$$\bar{N} = \nu N_t^U + (1-\nu)N_t^P, \quad (4.34)$$

where the left hand side is the aggregate endowment of entrepreneurs. By the ex-ante symmetry among entrepreneurs this is required to equal $\bar{N} = \bar{N}^j$ for all entrepreneurs j . Similarly, this ex-ante symmetry also implies the demand for capital by entrepreneurs is identical, or $K_t = K_t^j$ for all entrepreneurs j . I assume that capital can be transformed one for one from the final good and depreciates at rate δ^K . Therefore, capital evolves according to,

$$K_t = I_t + (1-\delta^K)K_{t-1}. \quad (4.35)$$

Since I do not include adjustment costs, the price of capital equals the price of the final good at all times, $q_t = p_t$. Adjustment costs in the production of capital could easily be added, as in BGG. However, in this model, they are not needed to generate an amplification effect. Instead, the asset price spirals occur from the redistribution of assets between agents as in KM. With this description of the model, the goods market clearing condition is satisfied

whenever,

$$Y_t = C_t + C_t^E + I_t + I_t^{BK} + \mu\Phi_{t-1}(\bar{\omega}_t) + \nu\mu^{R,U} \left(N_{t-1}^U(i)\right)^{\chi^{R,U}} + \nu\chi^{B,U} B_t^U, \quad (4.36)$$

which stipulates that the consumption good must be either consumed by the household or the entrepreneur, invested in bank capital, or used to audit or repossess the collateral of defaulting entrepreneurs. It is useful for the purpose of calibration and model inference to define GDP in this multi-sector model.

$$GDP_t = C_t + I_t + P_t^H H_t^{New} \quad (4.37)$$

4.2.3 Calibration

The model is calibrated to match characteristics of the U.S. economy from 1998 to 2012 and each time period is interpreted as one quarter. In order to numerically solve the model, there are 23 non-shock parameters and 15 shock parameters which must first be assigned values. Beginning with the household's parameters I calibrate $\beta = .99$ as to match up the steady state deposit rate in the model with the average rate on 3-Month U.S. Treasury Bills. I set the utility on non-housing leisure and housing leisure, $\eta^{L^H} = 7.43$ and $\eta^L = 1.88$ which matches the share of labor supplied in housing equal to 5%, the U.S. average using data from the BLS and the total share of time spent working equal to 1/3. Finally, the last of the preference parameters $\bar{\eta}^H = .2352$ calibrates the steady state real price of housing so that consumption's share of GDP $= \frac{C}{GDP} = .79$, which is the average ratio of personal consumption expenditures to personal consumption expenditures and private investment for the U.S. Similarly, setting $\delta^H = .021$ implies the share of housing wealth to annual GDP, $\frac{P^H H}{4 \times GDP} = 1.4$.

On the production side, I set the share of income going to labor in the goods producing sector, $1 - \alpha_G = .7$ and the same share in the housing sector $1 - \alpha_H = .8$ following Iacovello

and Neri (2010). I normalize the ‘land’ endowment of entrepreneurs, $\bar{N} = 1$. As for the financial accelerator parameters, I collectively set $\mu^M = .14$ and $\bar{\sigma}^\omega = .21$. The auditing cost parameter falls between the value from Christiano et al. (2013) and BGG and the steady state value of the variance of the idiosyncratic productivity shock implies an annual steady state default rate of $F(\bar{\omega}) = .01$ which is the average default rate on C & I loans secured by real-estate using data obtained from the St. Louis Fed’s FRED database.

With regards to the banking sector, I set the share of capital allocated to loan losses, $\psi = .25$, the average of loan-loss allocations to the equity of commercial banks over this period according to data obtained from FRED. I normalize each bank’s share of the population to be equal by setting $\nu = \frac{1}{2}$. The depreciation rate on capital is set at $\delta^{BK} = .08$ for a baseline calibration, following Gerali et al. (2010). The value for $\nu\chi^{B,U} = .0004954$ is obtained from Hafstead & Smith (2012) who create a time series of banking productivity in loan intermediation. I set the steady-state rate of return on entrepreneurial loans equal to the average prime loan rate, $r^E = 1.017$. This pins down the elasticity of substitution between bank loans, $\theta_B = 157.21$.⁹ There is little agreement over the real return on capital, I set it equal to equal to 10% per annum which is slightly below the real return on capital in the U.S. estimated by Oulton & Rincon-Aznar (2009).¹⁰ This value also matches the annualized return on small-cap stocks, representing firms who are likely to be financially constrained, using data from Morningstar.

I set $\bar{\gamma}^P = .003$ and $\bar{\gamma}^U = .001$. These values simply ensure the transfers made to the household for exposing equity to loan losses, is made up for with equity injections sufficient to guarantee a positive steady-state level of bank capital. Similarly, setting $\bar{\psi}^{N,P} = .12$ implies in steady state, the share of housing-secured assets held by the productive banks, $\frac{N^P}{N^P + N^U} = .92$, the average share of total credit exposure concentrated in large banks, as explained in Section (4.3.2). Additionally, I set $\chi^{R,U} = 1.06$, implying strictly convex cost

⁹Hafstead & Smith (2012) find a similarly high value for $\theta_B = 260$.

¹⁰Oulton & Rincon-Aznar (2009) estimate the average annual real return on capital to be 13%, however they acknowledge this estimate is potentially biased upwards. Therefore, I follow Hafstead & Smith (2012) and set the annual return to 10%.

of repossessing/liquidating collateral for the unproductive bank. This value is adjusted in the simulations below. This together with setting $\mu^{R,U} = 0.2755$ calibrates the steady state price of the housing-secured assets so that $\frac{P^N \bar{N} + B}{P^N \bar{N}} = 2.5$, or the average ratio of C&I loans plus total credit exposure to total credit exposure. Finally, normalizing $Z^R = 1$ and setting $\bar{Z}^H = 1.14$ implies the real-estate owned share, or REO share, $\frac{F(\bar{\omega}) Z^R \bar{N}}{F(\bar{\omega}) Z^R \bar{N} + H^{New}} = .1775$ which is the value in the data according to RealtyTrac.

As for the remaining policy parameters, steady state gross inflation is set equal to unity and $\rho_r, \psi_\pi = 1.5, \psi_{GDP} = 0.125$. The remaining shock parameters can not be pinned down by matching steady state values. The exogenous process, $\psi_t^{N,P}$, which governs the productive bank's moral hazard constraint is used to highlight the impact of asset redistribution between banks, and is largely not structural. Therefore, I set $\rho_{\psi^{N,P}} = .9$ and $\sigma_{\psi^{N,P}} = .01$.

The remaining exogenous processes are calibrated using a moments matching exercise. In particular, I choose $\rho_{Z^G} = 0.9338, \sigma_{Z^G} = 0.0157, \rho_{Z^H} = 0.6998, \sigma_{Z^H} = 0, \rho_{\eta^H} = 0.8959, \sigma_{\eta^H} = 0.0665, \rho_{\sigma^\omega} = 0.8898$ and $\sigma_{\sigma^\omega} = 0.0455$ to match the model's standard deviation and first-order autocorrelation of: the external finance premium (proxied by the spread between BAA corporate bond-rate and 10-year treasuries), real GDP¹¹, real private investment and real home prices. This exercise not only pins down values for the model's driving shocks, but since I do not restrict the calibration strategy to match the model's implied correlation matrix, it allows for an empirical examination of the model's performance.

The model fits the data reasonably well with all moments in the confidence interval. Comparing this model to the baseline BGG model augmented with a housing sector, it becomes clear why the celebrated BGG financial accelerator must be adjusted to analyze the financial crisis. The BGG financial contract assumes the borrowers wealth is liquid, therefore (real-estate) secured debt is absent in the model. This explains why BGG has difficulty capturing the dynamics between the EFP and both P^H and Investment. BGG's counter-

¹¹Real GDP is measured as the model equivalent. Hence, I sum personal consumption expenditures and private investment and deflate the resulting series by the civilian population over the age of 16 and the personal consumption expenditures excluding food and energy price deflator.

Table 4.1: Cyclical Properties of the Model

Correlation	5 Percent	Median	95 Percent	Model	BGG (1999)
EFP, GDP	-0.89	-0.68	-0.45	-0.82	0.03
EFP, P^H	-0.87	-0.60	-0.32	-0.68	0.18
EFP, Investment	-0.89	-0.63	-0.32	-0.88	-0.18
GDP, P^H	0.50	0.76	0.93	0.89	0.84
GDP, Investment	0.93	0.96	0.99	0.93	0.79
P^H , Investment	0.57	0.81	0.95	0.68	0.42

^a The data correlations and confidence intervals are computed using Jeffery's Prior and 5000 draws from the resulting posterior distribution of an estimated VAR(2).

factual correlation between GDP and the EFP stems largely from the documented puzzle that BGG's debt-contract generates an increases in the EFP following a positive technology shock. (Shen, 2011). These issues are absent in the model presented here since housing secured debt, and therefore home prices, play a critical role in the financial contract.

4.2.4 The Model's Amplifying Effect of Asset Redistribution

In this section I analyze the behavior of the following four variables:

1. External Finance Premium (EFP) = $\mu^{\frac{\phi_{t-1}(\bar{\omega}_t)}{B_{t-1}}}$
2. Real GDP = GDP_t
3. Real house prices = P_t^H
4. The share of housing-secured assets held by large banks = $\frac{(1-\nu)N_t^P}{\bar{N}}$

in response to the model's structural shocks. For each set of impulse response functions, I present the model's response when the productive bank's moral hazard constraint binds (the solid lines) and when this constraint is relaxed (the dashed lines). Notice that when the constraint is relaxed, the productive banks hold all of the housing-secured assets since they are significantly more productive. Hence, for this model, the *Large Bank Share* variable is constant.

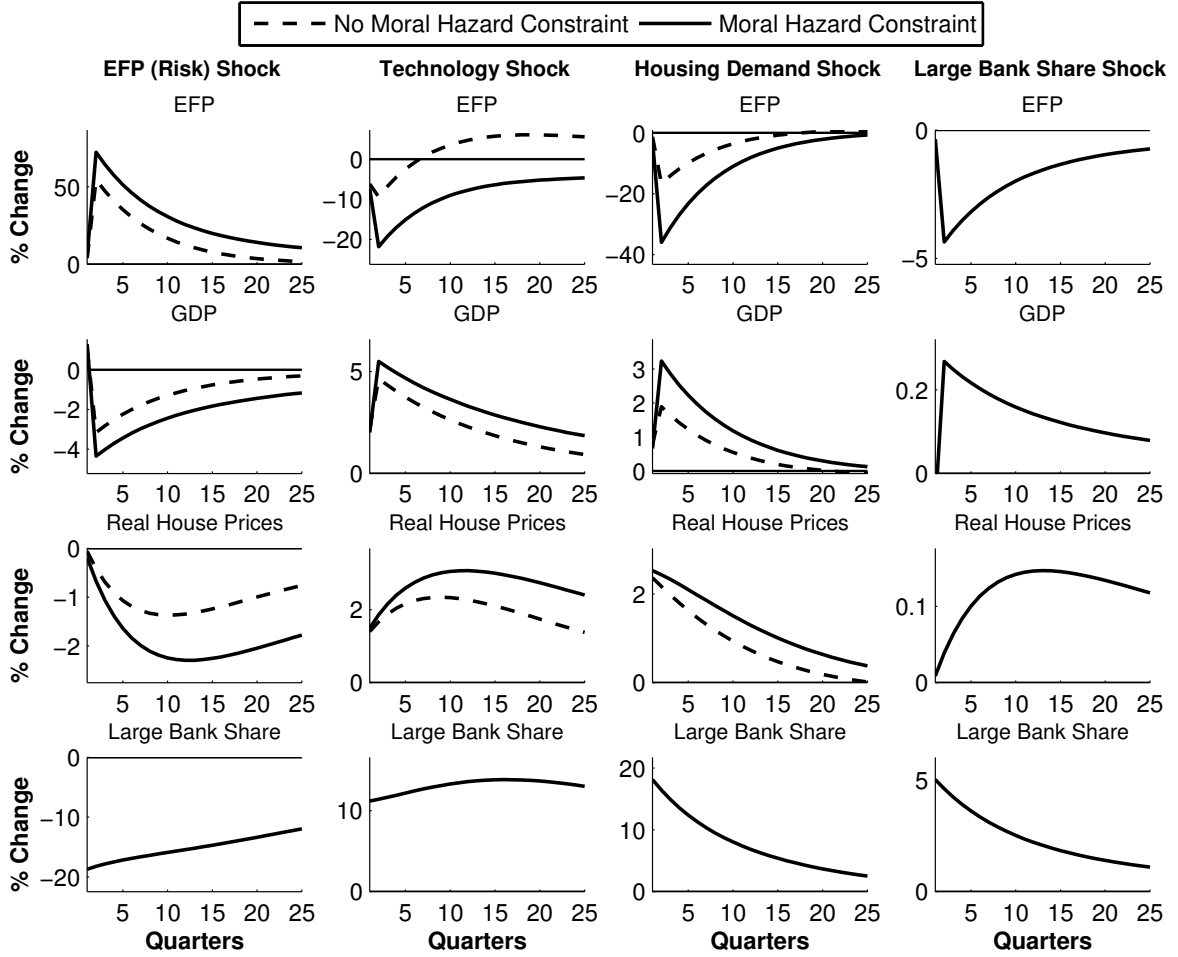


Figure 4.2: Impulse response functions from the equilibrium model. The solid lines denote the dynamic responses when the productive bank's moral hazard constraint binds and the dashed lines are the dynamics when this constraint is relaxed.

Figure 4.2 displays the equilibrium model's response of the endogenous variables to a detrimental risk shock, positive technology and housing demand shocks and an increase in the share of housing-secured assets held by the productive banks. The dynamics are noticeably different when the moral hazard constraint binds compared to the efficient allocation whereby large banks hold all of the housing-secured assets. In particular, the response of all the variables are *amplified*. Changes in the risk-characteristics of borrowers or the household's preferences towards housing are magnified by a factor of 2 when assets are redistributed between large and small banks. Even technology shocks raise GDP by 25% more at peak when

large banks are able to expand their housing-secured asset holdings. The key factor driving these changes are movements in home prices, which are themselves amplified.

I highlight this amplification effect in figure 4.3 which illustrates the differences between the two models simulated in the IRFs. The diagram shows the effect of a drop in home prices on the price of \bar{N} . In particular, the equation determining the price of housing-secured assets is given by the unproductive banks first order condition for N_t^U ,

$$P_t^N = \frac{1}{R_t^D} \mathbb{E}_t \left\{ Z^R P_{t+1}^H - \chi^{R,U} \mu^{R,U} N_t^U (\chi^{R,U} - 1) \right\} \quad (4.38)$$

which for $\chi^{R,U} > 1$ looks like a typical demand curve. If expected home prices fall, this will shift down the demand curve for these assets. Without any redistribution effect, asset prices fall from P_1^N to $P_1^{N'}$ - this is the dynamic captured in the figure on the left.

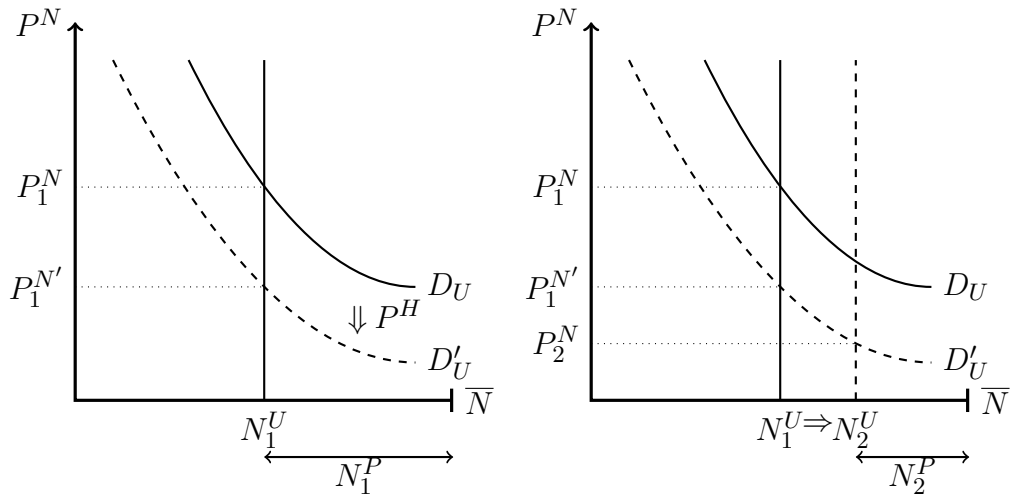


Figure 4.3: The graph on the left illustrates the impact on asset prices when home prices drop, without any redistribution effect. The graph on the right highlights the additional fall in asset prices that results when the productive bank must reduce its share of housing-secured secured assets due to the endogenously tightening moral hazard constraint.

To understand the amplification effect stemming from the redistribution of housing-secured assets, notice two things. (i) First, due to the positive marginal cost of liquidating collat-

eral, asset prices fall by *more* (in percentage terms) than expected home prices. That is, $\mathcal{E}_{P_t^N, \mathbb{E}_t\{P_{t+1}^H\}} > 1$.

$$\mathcal{E}_{P_t^N, \mathbb{E}_t\{P_{t+1}^H\}} = \frac{\partial P_t^N}{\partial \mathbb{E}_t\{P_{t+1}^H\}} \frac{\mathbb{E}_t\{P_{t+1}^H\}}{P_t^N} = \frac{Z^R \mathbb{E}_t\{P_{t+1}^H\}}{Z^R \mathbb{E}_t\{P_{t+1}^H\} - \chi^{R,U} \mu^{R,U} N_t^U (\chi^{R,U} - 1)} > 1 \quad (4.39)$$

(ii) Second, the debt contract described in section 4.2.2.2 shows that as the value of the entrepreneur's pledgeable assets falls, the probability of default increases.¹²

$$\frac{\partial F_t(\bar{\omega}_{t+1}^j)}{\partial P_t^N} = \frac{\partial F_t(\bar{\omega}_{t+1}^j)}{\partial \bar{\omega}_{t+1}^j} \frac{\partial \bar{\omega}_{t+1}^j}{\partial P_t^N} < 0 \quad (4.40)$$

These two effects, (i) and (ii) (Eqs. 4.39 and 4.40), both act to tighten the binding moral hazard constraint for the productive bank (Eq. (4.29)) and hence the large bank share falls. This is illustrated in the graph on the right of figure 4.3. In particular, a drop in home prices induces not only a direct fall in asset prices from P_1^N to $P_1^{N'}$ but a further drop to P_2^N due to an endogenous reduction in N_t^P (and the downward sloping demand for N_t^U due to the strictly convex costs). This is the beginning a multiplier effect of sorts. As P_t^N falls by more than $\mathbb{E}_t\{P_{t+1}^H\}$, the moral hazard constraint tightens further inducing further reductions in N_t^P . All the while, as these forces act to push down the value of secured debt, borrower's face an increasing external finance premium. This is the amplification effect highlighted by the difference between the 2 sets of IRFs in figure 4.2.

This static multiplier effect from asset redistribution may be easiest to spot when I exogenously increase the share of assets held by the productive bank. The dynamics following this exogenous shock are shown in the the last column of IRFs in figure 4.2. Although, the shock is set to increase N_t^P by only 1% the period the shock hits, N_t^P increases *more than 5 times as much*, above 5%. Again, this amplified response is driven by the asset redistribution illustrated in figure 4.3. When N_t^P increases there are two immediate effects, as explained

¹²The first partial derivative is positive due to the monotonicity of CDFs and the second partial derivative is negative due to the structure of the optimal debt contract.

by equations 4.39 and 4.40, both of which act to expound this increase in the productive bank's share of housing-secured assets.

One common theme through all the impulse response functions is an amplification effect stemming from asset redistribution. Although, I have highlighted the static amplification, there is also a dynamic feature at work which makes the moral-hazard constrained model more persistent. Since repossessed collateral ultimately is liquidated on the real-estate market, this increase in supply lowers home prices into the future to the extent that housing does not depreciate immediately. These effects re-enforce one another over time. Ultimately though, these dynamic amplification effects are powered by restarting, period by period, the engine that drives the static multiplier.

4.3 VAR Tests of the Model's Amplification Mechanism

In this section, I estimate a series of VAR models to examine the empirical plausibility of the model's predictions. The variables included in the VARs are the same variables examined in the model's impulse response functions: the external finance premium, real GDP, real home prices and the share of total credit exposure concentrated in large banks. All variables are available at a quarterly frequency from 1998:Q2 to 2012:Q4.¹³ As specified in log-levels, the Schwarz Bayesian information criterion selects 2 lags for the VAR. In what follows I first lay-out the model's testable predictions, then I go on to describe the data and the structural identification before presenting the impulse response functions for the various models.

4.3.1 The Model's Empirical Implications

The DSGE model developed in the previous section posits an amplification effect stemming from the redistribution of assets between large and small banks. In particular, the amplification mechanism posits that an initial economic downturn, in which home prices fall, causes

¹³The time series is limited by the availability of net credit exposure data from the OCC's Quarterly Derivatives Report. However, for the removal of trends, I use data going back to 1975:Q1

the credit exposure of large banks to fall which in turn causes finance premiums to rise.

1. A decrease in real home prices decreases the concentration of housing-secured assets in big banks.
2. A decrease in the concentration of housing-secured assets in big banks causes finance premiums to rise.
3. A rise in finance premiums lowers home prices and output.

This mechanism is self re-enforcing, 1. \rightarrow 2. \rightarrow 3. \rightarrow 1. \rightarrow 2. \rightarrow 3. \rightarrow 1. and so forth. At each completion of the cycle output falls; therefore, as the cycle feeds back on itself output falls further and further which deepens the recession. To test this qualitative aspect of the model, I use a factor structural vector auto-regression. The results provide further empirical support for the integrated housing and financial structure I put forth in the equilibrium model.

4.3.2 Data Description

One of the model's key variables, the external finance premium is unobservable. However, following the recent strategy of Christiano et al. (2013) and Carlstrom et al. (2012), I use the spread between BAA corporate bonds and 10-year Treasuries to proxy this unobservable variable. As for real GDP, I use the model equivalent definition. I sum personal consumption expenditures and private investment (the sum of residential and non-residential investment in the model) and divide the resulting series by the personal consumption expenditures excluding food and energy price deflator. I measure real home prices using the Case-Shiller National Composite Home Price Index divided by the personal consumption expenditures excluding food and energy price deflator. Both real GDP and real home prices reveal evidence of a unit root at the 10% confidence level using an ADF test. I therefore, remove any deterministic and/or stochastic trend by taking the difference between the log of the original series and a 25 quarter centered moving average of the logged series.

Finally, I follow Breuer (2000) and construct the large bank share variable using the Office of the Comptroller of the Currency's (OCC) Quarterly Derivatives Report which tracks the derivative activity of the 25 largest U.S. commercial banks (in terms of notional derivatives held) and the commercial banking sector as a whole. Specifically, I sum the total credit exposure of the largest commercial banks and divide this by the total credit exposure of all commercial banks. The banks I deem as 'large' are the six financial firms which consistently hold the largest amount of notional derivatives. These six banks include: JP Morgan Chase, Citibank, Bank of America, HSBC, Wachovia and Wells Fargo.¹⁴ In the baseline model, the variable Large Bank Share refers to the share of total credit risk held by these six banks relative to all U.S. commercial banks. These derivatives contracts are dominated by interest rate contracts, which were often secured by MBSs.¹⁵

As a robustness check, I also compute the large bank share using data obtained directly from the *FR Y-9C Consolidated Financial Statements for Bank Holding Companies* collected quarterly by the Federal Reserve.¹⁶ One advantage of using this data is the highly detailed handling of mergers and acquisitions by the National Information Center.

Ideally, I would like to obtain a time series of the amount of debt securities issued by bank holding companies which are secured by mortgage backed securities, as this most closely aligns with the interpretation of these variables in the model. For example, the Asset-Backed Commercial Paper market provides an ideal example of the type of debt banks issued on the behalf of firms/investment banks holding MBSs. Unfortunately this data has only been collected as of 2008. To get a rough measure of the debt banks have extended which are secured, either directly or indirectly, by housing I use the reported level of loans secured by real-estate. In the following IRF estimates which refer to 'Alternative Data,' the variable

¹⁴One challenge to tracking these firms over time is dealing with mergers, acquisitions and the financial crisis. I handle these issues by adding the off-balance sheet asset's of acquired banks to the acquiring bank's assets to create (as much as is possible) a consistent time series. See Table (4.3) for more details on how this group evolves over time.

¹⁵See for example: <http://www.icmagroup.org/Regulatory-Policy-and-Market-Practice/short-term-markets/Repo-Markets/frequently-asked-questions-on-repo/6-what-types-of-asset-are-used-as-collateral-in-the-repo-market/>

¹⁶I am indebted to Bob DeYoung for recommending this data source.

Large Bank Share refers to the share of loans secured by real estate held by the six banks listed above relative to all bank holding companies which file the Y-9C report.

4.3.3 Model A: A Factor Structural VAR

The model's implications described above calls for the identification of 3 distinct structural shocks using four variables. Since the number of desired structural shocks is less than the number of variables for which there are model implications, I employ a factor structural vector auto-regression following Gorodnichenko (2005). This approach is appropriately fitting here for a couple of reasons.

First, the four variables in the VAR behave qualitatively similar to technology and housing demand shocks in the equilibrium model as shown in Figure 4.2. For this reason, imposing timing restrictions at any horizon to distinguish these shocks proves difficult. Moreover, there is no need to disentangle these shocks to test the model's predictions that an aggregate expansion which increases real home prices, loosens both the supply and demand side financial frictions. Therefore, I choose to simply recognize macroeconomic disturbances as a single factor, which allows me to test the model's prediction along this dimension without imposing arbitrary timing restrictions on the behavior of output and real home prices. Blanchard & Quah (1989) make a similar argument that shocks can be aggregated when they elicit qualitatively similar dynamics.¹⁷ Second, the external finance premium in the model is unobservable. Hence, by using a factor-structural VAR, I can explicitly include measurement error terms to ensure this proxy for the external finance premium does not contaminate the structural shocks of study.

In addition to the macroeconomic factor discussed above, I identify a 'risk' shock, an increase in the cross sectional dispersion of borrowers in the model $\ln(\sigma_t^\omega)$ and an exogenous increase in the share of housing-secured assets held by large banks. The latter shock is the model equivalent of a decrease in $\ln(\psi_t^{N,P})$ which exogenously improves the moral hazard problem

¹⁷Their argument is a bit more formal. To summarize, they show that so long as the dynamic responses of the variables in the VAR to the aggregated shocks differ up to a scalar lag distribution (The responses need not be identical nor proportional) then the shocks can be aggregated.

between large banks and borrowers. To summarize, I identify an exogenous tightening of the demand-side financial friction (the ‘risk’ shock), a beneficial macroeconomic shock and an exogenous loosening of the supply-side financial friction. Identifying these three shocks is sufficient to test the three model predictions laid out above.

To identify these three shocks, I impose a recursive scheme that ensures global identification is achieved¹⁸ and allows for home prices and real GDP to behave symmetrically to all the shocks. In the model, the ex-post observable external finance premium in period t is determined by fundamentals in period $t - 1$. For this reason, I order the spread, which proxies the external finance premium first in the VAR to match this feature of the model. Next I order real GDP and then home prices. The ordering between these two variables is innocuous since they are treated symmetrically in the identification scheme. Finally, I order the share of total credit exposure held by large banks. This recursive ordering is consistent with the equilibrium model. The macroeconomic factor shocks in the DSGE model do not have a contemporaneous effect on the external finance premium but they do contemporaneously impact real GDP, real home prices and the large bank share. Additionally, the supply-side financial shock affects all variables (other than the large bank share) with a lagged response. To summarize the identification scheme, let e_t denote the 4×1 vector of reduced form VAR residuals. Let ϵ_t denote the 3×1 vector of structural shocks and let v_t denote the 4×1 vector of measurement errors which ensures the rank between the reduced form shocks and the identify structure match. The matrix A , is a 4×3 loading matrix which relates the structural factors to the reduced form residuals. Summarizing this,

$$e_t = A\epsilon_t + v_t \tag{4.1}$$

¹⁸Since the 3×3 sub-matrix of A excluding the last row is lower triangular, we can ensure global identification (Anderson, 2003; Anderson & Rubin, 1956).

where

$$A = \begin{bmatrix} a_{1,1} & 0 & 0 \\ a_{2,1} & a_{2,2} & 0 \\ a_{3,1} & a_{3,2} & 0 \\ a_{4,1} & a_{4,2} & a_{4,3} \end{bmatrix} \quad (4.2)$$

I assume the measurement error terms, v_t are independent of the structural factors ϵ_t at all leads and lags and I also assume the covariance matrix of the measurement errors, Ψ , is diagonal. Equation (4.1) is estimated using maximum likelihood techniques.¹⁹

4.3.4 Models for Robustness Checks: VAR Models B and C

To ensure the results obtained from the factor structural vector auto-regression are not driven by the reduced rank identification scheme, I examine the empirical implications of two alternative VAR models. Model B tests for the DSGE model's amplification mechanism using the approach employed in Ludvigson's 1998 *Journal of Money Credit and Banking* paper. The idea is to first identify the impact an expansionary housing demand disturbance (i.e. the shock to home prices) has on the large bank share variable. If this share falls with home prices, then proceed to step two which estimates the response of the external finance premium to an exogenous decrease in the large bank share when home prices are removed from the VAR. The idea is to examine how a change in the share of loans intermediated by large banks impacts the external finance premium independent of the endogenous response of home prices to this change in the composition of lending. Finally, if the finance premium increases when the large bank share decreases, I estimate the response of real GDP and real home prices to an exogenous increase in the external finance premium, excluding the large bank share. Again, the idea is to trace out the steps in the amplification mechanism while

¹⁹In particular, assuming e_t are *i.i.d. Normal*, the log-likelihood equation is given by,

$$\log(\mathcal{L}) = -\frac{4T\ln(\pi)}{2} - \frac{T}{2}\ln(|AA' + \Psi|) - \frac{1}{2}\sum_{t=1}^T e_t'(AA' + \Psi)^{-1}e_t.$$

controlling for the endogenous reaction of home prices and the share of credit held by large banks respectively.

Model C takes this idea one step further and estimates a series of bivariate VARs. The first VAR has home prices and the large bank share. If falling home prices causes large banks to withdraw relatively more credit than smaller banks, then I estimate a second bivariate VAR which includes the external finance premium and the share of credit held by large banks. If reductions in the large bank share drive-up finance premiums, then I estimate a third bivariate VAR with the external finance premium and GDP. If this VAR indicates that rising finance premiums causes real GDP to fall, then this provides evidence in support of the equilibrium model's amplification mechanism spelled out in Section (4.3.1). T

Throughout all of these VAR models, I order variables as follows: the external finance premium, real GDP, real home prices and the large bank share. The motivation for this ordering follows from the equilibrium model's impulse response functions in Figure (4.2). The most delicate issue of this ordering lies in the decision of whether to place real GDP before real home prices or vis-a-versa. The equilibrium model implies both have a non-zero response to housing demand and technology shocks. Fortunately all the results for the recursive models used in Model B and Model C are robust to reversing the ordering of real GDP and real home prices. Model A treats both of these variables symmetrically and for this reason is the preferred model.

4.3.5 Model A: IRFs

Impulse response functions trace out the path of the variables in periods $t = 0, 1, 2, \dots$ in response to a one time structural disturbance in period $t = 0$. Confidence bands are computed using Monte Carlo integration techniques assuming a normal likelihood and uninformative prior.

In order to test the model's amplification prediction the first step is to examine if a drop in home prices caused by an aggregate downturn tightens the large banks' moral hazard con-

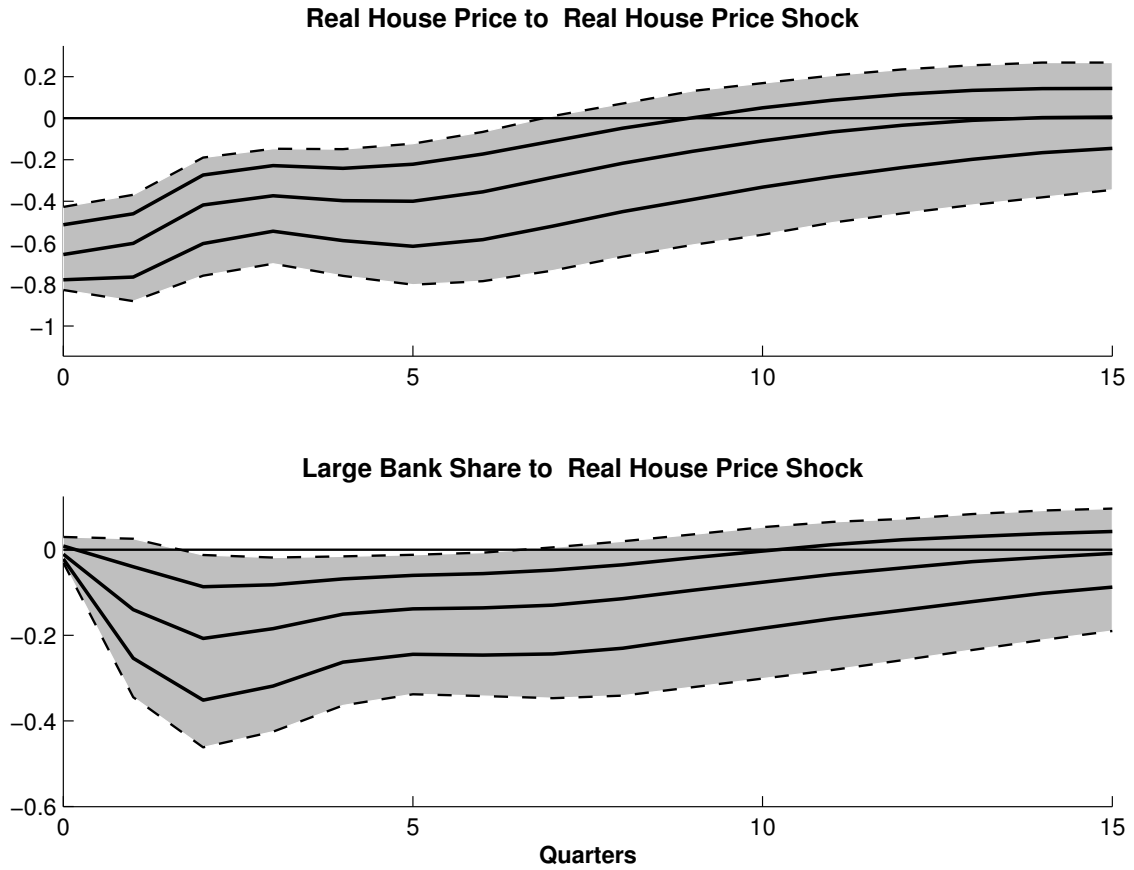


Figure 4.4: Model A: Median Impulse Response to a contractionary aggregate shock with 68% and 90% confidence bounds.

straint forcing them to shed assets relative to their smaller counterparts. This macroeconomic factor which simultaneously decreases real home prices and real GDP has the interpretation of either a housing demand disturbance or a technology shock in the DSGE model. Both shocks have been attributed as playing a driving role in the evolution of the real economy (See for example Liu et al. (2013) or Kydland & Prescott (1982)). As the model predicts, this aggregate downturn tightens the large banks' moral hazard constraint forcing them to shed assets relative to their smaller counterparts as shown in Figure (4.4).

The second step in the accelerator mechanism, and in fact the model's key prediction is that changes in the distribution of assets between productive and unproductive banks alters the

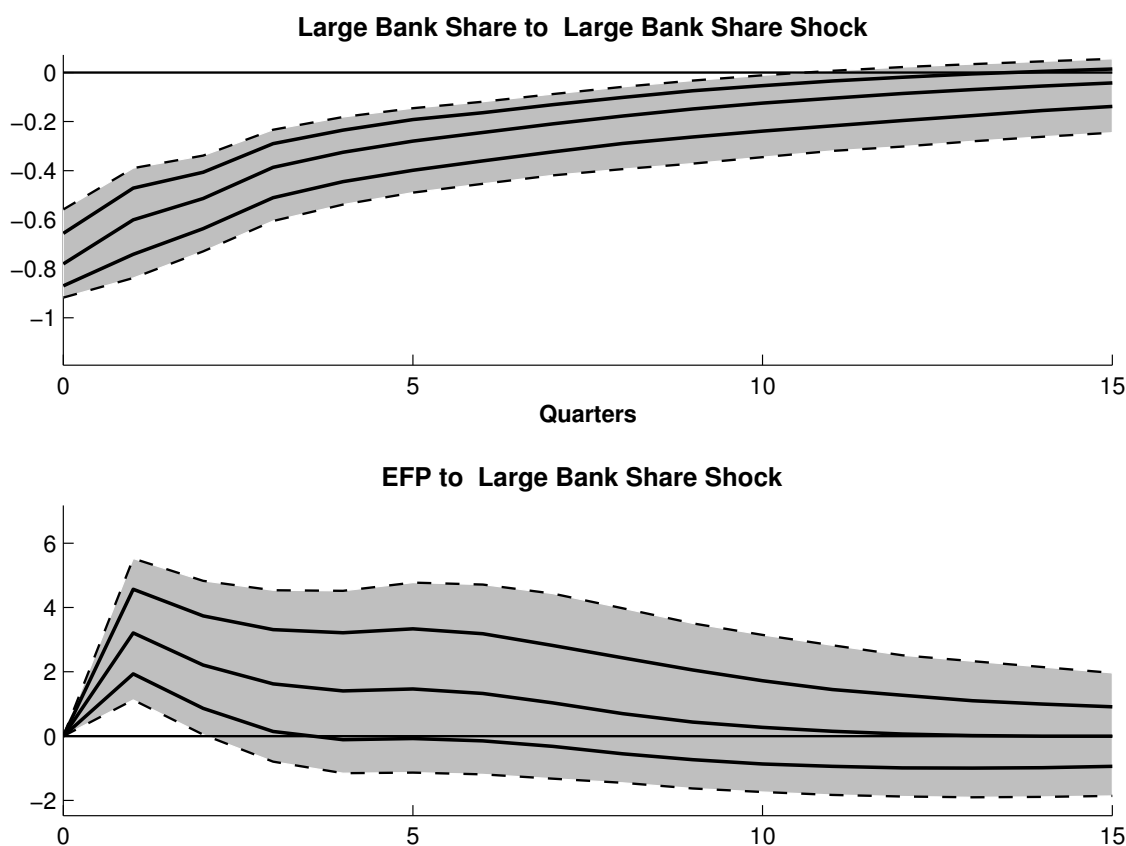


Figure 4.5: Model A: Median Impulse Response to an exogenous decrease in the large bank share with 68% and 90% confidence bounds.

finance terms offered to borrowers. For example, if the assets shift to less productive banks, which are the small banks in equilibrium, finance premiums will rise as the liquid value of the borrowers collateral falls when it is held by these unproductive intermediaries. The data confirms this prediction in Figure (4.5). An exogenous decrease in the share of assets held by large banks increases the external finance premium, suggesting financing conditions are worsening.

Since the VAR has confirmed the first two steps of the accelerator mechanism, I proceed to the third step to examine how rising finance premiums affect the macroeconomy. In particular, the model predicts that rising financing costs will lead to an increase in defaults and ultimately lower home price due to the increased supply of homes on the market. Moreover,

the existence of higher financing costs leads to less investment. Both of these movements exact a negative effect on real GDP. This is confirmed in Figure (4.6). As home prices fall so too does real GDP. Therefore the impulse response functions show that when home prices

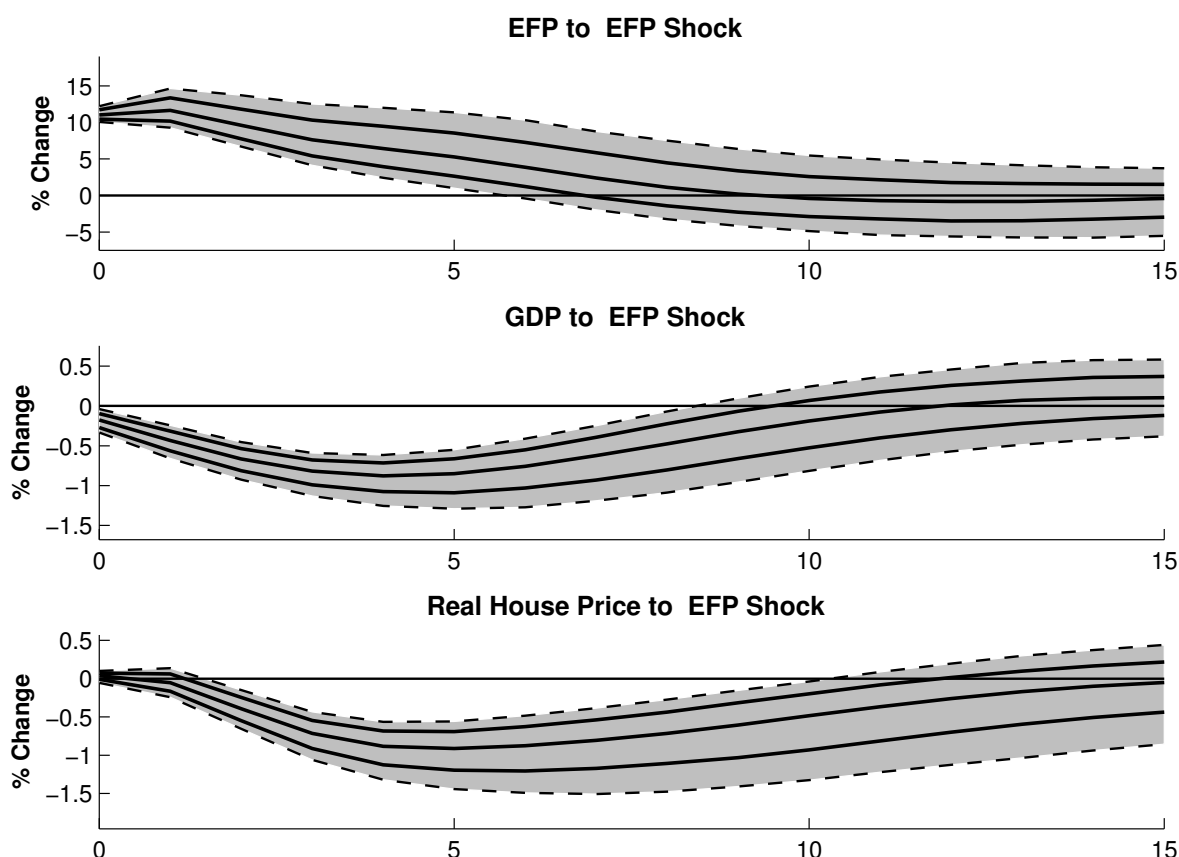


Figure 4.6: Model A: Median Impulse Response to an exogenous increase in the external finance premium with 68% and 90% confidence bounds.

fall, credit exposure shifts from large banks which magnifies the movement of finance premiums and, in turn, amplifies the movement of output and home prices, starting the cycle over. *In summary, the results above provide evidence which supports the hypothesis that the redistribution of housing-secured assets magnifies the movement of finance premiums, house prices and output across the business cycle, supporting the model's predictions.*

4.3.6 Model A: IRFs with Alternative Data

As mentioned above, I perform a robustness check of the above results by estimating the same factor-structural VAR with the large bank share variable measured by the share of loans secured by real estate from the Federal Reserve's Y-9C report. The results below show that both models behave qualitatively similar. The response of the finance premium to an

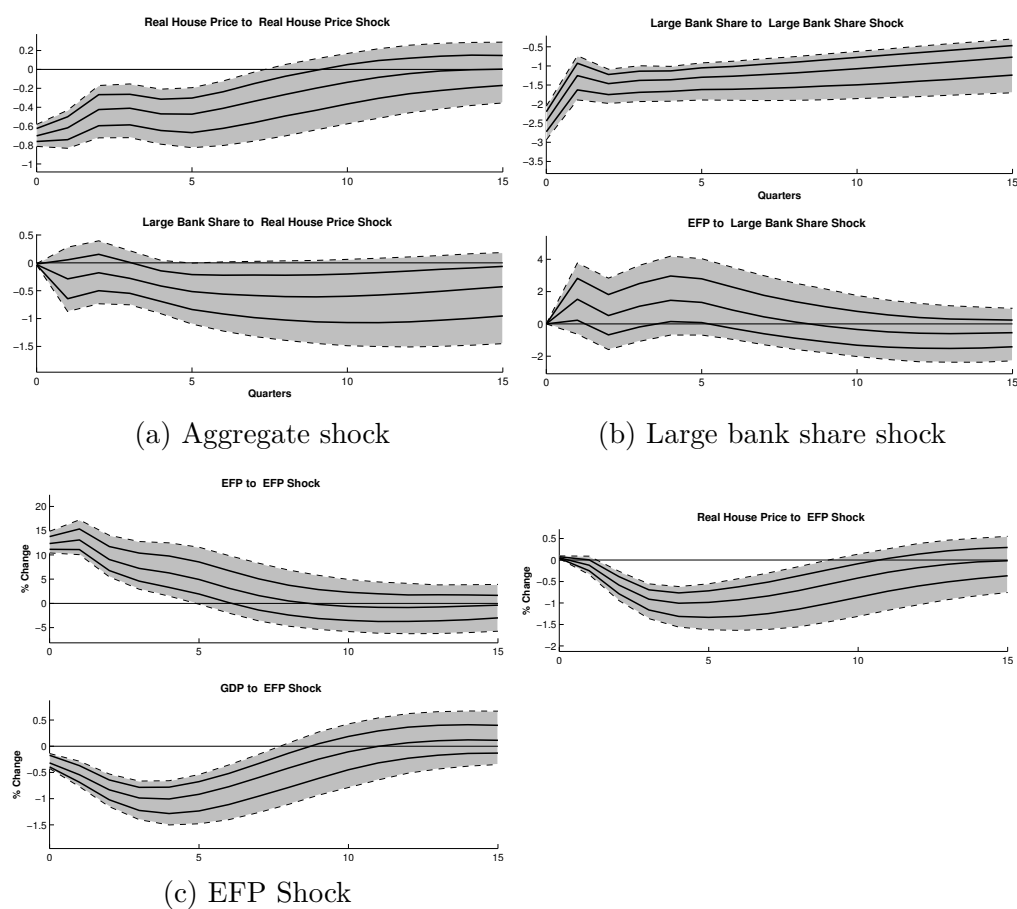


Figure 4.7: Model A with Alternative Data: Median Impulse Response to various contractionary shocks with 68% and 90% confidence bounds.

exogenous decrease in the share of real estate secured loans issued by large banks loses some statistical significance in this version. However, the median response is still positive for 8 quarters as is the majority of the posterior density for the first 6 quarters.

4.3.7 Model B: IRFs

Model A uses a reduced rank identification scheme, which may be driving the results from the previous section. To examine the implications of the reduced rank identification strategy, I also test the model by estimating a sequence of VAR models where the response variables are restricted from the next model (Ludvigson, 1998). I call these sequence of VARs Model B. The results confirm the equilibrium model's amplification mechanism is a robust feature

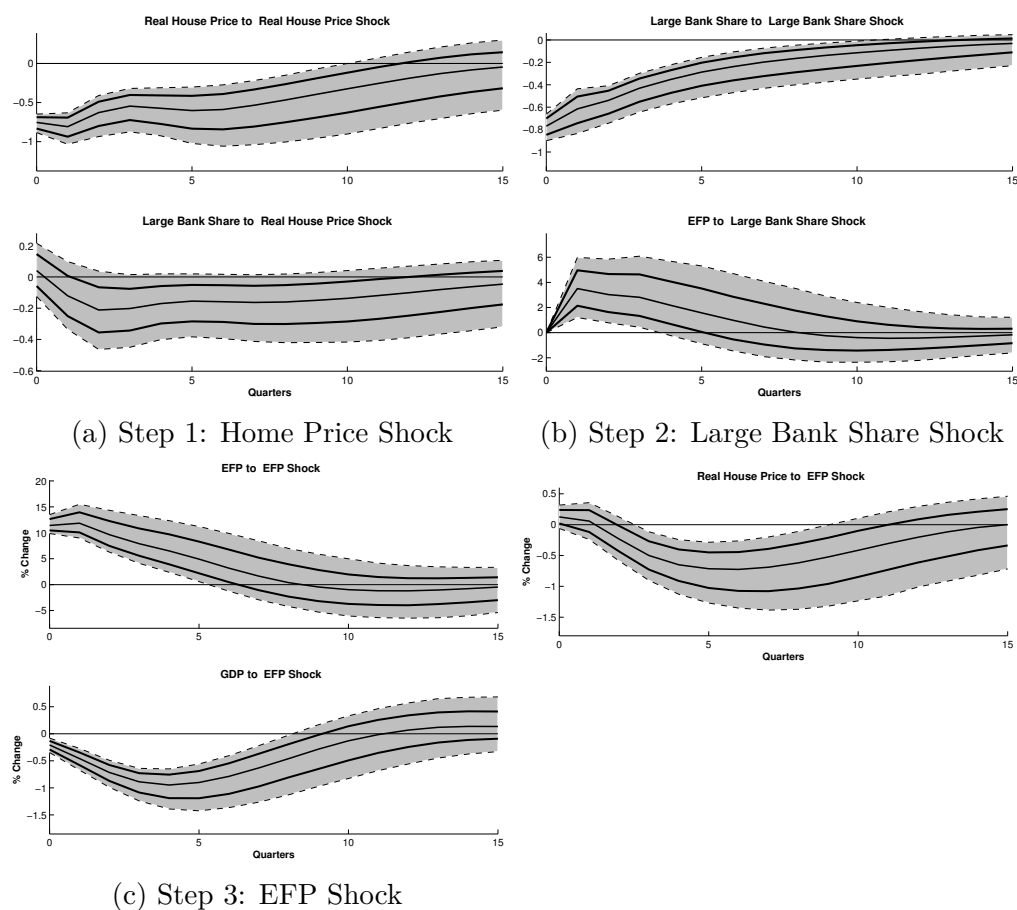


Figure 4.8: Model B: Median Impulse Response to various contractionary shocks with 68% and 90% confidence bounds.

of the data, even when the VAR model has full rank and the sequence of estimated models restricts endogenous responses of real home prices and the large bank share.

4.3.8 Model B: IRFs with Alternative Data

In this section, I repeat Ludvigson's (1998) approach to testing for a financial accelerator mechanism using the alternative measure of the large bank share variable measured by the share of loans secured by real estate from the Federal Reserve's Y-9C report. The results below show that both models behave qualitatively similar. Given the similar behavior of

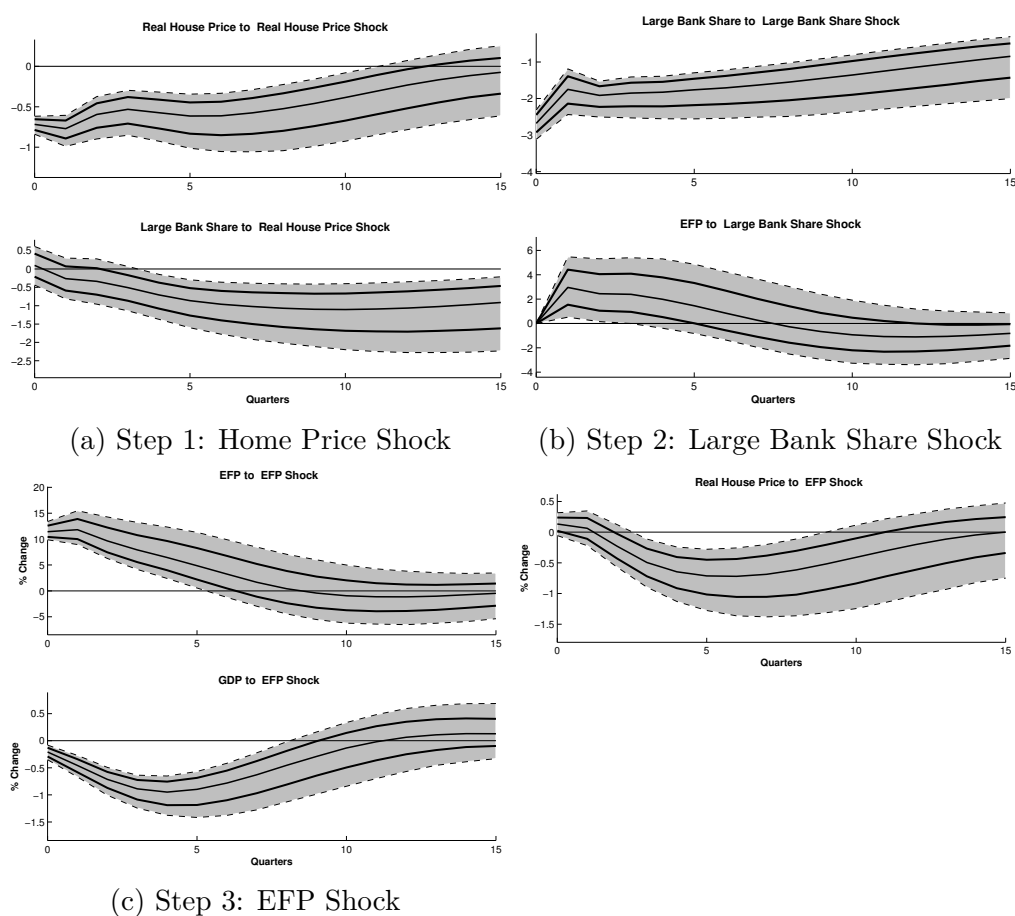


Figure 4.9: Model B with Alternative Data: Median Impulse Response to various contractionary shocks with 68% and 90% confidence bounds.

Model A with both measures of the large bank share variable, this section further confirms the empirical results are not excessively fragile to the measure of large bank asset concentration.

4.3.9 Model C: IRFs

In this section, I perform a third robustness check to the VAR model used to test for the accelerator mechanism spelled out in Section (4.3.1). In Model C, I estimate a sequence of bivariate VARs. Although this provides a simplistic interpretation of the data, the bivariate models ensure the IRFs are not influenced by the endogenous responses of variables. For example, these results confirm *(i)* the drop in the large bank share following a drop in home prices and *(ii)* the response of the EFP to an exogenous drop in the large bank share is not driven by real GDP. Finally, real GDP falls in response to rising finance premiums even when there is no explicit real home price channel.

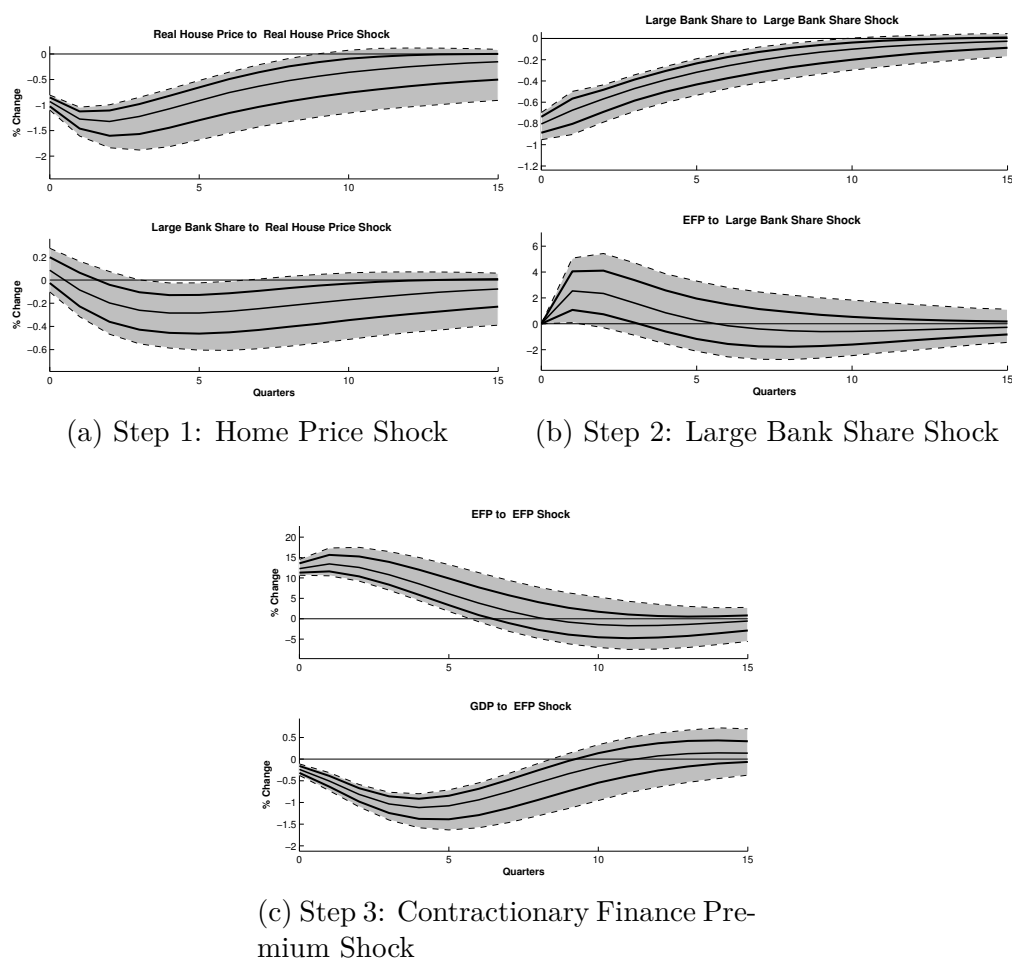
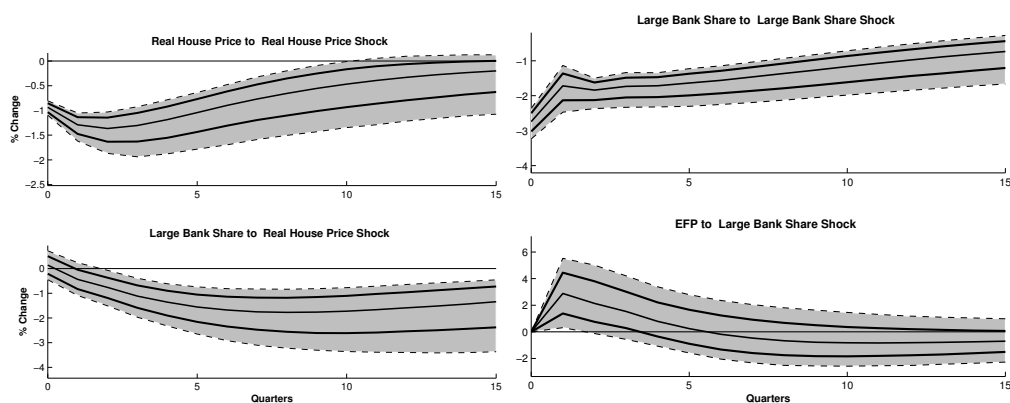


Figure 4.10: Model C: Median Impulse Response to various contractionary shocks with 68% and 90% confidence bounds.

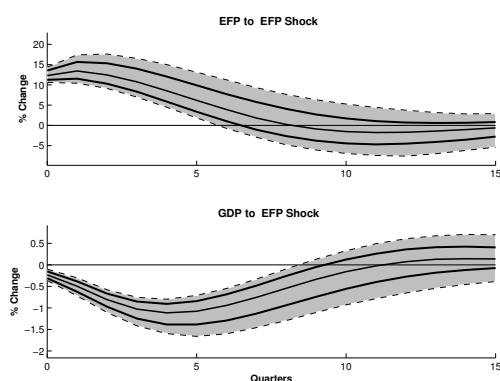
4.3.10 Model C: IRFs with Alternative Data

This section performs one final robustness check by re-estimating Model C with the alternative measure of the large bank share. As was the case for Model A and Model B, the results are not sensitive to how the large bank share variable is measured. The cumulative evidence of these alternative models shows that over the last 14 years, there has been a tight relationship between home prices, the concentration of assets in the largest financial firms, finance premiums and the real economy. The following section returns to the equilibrium model to study the impact of actions taken by policy makers to break this relationship.



(a) Step 1: Home Price Shock

(b) Step 2: Large Bank Share Shock



(c) Step 3: EFP Shock

Figure 4.11: Model C with Alternative Data: Median Impulse Response to various contractionary shocks with 68% and 90% confidence bounds.

4.4 Unconventional Monetary Policy

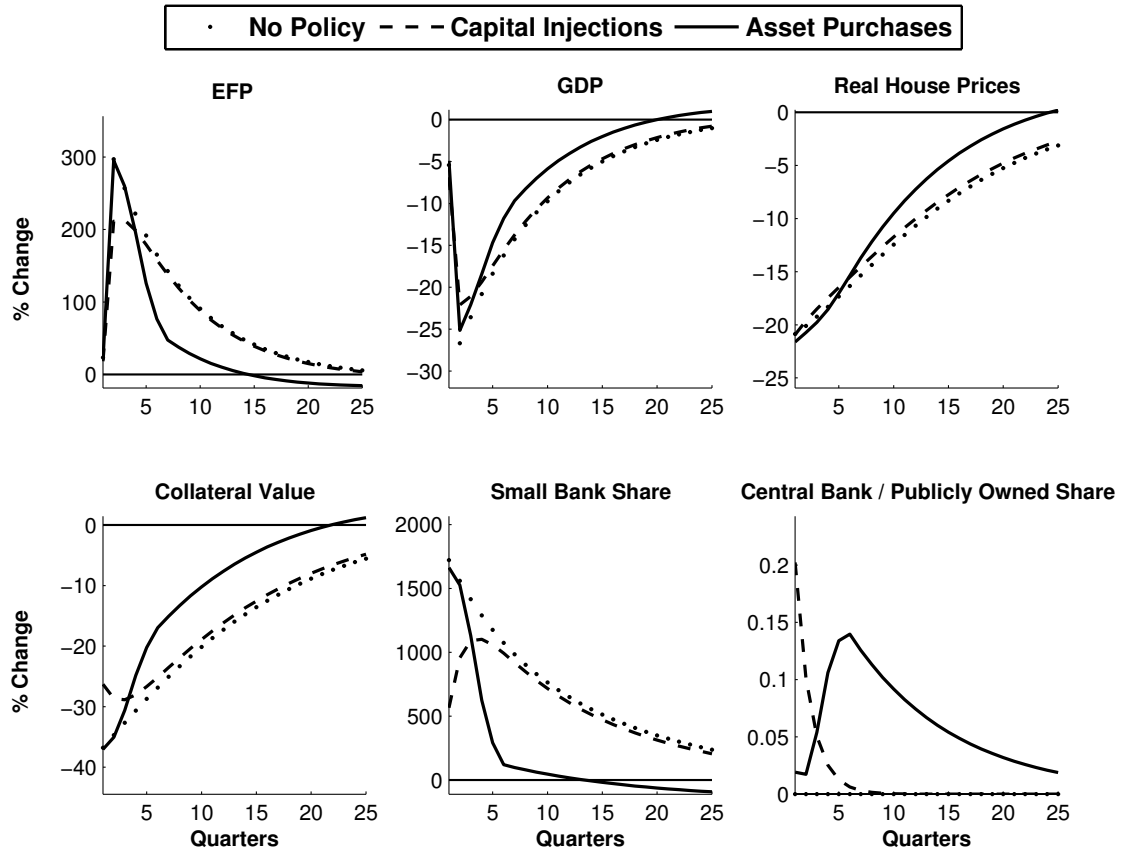


Figure 4.12: Impulse responses to large contraction in housing demand. The dashed lines represent the dynamics under no policy response, the solid lines are the dynamics following a short-term equity injection and the dotted line displays the dynamics under a persistent central bank asset purchase policy.

I now turn the focus to analyzing the unprecedented actions taken by policy makers in the wake of the 2007 Financial Crisis through the lens of this empirically verified model. Since the model features large financial firms ('Too Big to Fail' banks) and housing secured debt (Mortgage Backed Securities), it can be used to analyze the relative effectiveness of:

- (i) Equity injections into big banks similar to TARP
- (ii) Central Bank purchases of mortgage backed securities such as QE1 and QE3

To simulate these policies I hit the economy with a large decrease in housing demand which generates a 20% drop in real home prices, real GDP and a 3 to 4 fold increase in the external finance premium – all similar to the magnitude of the Great Recession with the exception of output which fell by less during the recession, although I am disregarding any expansionary effects of fiscal policy in the model. I consider three alternative unconventional monetary policy regimes including no policy as simulated in Figure (4.2). The alternative two regimes are described below.

Equity Injections To simulate equity injections into ‘Too Big to Fail’ banks I assume the Central Bank uses a lump-sum tax, T_t , to raise money from the household and provide an equity injection, INJ_t^{CB} , to the productive banks. The banks can use the capital immediately but must pay it back in its entirety. This policy is modeled by adding/augmenting the following equations to the model.

$$INJ_t^{CB} = \theta_{BK} \varepsilon_t^H \quad (4.1)$$

$$T_t = INJ_t^{CB} - \delta^{CB} \frac{BK_{t-1}^{CB}}{\pi_t} \quad (4.2)$$

$$BK_t^{CB} = INJ_t^{CB} + (1 - \delta^{CB}) \frac{BK_{t-1}^{CB}}{\pi_t} \quad (4.3)$$

In any period t , the productive banks now have total capital equal to $BK_t^P + BK_t^{CB}$. This policy rule only has two parameters to calibrate. I set $\theta_{BK} = .18$ which calibrates the size of the initial equity injection and $\delta^{CB} = .5$. This calibration is set so that the equity injections comprise 20

Quantitative Easing: To simulate large scale asset purchases of mortgage backed securities (MBS) I assume the Central Bank uses a lump-sum tax, T_t , to raise money from the household and then purchases $P_t^N N_t^{CB}$ units of collateral. As the policy persists, the household will be responsible for any losses or profits from the central bank’s holding of these assets. This policy is modeled by adding/augmenting the following equations

to the model.

$$N_t^{CB} = \rho^N N_{t-1}^{CB} + \sum_{j=0}^4 \theta_j^N \varepsilon_{t-j}^{\eta^H} \quad (4.4)$$

$$T_t = P_t^N N_t^{CB} - (1 - F_{t-1}(\bar{\omega}_t)) p_{t-1}^N N_{t-1}^{CB} r_{t-1}^D - F_{t-1}(\bar{\omega}_t) P_t^H Z^R N_{t-1}^{CB} \quad (4.5)$$

In any period t , the market clearing condition for secured lending is now given by,

$$\bar{N} = \nu N_t^U + (1 - \nu) N_t^P + N_t^{CB}. \quad (4.6)$$

This policy rule has six parameters to calibrate. I set $\theta_0^N = \theta_4^N = .5$, $\theta_1^N = \theta_3^N = 1$ and $\theta_2^N = 1.4$ which calibrates the flow of central bank asset purchases to have a hump-shaped pattern with the flow of purchases of Mortgage-Backed Securities held outright by the central bank (using data from the Flow of Funds H.4.1, Factors Affecting Reserve Balances) peaking at 14% of the total credit exposure of all commercial banks.²⁰ and I set $\rho^N = .9$ to capture the prolonged nature of the Quantitative Easing policies implemented by the Federal Reserve. In particular, with $\rho^N = .9$ the purchases last for more than 8 years. In the welfare section I vary this.

Equity Injections vs. Quantitative Easing

The simulations of the large fall in housing demand are illustrated in Figure (4.12) under all three unconventional policy regimes – including no policy intervention. The results reveal that *both* equity injections and large-scale asset purchases (LSAP's) mitigate the crisis on impact. However, the duration of the unconventional policies plays a critical role in determining the time it takes to recover from the recession. Since the equity injections are paid back after 7 quarters, the dynamics under this regime converge towards the no-policy regime. The length of the recession is essentially unchanged – although the initial severity is

²⁰Credit exposure is computed as explained in Section (4.3.2) using data from the OCC's Quarterly Derivatives Reports

lessened. Comparing equity injections to LSAP's, the equity injections better mitigate the initial impact of the crisis; however, the persistence of the LSAP policy serve to speed-up the recovery.

Which policy is preferred? To answer this question, I analyze *(i)* the total costs of the policies and importantly *(ii)* the welfare rankings of the policies using the household's utility function. Table (4.2) reveals that although both unconventional policy regimes are preferred

Table 4.2: Equity Injections vs. Quantitative Easing

Policy Rule	Length of Recession (Quarters)	Taxpayer Revenue (% of GDP)	Welfare Cost ¹ (% of Consumption)
No Policy	33	0.000%	0.71%
Equity Injections	31	0.000%	0.63%
QE, 5 Quarters	32	0.016%	0.63%
QE, 6 Quarters	31	0.022%	0.61%
QE, 7 Quarters	31	0.025%	0.59%
QE, 8 Quarters	31	0.029%	0.57%
QE, 9 Quarters	30	0.036%	0.55%
QE, 11 Quarters	29	0.046%	0.51%
QE, 13 Quarters	28	0.064%	0.44%
QE, 18 Quarters	25	0.107%	0.32%
QE, 34 Quarters	18	0.300%	0.00%

¹ Welfare is computed by evaluating $\frac{\partial \mathcal{W}_t}{\partial \varepsilon_t^H}$ where $\mathcal{W}_t = u_t + \beta \mathbb{E}_t\{\mathcal{W}_{t+1}\}$ and u_t is the household's period utility function. the cost is the % of steady-state consumption under the optimal policy the household would have give-up under a prolonged QE policy to be indifferent to the inferior policy.

to no policy intervention, the quantitative easing policy outperforms the equity injections in terms of *both* cost and welfare. The central bank's asset purchases actually result in profits for tax-payers – consistent with the U.S. experience in which the Federal Reserve has made record transfers to Treasury in light of its balance-sheet expansion. Moreover, this policy is preferred from a welfare standpoint. Independent of the transfers to the household resulting from the QE policy, the persistence of the policy speeds-up the recovery from the recession which improves welfare.

In addition to these quantitative factors, there is a political aspect to the comparison of these policies as well. Although not modeled here, equity injections carry a stigma of *(i)* Directly benefiting Wall Street as opposed to Main Street and *(ii)* The government taking an ownership stake in a private firm. For example, in Figure (4.12) the variable ‘Publicly Owned Share’ captures the government’s equity stake in the big banks. One could easily argue the welfare rankings between equity injections and QE could reverse if the equity injections are larger or more persistent. However, the political ramifications of equity injections makes such adjustments unrealistic. In totality, QE policies are preferred as they provide sufficient stimulus to offset the crisis without carrying political costs.

Were equity injections necessary?

The above conclusion that a persistent QE policy outperforms a short-lived equity injection into big banks raises a natural question: Were the equity injections necessary? In other words, the results above suggest the economy would be in roughly the same condition with or without the TARP injections. Recall however the circumstances under which TARP was passed.²¹ The economy was quickly deteriorating and urgent action was required to prevent a total collapse of the financial system. Chairman Ben Bernanke’s reply to a weekend extension for congress to debate the equity injections was famously recounted as, “If we don’t do this [pass TARP] tomorrow, we won’t have an economy on Monday.” This paper’s inclusion of heterogeneous banks allows me to analyze the stabilizing role of capitalizing big banks.

In particular, the strength of the asset-price spiral from the redistribution of assets is calibrated by the slope of the unproductive bank’s demand for housing-backed assets, $\chi^{R,U}$. The baseline calibration is set to $\chi^{R,U} = 1.06$. As this value increases the impact of asset redistribution is strengthened. In fact, for slightly larger values of $\chi^{R,U}$ the model becomes indeterminate so that no unique rational expectations equilibrium exists. The amplification effect becomes so strong that without counter-cyclical policy there are multiple equilibria.

Interestingly, this indeterminacy can be remedied with counter-cyclical equity injections. In

²¹Outlined nicely in Andrew Ross Sorkin’s book, *Too Big to Fail*

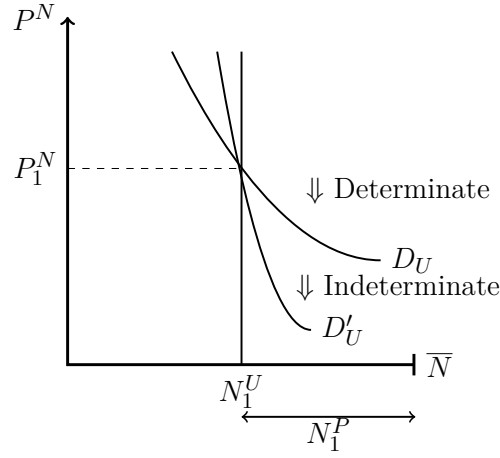


Figure 4.13: If the asset-price spiral is strong enough, the economy becomes indeterminate. Determinacy can be restored with counter-cyclical equity injections.

particular,

$$INJ_t^{CB} = \theta_{BK} \left(\log(P_t^N) - \log(\bar{P}^N) \right)$$

with $\theta_{BK} < 0$ is sufficient to restore determinacy. This systematic recapitalization of big banks eliminates the multiple equilibria induced by a strong asset-price spiral. In this sense, the ranking provided in Table (4.2) should serve as a lower bound of the value of various unconventional policies.

4.5 Conclusion

The paper presents a financial mechanism which is able to capture salient features of the U.S. economy stemming from the interconnectedness of housing and financial markets. The interaction between the two sectors allows for a more natural interpretation of what set-off the Great Recession, a drop in housing demand, as opposed to the “Financial Shocks” that other papers consider. Beyond simply being the source of the initial downturn, the housing market also plays a fundamental role in prolonging the downturn. Since housing is a durable, when defaults begin rising due to a drop in home prices, the future price of housing will remain depressed as it takes time for the stock of homes to return to normal levels. This mechanism captures the observed empirical relationship between the external finance premium and home prices, output and investment; a relationship which the traditional Bernanke et al. (1999) financial accelerator fails to capture.

In addition to integrating housing and financial markets, in this paper I provide insights for policy makers regarding the treatment of “Too Big to Fail” banks. By eschewing the assumption that all banks are the same (or similarly, banks face no agency problems), I am able to show why big banks withdrew relatively more credit during the Great Recession. The model’s qualitative predictions for the joint behavior of the asset concentration in large banks, home prices, output and finance premiums are confirmed using an estimated VAR. Beyond capturing this empirical observation, the model shows the differential behavior between big banks and their smaller counter parts served to strengthen the Great Recession. Therefore, the equity injections into the largest financial firms in 2008 are well justified as they mitigated the severity of the downturn and prevented a potential collapse. Although this thwarted the initial downturn, policies such as “QE 1/3,” the Fed’s Mortgage Backed Security purchase programs, play an important role in the model by speeding up the recovery.

Future Work

Although this paper explains why big banks withdrew relatively more credit when home prices plunged in 2007 and provides a transmission channel for unconventional monetary policy, much work lies ahead. In particular, the policy options in this paper are designed to clean-up after the crash instead of preventing it in the first place. Moving beyond crisis management, the empirically validated financial structure presented in this paper can be used to explore the efficacy of macroprudential policy in preventing a build-up of assets in large financial firms when home prices rise.

A question of empirical importance arises as well from the model. Christiano et al. (2013) show that “risk shocks” have played a substantial role in shaping the business cycle for the last 20 years. However, they point out their finding is driven by the ability of risk shocks to generate the empirical observation that credit contracts during recessions. Due to the novel debt contract I introduce in this paper, credit contracts following adverse technology and demand shocks – the usual business cycle suspects. Given the policy implications, it is worth while to examine the robustness of their results in this housing-centered financial structure.

References

- Adrian, T. & Shin, H. S. (2010). Financial intermediaries and monetary economics. In B. M. Friedman & M. Woodford (Eds.), *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics* chapter 12, (pp. 601–650). Elsevier.
- Anderson, T. (2003). *An introduction to multivariate statistical analysis*. Wiley series in probability and mathematical statistics. Probability and mathematical statistics. Wiley.
- Anderson, T. W. & Rubin, H. (1956). Statistical inference in factor analysis. In *Proceedings of the third Berkeley symposium on mathematical statistics and probability*, volume 5 (pp. 1–1).
- Bernanke, B. S., Gertler, M., & Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In J. B. Taylor & M. Woodford (Eds.), *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics* chapter 21, (pp. 1341–1393). Elsevier.
- Blanchard, O. J. & Quah, D. (1989). The dynamic effects of aggregate demand and supply disturbances. *American Economic Review*, 79(4), 655–73.
- Bos, J. W. & Kolari, J. W. (2013). *Optimal size in banking*, (pp. 77–94). John Wiley & Sons, Ltd.
- Breuer, P. (2000). Measuring off-balance-sheet leverage. *IMF Working Paper*.

- Carlstrom, C. T. & Fuerst, T. S. (1997). Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review*, 87(5), 893–910.
- Carlstrom, C. T., Fuerst, T. S., Ortiz, A., & Paustian, M. (2012). *Estimating contract indexation in a financial accelerator model*. Technical report.
- Christiano, L., Motto, R., & Rostagno, M. (2013). *Risk Shocks*. NBER Working Papers 18682, National Bureau of Economic Research, Inc.
- Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1), 1–45.
- Davis, M. A. & Heathcote, J. (2005). Housing and the business cycle. *International Economic Review*, 46(3), 751–784.
- Demirguc-Kunt, A., Detragiache, E., & Merrouche, O. (2013). Bank capital: Lessons from the financial crisis. *Journal of Money, Credit and Banking*, 45(6), 1147–1164.
- Gerali, A., Neri, S., Sessa, L., & Signoretti, F. M. (2010). Credit and banking in a dsge model of the euro area. *Journal of Money, Credit and Banking*, 42(s1), 107–141.
- Gertler, M. & Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1), 17–34.
- Gertler, M. & Kiyotaki, N. (2010). Financial intermediation and credit policy in business cycle analysis. In B. M. Friedman & M. Woodford (Eds.), *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics* chapter 11, (pp. 547–599). Elsevier.
- Gorodnichenko, Y. (2005). *Reduced-Rank Identification of Structural Shocks in VARs*. Macroeconomics 0512011, EconWPA.
- Hafstead, M. & Smith, J. (2012). Financial shocks, bank intermediation, and monetary policy in a dsge model. *Unpublished Manuscript*.

- Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review*, 95(3), 739–764.
- Iacoviello, M. & Neri, S. (2010). Housing market spillovers: Evidence from an estimated dsge model. *American Economic Journal: Macroeconomics*, 2(2), 125–64.
- Kiyotaki, N. & Moore, J. (1997). Credit cycles. *Journal of Political Economy*, 105(2), 211–48.
- Kydland, F. E. & Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6), 1345–70.
- Liu, Z., Wang, P., & Zha, T. (2013). Land-price dynamics and macroeconomic fluctuations. *Econometrica*, 81(3), 1147–1184.
- Ludvigson, S. (1998). The channel of monetary transmission to demand: evidence from the market for automobile credit. *Journal of Money, Credit and Banking*, (pp. 365–383).
- Oulton, N. & Rincon-Aznar, A. (2009). *Rates of Return and Alternative Measures of Capital Input: 14 Countries and 10 Branches, 1971-2005*. Cep discussion papers, Centre for Economic Performance, LSE.
- Shen, G. (2011). News shocks and the external finance premium. *The B.E. Journal of Macroeconomics*, 11(1), 1–27.
- Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2), 265–293.
- Wheelock, D. C. & Wilson, P. W. (2012). Do large banks have lower costs? new estimates of returns to scale for u.s. banks. *Journal of Money, Credit and Banking*, 44(1), 171–199.

4.A Appendix: VAR Data and Complete Results

Table 4.3: Bank Mergers and Acquisitions

Banks Included in the set of <i>Large</i> Commercial Banks									
1998:Q2-1999:Q2	1999:Q3 ¹	1999:Q4 ²	2000:Q1-2001:Q3 ³	2001:Q4 - 2002:Q1 ⁴	2002:Q2 - 2004:Q3 ⁵	2004:Q4 - 2009:Q1 ⁶	2009:Q2 - 2012:Q4 ⁹		
Citibank	Citibank	Citibank	Citibank	Citibank	Citibank	Citibank	Citibank		
Wells Fargo Bank NA	Wells Fargo Bank NA	Wells Fargo Bank NA	Wells Fargo Bank NA	Wells Fargo Bank NA	Wells Fargo Bank NA	Wells Fargo Bank NA	Wells Fargo Bank NA		
First Union NB	First Union NB	First Union NB	First Union NB	First Union NB	Wachovia Bank NA	Wachovia Bank NA	JP Morgan Chase Bank		
Wachovia Bank NA	Wachovia Bank NA	Wachovia Bank NA	Wachovia Bank NA	Wachovia Bank NA	JP Morgan Chase Bank	JP Morgan Chase Bank ⁸	HSBC Bank USA		
Chase Manhattan Bank	Chase Manhattan Bank	Chase Manhattan Bank	Chase Manhattan Bank	JP Morgan Chase Bank	Bank One Natl. ASSN	HSBC Bank USA	Bank of America NA		
Morgan Guaranty NY	Morgan Guaranty NY	Morgan Guaranty NY	Morgan Guaranty NY	Bank One Natl. ASSN	HSBC Bank USA	Bank of America NA			
First NB of Chicago	First NB of Chicago	First NB of Chicago	Bank One Natl. ASSN	HSBC Bank USA	Bank of America NA				
Bank One NA	Bank One NA	Bank One of NA	HSBC Bank USA						
Republic NB NY	Republic NB NY	HSBC Bank USA	Bank of America NA						
Bank of America NT&SA	Bank of America NA								
Nationsbank NA									

Banks Excluded from the set of Commercial Banks

2008:Q4 - 2012:Q4 ⁷
Goldman Sachs

¹ Bank of America NT and SA merges with Nationsbank NA

² HSBC enters the U.S. commercial bank market by acquiring Republic NB of New York

³ Bank One of NA based in Ohio merges with First NB of Chicago, forming Bank One National ASSN based in Chicago, Illinois.

⁴ Chase Manhattan Bank and Morgan Guaranty TR CO of NY merge to form JP Morgan Chase Bank

⁵ Wachovia acquires First Union National Bank

⁶ JP Morgan Chase acquires Bank One National ASSN

⁷ Goldman Sachs becomes a commercial bank

⁸ JP Morgan Chase acquired Bear Stearns due to extreme financial distress. Beginning in 2008:Q1 JP Morgan Chase experienced increase exposure through repo transactions with Bear Stearns and the ultimate acquisition of Bear Stearns and all exposures. This is very difficult to account for since Q-10 SEC filings report Notional values, not total credit exposure. Hence, from 2008:Q1-2008:Q4 I use a linear interpolation to assess JP Morgan Chase's total credit exposure.

⁹ HSBC Bank USA acquires Wachovia Bank NA under financial distress.

Below, in Figure (4.14) I provide the complete set of impulse response functions for the VAR model described in Section (4.3). I also provide historical decompositions of all three structural shocks. Since the model is under-identified in the sense that there are less shocks than variables, projecting the variables on to the space of structural shocks does not yield the full set of variables. However, this serves as a useful tool to diagnose the extent to which the structural shocks can explain the observed movement of the endogenous variables. The inability to explain much of the variation in these variables would seriously call in to question the identification scheme. The results of this exercise are reported in Figure (5.6) below which show the three structural shocks can explain nearly all of the movement of the endogenous variables in the VAR.

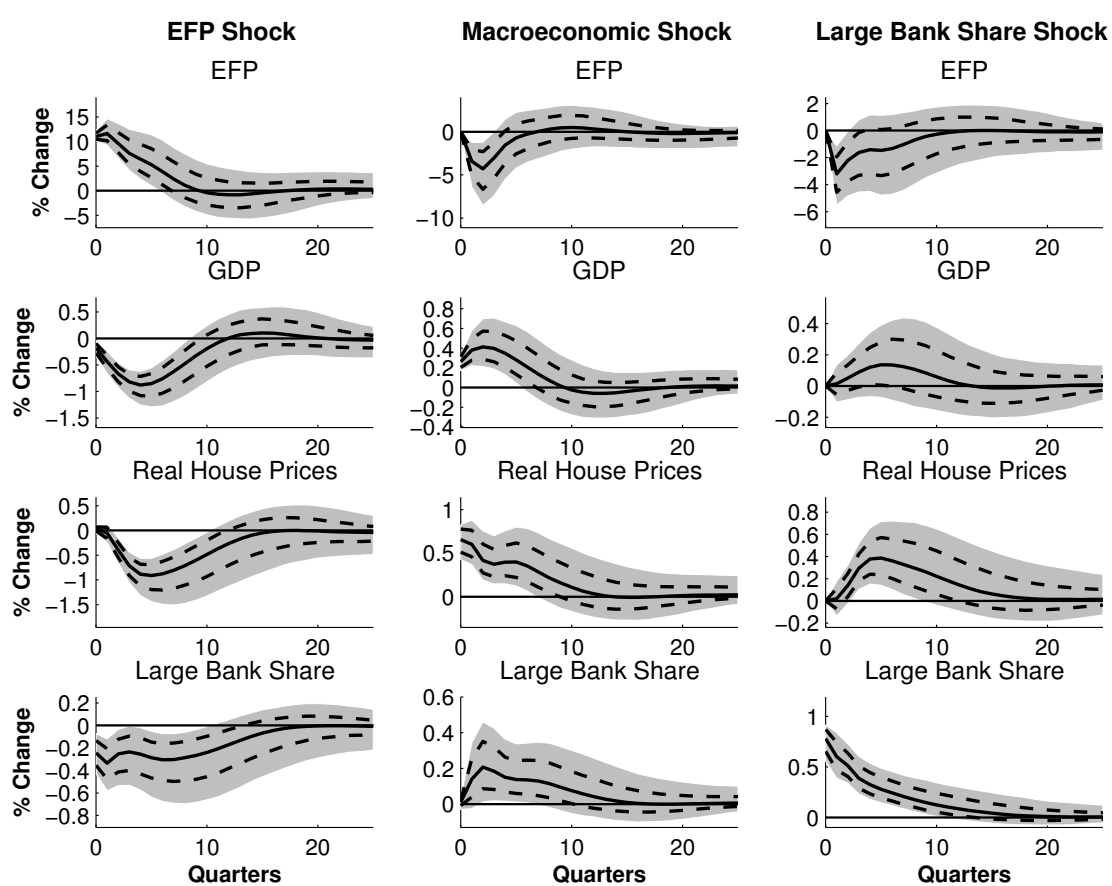


Figure 4.14: Model A: Median Impulse Response Functions with 68% and 90% confidence bounds computed with 5000 draws from the posterior distribution using Jeffery's Prior.

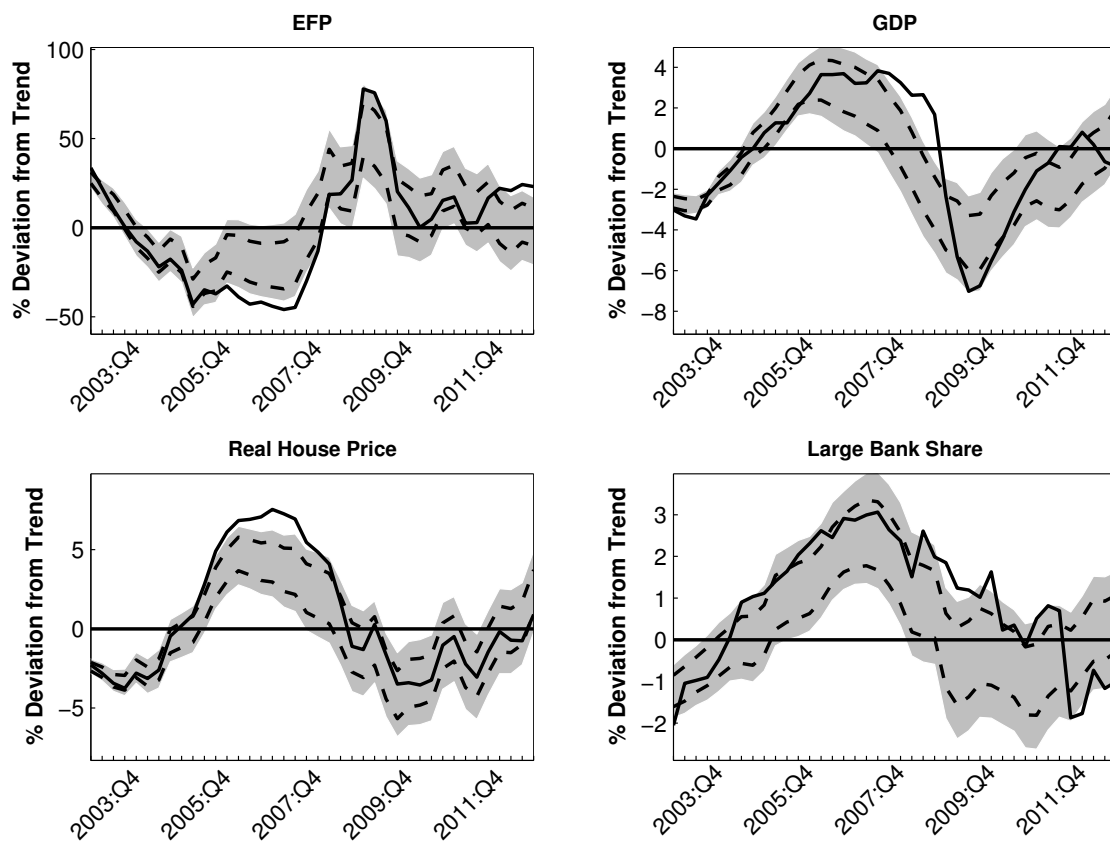


Figure 4.15: Projected historical decompositions with 68% and 90% confidence bounds computed with 5000 draws from the posterior distribution using Jeffery's Prior. Solid lines denote the actual data.

4.B Appendix: The Stationary, Non-Linear Model

In this section I provide the full set of equations which defines the dynamic equilibrium model. All upper case variables are real and lower case variables are nominal - including interest rates. In particular for any nominal interest rate r_t , the real interest rate is given via the Fisher equation, $R_t = \mathbb{E}_t \left\{ \frac{r_t}{\pi_{t+1}} \right\}$.

Household - 5

$$\frac{W_t^H}{C_t} = \frac{\eta^{L^H}}{1 - L_t^{New}} \quad (4.B.1)$$

$$\frac{W_t^{G,C}}{C_t} = \frac{\eta^L}{1 - L_t^{G,C}} \quad (4.B.2)$$

$$\frac{\eta_t^H}{H_t} = \frac{P_t^H}{C_t} - \beta \mathbb{E}_t \left\{ \frac{(1 - \delta^H) P_{t+1}^H}{C_{t+1}} \right\} \quad (4.B.3)$$

$$\frac{1}{r_t^D} = \mathbb{E}_t \{ \Lambda_{t+1} \} \quad (4.B.4)$$

$$\Lambda_t = \beta \frac{C_{t-1}}{\pi_t C_t} \quad (4.B.5)$$

Aggregate Goods Production - 7

$$Y_t = Z_t^G (K_{t-1})^{\alpha_G} (L_t^G)^{1-\alpha_G} \quad (4.B.6)$$

$$L_t^G = (L_t)^{(1-\alpha_E)} (L_t^E)^{\alpha_E} \quad (4.B.7)$$

$$W_t = (1-\alpha_G)(1-\alpha_E) \frac{Y_t}{L_t} \quad (4.B.8)$$

$$W_t^E = (1-\alpha_G)\alpha_E \frac{Y_t}{L_t^E} \quad (4.B.9)$$

$$r_t^K = \frac{\alpha_G \frac{Y_t}{K_t} + (1-\delta^K)Q_t}{Q_{t-1}} \quad (4.B.10)$$

$$C_t^E = (1-\alpha_t^N)(1-\Gamma_{t-1}(\bar{\omega}_t)) R_t^K K_{t-1} \quad (4.B.11)$$

$$\alpha_t^N = \frac{(1-F_{t-1}(\bar{\omega}_t)) P_{t-1}^N R_{t-1}^D \bar{N}}{(1-\Gamma_{t-1}(\bar{\omega}_t)) R_t^K Q_{t-1} K_{t-1}} \quad (4.B.12)$$

Capital Producers - 1

$$I_t = K_t - (1-\delta^K)K_{t-1} \quad (4.B.13)$$

Goods Production: Firm-level Debt Contract - 9

In this section I provide the equations which determine the debt contract. Moreover, I provide a description of the individual entrepreneur's problem which leads to the aggregate entrepreneur's consumption rule defined in equation (4.16). In particular, suppose entrepreneur j has preferences over consumption and housing given by a Cobb-Douglas utility function,

$$U_t(j) = C_t(j)^{1-\alpha_t^N} N_t(j)^{\alpha_t^N}$$

with

$$\alpha_t^N = \frac{(1 - F_{t-1}(\bar{\omega}_t)) P_{t-1}^N R_{t-1}^D \bar{N}}{(1 - \Gamma_{t-1}(\bar{\omega}_t)) R_t^K Q_{t-1} K_{t-1}}.$$

Since entrepreneur j takes the aggregate default rate $F_{t-1}(\bar{\omega}_t)$ and the aggregate choice of capital, K_t as given, this Walrasian demand bundles given these preferences has the well-known property of constant expenditure shares on consumption and housing.

$$C_t(j) = (1 - \alpha_t^N) (1 - \Gamma_{t-1}(\bar{\omega}_t^j)) R_t^K K_{t-1}^j Q_{t-1}$$

Aggregating over this equation implies

$$\begin{aligned} C_t &= \int_0^\infty (1 - \alpha_t^N) (1 - \Gamma_{t-1}(\bar{\omega}_t^j)) R_t^K K_{t-1}^j Q_{t-1} dj \\ &= (1 - \alpha_t^N) (1 - \Gamma_{t-1}(\bar{\omega}_t)) R_t^K K_{t-1} Q_{t-1} \\ &= (1 - \Gamma_{t-1}(\bar{\omega}_t)) R_t^K K_{t-1} Q_{t-1} - (1 - F_{t-1}(\bar{\omega}_t)) P_{t-1}^N R_{t-1}^D \bar{N}. \end{aligned}$$

The second equality follows from the ex-ante homogeneity among entrepreneurs implying they all will choose the same default cut-off, $\bar{\omega}_t^j$, and the same level of capital expenditures, K_t^j .

$$\mathbb{E}_t \left\{ \Gamma'_t(\bar{\omega}_{t+1}) R_{t+1}^K Q_t K_t \right\} = \lambda_t^E \mathbb{E}_t \left\{ \left(\Gamma'_t(\bar{\omega}_{t+1}) - \mu^M G'_t(\bar{\omega}_{t+1}) \right) R_{t+1}^K Q_t K_t \right\} \quad (4.B.14)$$

$$\begin{aligned} \mathbb{E}_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] R_{t+1}^K \right\} &= \lambda_t^E \mathbb{E}_t \left\{ R_t^E - \Gamma_t(\bar{\omega}_{t+1}) R_{t+1}^K \right\} \\ &+ \lambda_t^E \mathbb{E}_t \left\{ \mu^M G_t(\bar{\omega}_{t+1}) R_{t+1}^K \right\} \end{aligned} \quad (4.B.15)$$

$$R_t^E \left(Q_t K_t - P_t^N \bar{N} - W_t^E \right) = \mathbb{E}_t \left\{ \left(\Gamma_t(\bar{\omega}_{t+1}) - \mu^M G_t(\bar{\omega}_{t+1}) \right) R_{t+1}^K Q_t K_t \right\} \quad (4.B.16)$$

$$B_t = Q_t K_t - P_t^N \bar{N} - W_t^E \quad (4.B.17)$$

$$z_{t+1} = \frac{\ln(\bar{\omega}_{t+1}) + .5(\sigma_t^\omega)^2}{\sigma_t^\omega} \quad (4.B.18)$$

$$G_t(\bar{\omega}_{t+1}) = \Phi^{\mathcal{N}}(z_{t+1} - \sigma_t^\omega) \quad (4.B.19)$$

$$\Gamma_t(\bar{\omega}_{t+1}) = \Phi^{\mathcal{N}}(z_{t+1} - \sigma_t^\omega) + \bar{\omega}_{t+1} \left(1 - \Phi^{\mathcal{N}}(z_{t+1}) \right) \quad (4.B.20)$$

$$G'_t(\bar{\omega}_{t+1}) = \left(\frac{1}{\sigma_t^\omega \sqrt{2\pi}} \right) e^{-\frac{z_{t+1}^2}{2}} \quad (4.B.21)$$

$$\Gamma'_t(\bar{\omega}_{t+1}) = 1 - \Phi^{\mathcal{N}}(z_{t+1}) \quad (4.B.22)$$

Housing Production: Aggregate Behavior - 2

$$H_t^{New} = Z_t^H (L_t^H)^{(1-\alpha_H)} \quad (4.B.23)$$

$$W_t^H = P_t^H (1 - \alpha_H) \frac{H_t^{New}}{L_t^H} \quad (4.B.24)$$

Productive Bank - 10

$$\left\{ (1 - \psi_t F_t(\bar{\omega}_{t+1})) BK_t^P \right\} = N_t^P \mathbb{E}_t \left\{ \frac{Z^R P_{t+1}^H - (1 - F_t(\bar{\omega}_{t+1})) P_t^N R_t^D}{P_t^N} \right\} \quad (4.B.25)$$

$$\mathbb{E}_t \left\{ \Lambda_{t+1} \bar{r}_t^{E,P} \right\} = \mathbb{E}_t \left\{ \Lambda_{t+1} r_t^D \right\} \quad (4.B.26)$$

$$\begin{aligned} \mathbb{E}_t \left\{ \Lambda_{t+1} \bar{r}_t^{E,P} \right\} &= \left(\frac{\theta_B - 1}{\theta_B} \right) \mathbb{E}_t \left\{ \Lambda_{t+1} (1 - F_t(\bar{\omega}_{t+1})) \right\} r_t^{B,P} \\ &\quad + \mathbb{E}_t \left\{ \Lambda_{t+1} \pi_{t+1} \frac{(1 - \mu^M) \Phi_t(\bar{\omega}_{t+1})}{B_t} \right\} \end{aligned} \quad (4.B.27)$$

$$B_t^P = \left(\frac{r_t^{B,P}}{r_t^B} \right)^{-\theta_B} B_t \quad (4.B.28)$$

$$B_t^P = D_t^P + \frac{BK_{t-1}^P}{\pi_t} - P_t^N N_t^P \quad (4.B.29)$$

$$\begin{aligned} \Pi_t^P &= (1 - F_{t-1}(\bar{\omega}_t)) \frac{r_{t-1}^{B,P}}{\pi_t} B_{t-1}^P \\ &\quad + (1 - \mu^M) \frac{B_{t-1}^P}{B_{t-1}} \Phi_t(\bar{\omega}_{t+1}) \\ &\quad + (1 - F_{t-1}(\bar{\omega}_t)) R_{t-1}^D P_{t-1}^N N_{t-1}^P \\ &\quad + F_{t-1}(\bar{\omega}_t) P_t^H Z^R N_{t-1}^P - P_t^N N_t^P \\ &\quad - R_{t-1}^D D_{t-1}^P + D_t^P - B_t^P \end{aligned} \quad (4.B.30)$$

$$DIV_t^P = \Pi_t^P \quad (4.B.31)$$

$$TRANS_t^P = F_{t-1}(\bar{\omega}_t) \psi_{t-1} \frac{BK_{t-1}^P}{\pi_t} \quad (4.B.32)$$

$$INV_t^P = T^{B,P} - TRANS_t^P \quad (4.B.33)$$

$$BK_t^P = INV_t^P + (1 - \delta^{BK,P}) \frac{BK_{t-1}^P}{\pi_t} \quad (4.B.34)$$

Unproductive Bank - 10

$$P_t^N = \frac{1}{R_t^D} \mathbb{E}_t \left\{ Z^R P_{t+1}^H - \chi^{R,U} \mu^{R,U} N_t^U (\chi^{R,U})^{-1} \right\} \quad (4.B.35)$$

$$\mathbb{E}_t \left\{ \Lambda_{t+1} \bar{r}_t^{E,U} \right\} = \mathbb{E}_t \left\{ \Lambda_{t+1} r_t^D + \chi^{B,U} \right\} \quad (4.B.36)$$

$$\begin{aligned} \mathbb{E}_t \left\{ \Lambda_{t+1} \bar{r}_t^{E,U} \right\} &= \left(\frac{\theta_B - 1}{\theta_B} \right) \mathbb{E}_t \left\{ \Lambda_{t+1} (1 - F_t(\bar{\omega}_{t+1})) \right\} r_t^{B,U} \\ &+ \mathbb{E}_t \left\{ \Lambda_{t+1} \pi_{t+1} \frac{(1 - \mu^M) \Phi_t(\bar{\omega}_{t+1})}{B_t} \right\} \end{aligned} \quad (4.B.37)$$

$$B_t^U = \left(\frac{r_t^{B,U}}{r_t^B} \right)^{-\theta_B} B_t \quad (4.B.38)$$

$$B_t^U = D_t^U + \frac{BK_{t-1}^U}{\pi_t} - P_t^N N_t^P \quad (4.B.39)$$

$$\begin{aligned} \Pi_t^U &= (1 - F_{t-1}(\bar{\omega}_t)) \frac{r_{t-1}^{B,U}}{\pi_t} B_{t-1}^U \\ &+ (1 - \mu^M) \frac{B_{t-1}^U}{B_{t-1}} \Phi_t(\bar{\omega}_{t+1}) \\ &+ (1 - F_{t-1}(\bar{\omega}_t)) R_{t-1}^D P_{t-1}^N N_{t-1}^U \\ &+ F_{t-1}(\bar{\omega}_t) \left[P_t^H Z^R N_{t-1}^U - \mu^{R,U} (N_{t-1}^U)^{\chi^{R,U}} \right] \\ &- P_t^N N_t^U - R_{t-1}^D D_{t-1}^U + D_t^U - B_t^U (1 + \chi^{B,U}) \end{aligned} \quad (4.B.40)$$

$$DIV_t^U = \Pi_t^U \quad (4.B.41)$$

$$TRANS_t^U = F_{t-1}(\bar{\omega}_t) \psi_{t-1} \frac{BK_{t-1}^U}{\pi_t} \quad (4.B.42)$$

$$INV_t^U = T^{B,U} - TRANS_t^U \quad (4.B.43)$$

$$BK_t^U = INV_t^U + (1 - \delta^{BK,U}) \frac{BK_{t-1}^U}{\pi_t} \quad (4.B.44)$$

Aggregate Bank - 10

$$\bar{r}_t^E = \left[\nu \left(\bar{r}_t^{E,U} \right)^{1-\theta_B} + (1-\nu) \left(\bar{r}_t^{E,P} \right)^{1-\theta_B} \right]^{\frac{1}{1-\theta_B}} \quad (4.B.45)$$

$$r_t^E = \left(\frac{\theta_B}{\theta_B - 1} \right) \bar{r}_t^E - \left(\frac{1}{\theta_B - 1} \right) (1-\mu^M) \frac{\mathbb{E}_t \left\{ \pi_{t+1} \Phi_{t+1}^{H.B.} \right\}}{B_t} \quad (4.B.46)$$

$$R_t^E = \frac{r_t^E}{\pi_{t+1}} \quad (4.B.47)$$

$$R_t^D = \frac{r_t^D}{\pi_{t+1}} \quad (4.B.48)$$

$$r_t^B = \left[\nu \left(r_t^{B,U} \right)^{1-\theta_B} + (1-\nu) \left(r_t^{B,P} \right)^{1-\theta_B} \right]^{\frac{1}{1-\theta_B}} \quad (4.B.49)$$

$$D_t = \nu D_t^U + (1-\nu) D_t^P \quad (4.B.50)$$

$$DIV_t = \nu DIV_t^U + (1-\nu) DIV_t^P \quad (4.B.51)$$

$$TRANS_t = \nu TRANS_t^U + (1-\nu) TRANS_t^P \quad (4.B.52)$$

$$INV_t = \nu INV_t^U + (1-\nu) INV_t^P \quad (4.B.53)$$

$$\Phi_{t-1}(\bar{\omega}_t) = G_{t-1}(\bar{\omega}_t) R_t^K Q_{t-1} K_{t-1} \quad (4.B.54)$$

Market Clearing - 4

$$H_t = H_t^{New} + F_{t-1}(\bar{\omega}_t) Z^R \bar{N} + (1-\delta^H) H_{t-1} \quad (4.B.55)$$

$$\bar{N} = \nu N_t^P + (1-\nu) N_t^U \quad (4.B.56)$$

$$Y_t = C_t + C_t^E + I_t + I_t^{BK} + \mu^M \Phi_{t-1}(\bar{\omega}_t) + \nu F_{t-1}(\bar{\omega}_t) \mu^{R,U} \left(N_{t-1}^U \right)^{\chi^{R,U}} + \nu \chi^{B,U} B_t^U \quad (4.B.57)$$

$$GDP_t = C_t + I_t + P_t^H H_t^{INV} \quad (4.B.58)$$

Monetary Policy - 2

$$\psi_t = \bar{\psi} \quad (4.B.59)$$

$$\frac{r_t^D}{\bar{r}^D} = \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \quad (4.B.60)$$

Exogenous Shocks - 5

$$\ln(\eta_t^H) = (1 - \rho_{\eta^H}) \bar{\eta}^H + \rho_{\eta^H} \ln(\eta_{t-1}^H) + \varepsilon_t^{\eta^H} \quad (4.B.61)$$

$$\ln(Z_t^G) = (1 - \rho_{Z^G}) \bar{Z}^G + \rho_{Z^G} \ln(Z_{t-1}^G) + \varepsilon_t^{Z^G} \quad (4.B.62)$$

$$\ln(\sigma_t^\omega) = (1 - \rho_{\sigma^\omega}) \bar{\sigma}^\omega + \rho_{\sigma^\omega} \ln(\sigma_{t-1}^\omega) + \varepsilon_t^{\sigma^\omega} \quad (4.B.63)$$

$$\ln(Z_t^H) = (1 - \rho_{Z^H}) \bar{Z}^H + \rho_{Z^H} \ln(Z_{t-1}^H) + \varepsilon_t^{Z^H} \quad (4.B.64)$$

$$\ln(\psi_t^{N,P}) = (1 - \rho_{\psi^{N,P}}) \bar{\psi}^{N,P} + \rho_{\psi^{N,P}} \ln(\psi_{t-1}^{N,P}) + \varepsilon_t^{\psi^{N,P}} \quad (4.B.65)$$

Chapter 5

The Foreclosure Accelerator versus the Financial Accelerator: Housing and Borrower's Net Worth

5.1 Introduction

Newspaper articles, documentaries and general discussions about the 2008 financial crisis center largely on the housing boom and bust now synonymous with the Great Recession. However, much of the recent business cycle literature which has focused on integrating financial factors into general equilibrium models don't feature housing markets (Christiano et al., 2013; Justiniano et al., 2011). Figure (5.1) highlights the connection between the timing of the fall in home prices, drop in output and pre-recession trough of finance premiums.

The disconnect between the general narrative regarding the role of housing in the worst postwar recession and the analysis of business cycles is bridged in this paper. In order to capture the above relationships, I develop a novel financial mechanism whereby firms rely on housing secured debt instruments to finance their investment projects. When home prices fall, the amount of secured debt firms can issue based upon a fixed amount of collateral

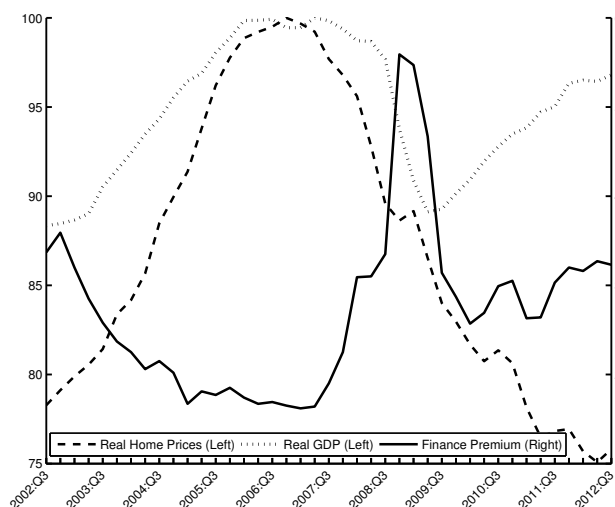


Figure 5.1: Housing, Finance Premiums and Real GDP

falls, forcing borrowers to turn to costly unsecured debt. This drives up finance premiums as default is more likely given the diminished financial position of the borrower. The unfavorable loan terms drive down investment and output. This mechanism is self re-enforcing since debt is secured by housing. When loan terms worsen, defaults increase resulting in more liquidated homes, further decreasing home prices and collateral values. I call this the *foreclosure accelerator*.

When I confront the data with this model, I find that home prices have played a significant role in shaping finance premiums, secured debt and output over the business cycle. Furthermore, unlike Christiano et al. (2013), I find that risk shocks have played only a marginal role in driving the business cycle. Instead, these shocks only explain high frequency movements in finance premiums. I additionally explore the implication for monetary policy makers and find the argument that the Fed should have tightened during the run-up in home prices has little empirical footing. Conversely, contractionary deviations from the estimated Taylor rule due to the binding zero lower bound caused a sharp contraction in output and spike in finance premiums.

5.2 A DSGE Model with Integrated Housing and Financial Markets

Here I take a medium-sized DSGE model and compliment it with a financial mechanism which integrates housing and financial markets. The basic model, features sticky prices, sticky wages, costly capital utilization, investment adjustment costs and habit-formation in consumption. These factors significantly improve the model's fit when it is confronted with the data (See for e.g. Christiano et al. (2005); Smets & Wouters (2007)).

5.2.1 Model Description

The model consists of a continuum of infinitely lived households, a continuum of short-lived entrepreneurs who produce the wholesale good using capital. New capital is produced by perfectly competitive firms who use inputs from the perfectly competitive investment goods producing sector. The introduction of these agents allows for time variation in the price of investment goods and the price of capital. The financial sector is comprised by a continuum of competitive banks which finances investment for goods producers. Lastly a central bank is modeled. In this section I will describe the behavior of each agent in turn. I follow the convention throughout the model that lower case variables are stationary and upper-case variables are non-stationary.

5.2.1.1 Household

There are a continuum of households indexed by $i \in [0, 1]$ where each household i supplies a differentiated labor. I assume as in Erceg et al. (2000) that household's have access to a complete state-contingent securities market so that households are homogeneous with respect to consumption and asset holdings, but possibly heterogeneous with respect to their labor income. This is reflected in my notation as I will index each household type i 's wage rate and labor hours supplied, but not other household level variables.

The household earns labor income by renting labor to goods producers $l_t(i)$ and home builders

$l_t^h(i)$. Since household i supplies a differentiated type of labor they have some degree of market power and therefore face the downward sloping demand for their labor variety:

$$l_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\theta_l} l_t \quad l_t^h(i) = \left(\frac{W_t^h(i)}{W_t^h} \right)^{-\theta_l} l_t^h \quad (5.1)$$

In order to introduce sticky nominal wages I assume that households must pay an adjustment cost similar to Rotemberg (1982). The specification of adjustment costs allows for the partial indexation of wages to lagged wage inflation, π_{t-1}^W and $\pi_{t-1}^{W,h}$, and the central bank's inflation target, $\bar{\pi}_t$.

Households earn non-labor income from their ownership stake in retail firms, $P_t s_t$, and principal plus interest payments $P_{t-1} d_{t-1} r_{t-1}^D$ on last periods deposits. This income can be saved in the form of bank deposits $P_t d_t$ or spent on consumption $P_t c_t$ and housing $P_t^h h_t$. Also, any non-depreciated housing stock, $(1 - \delta) h_{t-1}$, can be resold at the market price P_t^h . The resulting budget constraint in any period $t = 0, 1, 2, \dots$ is given by:

$$\begin{aligned} P_t c_t + P_t d_t + \frac{\phi_W}{2} \left[\frac{W_t(i)}{W_{t-1}(i) \pi_{t-1}^{W \varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right]^2 P_t l_t + \frac{\phi_W}{2} \left[\frac{W_t^h(i)}{W_{t-1}^h(i) \pi_{t-1}^{W,h \varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right]^2 P_t l_t^h \\ = W_t(i) l_t(i) + W_t^h(i) l_t^h(i) + P_{t-1} d_{t-1} r_{t-1}^d - P_t^h [h_t - (1 - \delta^h) h_{t-1}] + P_t s_t. \end{aligned}$$

The household maximizes their lifetime expected utility by choosing sequences for the variables $\{c_t, h_t, d_t, l_t(i), l_t^h(i), W_t(i), W_t^h(i)\}_{t=0}^\infty$ while satisfying the flow budget constraint along with the demands for their variety of labor. The household's lifetime expected utility is specified by:

$$U(i) = \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left\{ \eta_t^c \left[\ln(c_t - \varrho c_{t-1}) + \eta_t^h \ln(h_t) - \eta_{l,t} \frac{l_t(i)^{1+\nu_l}}{1+\nu_l} - \eta_{l,t}^h \frac{l_t^h(i)^{1+\nu_l}}{1+\nu_l} \right] \right\},$$

η_t^c , η_t^h , $\eta_{l,t}$ and $\eta_{l,t}^h$ represent shifts in the preferences over consumption goods, housing and leisure. I specify these as exogenous processes, each of which follows a first order auto-

regressive process:

$$\ln(\eta_t^c) = \rho_{\eta^c} \ln(\eta_{t-1}^c) + \varepsilon_t^{\eta^c} \quad \varepsilon_t^{\eta^c} \sim \mathcal{N}(0, \sigma_{\eta^c}), \quad (5.2)$$

$$\ln(\eta_t^h) = (1 - \rho_{\eta^h}) \ln(\bar{\eta}^h) + \rho_{\eta^h} \ln(\eta_{t-1}^h) + \varepsilon_t^{\eta^h} \quad \varepsilon_t^{\eta^h} \sim \mathcal{N}(0, \sigma_{\eta^h}), \quad (5.3)$$

$$\ln(\eta_{l,t}) = (1 - \rho_{\eta_l}) \ln(\bar{\eta}_l) + \rho_{\eta_l} \ln(\eta_{l,t-1}) + \varepsilon_t^{\eta_l} \quad \varepsilon_t^{\eta_l} \sim \mathcal{N}(0, \sigma_{\eta_l}), \quad (5.4)$$

$$\ln(\eta_{l,t}^h) = (1 - \rho_{\eta_l^h}) \ln(\bar{\eta}_l^h) + \rho_{\eta_l^h} \ln(\eta_{l,t-1}^h) + \varepsilon_t^{\eta_l^h}. \quad (5.5)$$

Although I follow Iacoviello (2005) and Iacoviello & Neri (2010) by modeling a housing market, I do not include household home equity borrowing constraints. I focus more so on real-estate secured assets as collateral for investment and the interaction of house prices and these assets. That being said, in general equilibrium changes in home prices can impact consumption via traditional income and substitution effects, but not through the home equity channel highlighted by Iacoviello (2005).

$$\lambda_t = \mathbb{E}_t \left\{ \frac{\eta_t^c}{c_t - \varrho c_{t-1}} - \beta \varrho \frac{\eta_{t+1}^c}{c_{t+1} - \varrho c_t} \right\} \quad (5.6)$$

$$\lambda_t p_t^h = \frac{\eta_t^c \eta_t^h}{h_t} + \beta(1 - \delta^h) \mathbb{E}_t \left\{ \lambda_{t+1} p_{t+1}^h \right\} \quad (5.7)$$

$$\lambda_t = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} r_t^d}{\pi_{t+1}} \right\} \quad (5.8)$$

$$(1 - \theta_l) w_t + \frac{\eta_{l,t} l_t(i)^{\nu_l}}{\lambda_t} \theta_l = \phi_W \left[\frac{\pi_t^W(i)}{\pi_{t-1}^{W\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right] \frac{\pi_t^W(i)}{\pi_{t-1}^{W\varphi_l} \bar{\pi}_t^{1-\varphi_l}} \frac{W_t}{W_t(i)} - \beta \phi_W \mathbb{E}_t \left\{ \frac{\lambda_{t+1} l_{t+1}}{\lambda_t l_t} \left[\frac{\pi_{t+1}^W(i)}{\pi_t^{W\varphi_l} \bar{\pi}_{t+1}^{1-\varphi_l}} - 1 \right] \frac{\pi_{t+1}^W(i)}{\pi_t^{W\varphi_l} \bar{\pi}_{t+1}^{1-\varphi_l}} \frac{W_t}{W_t(i)} \right\} \quad (5.9)$$

$$(1 - \theta_l) w_t^h + \frac{\eta_{l,t}^h l_t^h(i)^{\nu_l}}{\lambda_t} \theta_l = \phi_W \left[\frac{\pi_t^{W,h}(i)}{\pi_{t-1}^{W,h\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right] \frac{\pi_t^{W,h}(i)}{\pi_{t-1}^{W,h\varphi_l} \bar{\pi}_t^{1-\varphi_l}} \frac{W_t^h}{W_t^h(i)} - \beta \phi_W^h \mathbb{E}_t \left\{ \frac{\lambda_{t+1} l_{t+1}}{\lambda_t l_t} \left[\frac{\pi_{t+1}^{W,h}(i)}{\pi_t^{W,h\varphi_l} \bar{\pi}_{t+1}^{1-\varphi_l}} - 1 \right] \frac{\pi_{t+1}^{W,h}(i)}{\pi_t^{W,h\varphi_l} \bar{\pi}_{t+1}^{1-\varphi_l}} \frac{W_t^h}{W_t^h(i)} \right\} \quad (5.10)$$

5.2.1.2 The Financial Sector

I assume a continuum of banks populate the financial sector. These banks, indexed by z , make secured and unsecured loans to entrepreneurs who produce the consumption good. These loans are funded entirely by deposits raised from the household. However, households can not loan directly to entrepreneurs because they don't have access to the monitoring and liquidation technologies. Banks on the other hand are assumed to have access to technologies which allow them to (1) monitor loan projects in order to verify default on unsecured loans and (2) repossess and liquidate collateral posted for secured loans.

Since every bank z produces homogeneous outputs, no single bank has any degree of market power. Thus, banks take as given the interest rate paid on deposits along with the interest rate charged on secured and unsecured loans. Furthermore, I assume each bank can perfectly diversify any idiosyncratic risk and therefore depositors can be guaranteed the same rate of return across all banks. More specifically, borrowers who default on loans made in period $t-1$, which occurs with probability $F_{t-1}(\bar{\omega}_t)$, have their assets seized by a collections agency who distributes the proceeds, $\Upsilon_{t-1}(\bar{\omega}_t)$, to banks in proportion to their loan market share b_{t-1}^z/b_{t-1} . Banks still bear the monitoring costs of confirming repayment was not possible and therefore net only a fraction $1-\mu$ of these proceeds.

Meanwhile, borrowers who default on their secured loan, which also occurs with probability $F_{t-1}(\bar{\omega}_t)$, have their collateral repossessed by the bank and liquidated in the housing market. Banks have access to technology which enables them to transform repossessed collateral into housing at rate z^r while bearing the time-varying real marginal liquidation costs of χ_t^r :

$$\ln(\chi_t^r) = (1 - \rho_{\chi^r})\bar{\chi}^r \rho_{\chi^r} \ln(\chi_{t-1}^r) + \varepsilon_t^{\chi^r} \quad \varepsilon_t^{\chi^r} \sim \mathcal{N}(0, \sigma_{\chi^r}). \quad (5.11)$$

Movements in χ_t^r provide a proxy for financial innovation or differences in collateral from its fundamental value that may arise in times of market illiquidity.

These receipts and outlays are summarized in bank z 's period t profit function below:

$$\begin{aligned}\Pi_t^z &= (1 - F_{t-1}(\bar{\omega}_t)) r_{t-1}^b P_{t-1} b_{t-1}^z + (1 - \mu) \Upsilon_{t-1}(\bar{\omega}_t) \frac{b_{t-1}^z}{b_{t-1}} \\ &\quad + (1 - F_{t-1}(\bar{\omega}_t)) P_{t-1}^n n_{t-1}^z r_{t-1}^d + F_{t-1}(\bar{\omega}_t) \left[P_t^h z^r n_{t-1}^z - P_t \chi_{t-1}^r n_{t-1}^z \right] \\ &\quad - P_{t-1} d_{t-1}^z r_{t-1}^d + P_t d_t^z - P_t^n n_t^z - P_t b_t^z.\end{aligned}$$

Bank z seeks to maximize the discounted present value of their profits subject to the balance sheet constraint which stipulates that assets equal liabilities at all times. More formally, the bank solves:

$$\max_{\{b_t^z, n_t^z, d_t^z\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 \left\{ \mathcal{M}_{0|t} \frac{\Pi_t^z}{P_t} \right\} \quad \text{subject to} \quad P_t b_t^z + P_t^n n_t^z = P_t d_t^z,$$

where $\mathcal{M}_{0|t} = \beta^t \frac{\lambda_t}{\lambda_0}$ is the stochastic discount factor derived from the household's optimization problem. The first order conditions from the bank's problem will be useful when describing the entrepreneurs in the following section. Therefore, they are provided below.

$$\mathbb{E}_t \left\{ \mathcal{M}_{t|t+1} \frac{r_t^d}{\pi_{t+1}} b_t \right\} = \mathbb{E}_t \left\{ \mathcal{M}_{t|t+1} \left((1 - F_t(\bar{\omega}_{t+1})) \frac{r_t^b}{\pi_{t+1}} b_t + (1 - \mu) \frac{\Upsilon_t(\bar{\omega}_{t+1})}{P_{t+1}} \right) \right\} \quad (5.12)$$

$$\mathbb{E}_t \left\{ \mathcal{M}_{t|t+1} \frac{r_t^d}{\pi_{t+1}} p_t^n \right\} = \mathbb{E}_t \left\{ \mathcal{M}_{t|t+1} (p_{t+1}^H z^r - \chi_t^r) \right\} \quad (5.13)$$

5.2.1.3 Wholesale Goods Production

The goods producing sector is comprised of a continuum of entrepreneurs. Entrepreneurs have limited resources to finance capital required to produce the final good so they must borrow from banks. A financial friction arises whereby entrepreneurs borrow funds from banks this period to purchase capital used in production next period. Their output next period is subject to idiosyncratic productivity disturbances only observable by banks after paying a monitoring cost.

Entrepreneur's Debt Contract: Housing Secured Debt

Entrepreneurs are risk-neutral agents who supply wholesale goods to retailers using capital and labor. Here I use the large-family metaphor and assume that each household type i has a large number of entrepreneurs.¹ These entrepreneurs are indexed by $j \in \mathbb{R}_+$, where the index corresponds to their liquid assets, denoted a_t^j at the beginning of period t . In addition to these liquid assets, each entrepreneur is endowed with 1 unit of labor supply in period t and \bar{n} units of an asset which can be transformed into housing only by entrepreneurs and banks in period $t+1$. However, capital must be purchased in period t to be useful in period $t+1$. Denote the quantity of capital purchased in period t by entrepreneur j by k_t^j and denote the period t nominal price of capital by Q_t . To purchase this capital the entrepreneur will receive financing from the banking sector. More specifically, the entrepreneur use their liquid assets (which includes their wages earned in period t) and pledges their $t+1$ housing security endowment \bar{n} as collateral for a secured loan in the amount $P_t^n \bar{n}$. The remaining portion of the capital purchase is financed with an unsecured loan in the amount $P_t b_t^j$. More concretely,

$$Q_t k_t^j = P_t^n \bar{n} + P_t b_t^j + P_t a_t^j \quad (5.14)$$

is entrepreneur j 's budget constraint.

Without default, distinguishing between secured and unsecured loans is trivial. However, in period $t+1$, entrepreneurs are subjected to an idiosyncratic productivity shock ω_{t+1}^j which is *i.i.d.* across entrepreneurs and time. I assume throughout the analysis in this paper, $\omega_{t+1}^j \sim \ln \mathcal{N} \left(-\frac{(\sigma_t^\omega)^2}{2}, \sigma_t^\omega \right)$ with CDF at time t denoted by $F_t(\omega_{t+1})$. The choice of parameters implies $\mathbb{E} \{ \omega_{t+1}^j \} = 1$ so that in the aggregate this idiosyncratic shock has no direct impact on production, but the existence of uncertainty at the firm level impacts aggregate output through financial imperfections (BGG).

¹This assumption, as in Christiano et al. (2013), makes only 1 minor change to the equilibrium equations when compared to the set-up in Bernanke et al. (1999) who don't assume the entrepreneurs live with the household.

To capture exogenous changes in the cross-sectional dispersion of idiosyncratic productivity shocks I allow σ_t^ω to vary over time. I posit the simple auto-regressive process,

$$\ln(\sigma_t^\omega) = (1 - \rho_{\sigma^\omega})\ln(\bar{\sigma}^\omega) + \rho_{\sigma^\omega}\ln(\sigma_{t-1}^\omega) + \varepsilon_t^{\sigma^\omega} \quad \varepsilon_t^{\sigma^\omega} \sim \mathcal{N}(0, \sigma_{\sigma^\omega}) \quad (5.15)$$

for this demand-side *risk* shock. Christiano et al. (2013) show that such shocks have played a significant role in shaping the U.S. business cycle.

Since projects are financed before the idiosyncratic productivity shock can be observed by either the entrepreneur or the bank, entrepreneurs who receive a low productivity value will default upon their loan. Denote the gross interest rate on unsecured loans paid by entrepreneur j by $r_t^{b,j}$ and denote the gross return on capital common to all entrepreneurs by r_{t+1}^k . Then for any entrepreneur j , we can define the cut-off value of $\bar{\omega}_{t+1}^j$ by the equation

$$\bar{\omega}_{t+1}^j r_{t+1}^k k_t^j Q_t = P_t b_t^j r_t^{b,j}. \quad (5.16)$$

This equation defines the minimum level of productivity needed to pay back the unsecured loan. For entrepreneur j , the loan will be repaid if $\omega_{t+1}^j \geq \bar{\omega}_{t+1}^j$ and will otherwise be defaulted upon. However, the bank can not observe the level of productivity without paying an auditing cost in proportion $\mu \in (0,1)$ to the entrepreneur's revenue.² Banks who do not pay for auditing never find out if the entrepreneur actually received a low productivity draw or if they simply chose to renege on their loan. Given this arrangement, the optimal debt-contract from the borrowers perspective dictates that banks will audit only defaulting entrepreneurs and only entrepreneurs who receive a bad-draw will default on their loans.

Although the contract is optimal from the borrower's perspective, the entrepreneur must ensure the bank's profit maximization condition from (5.12) is satisfied. More strictly, the debt-contract must be individually rational from the bank's perspective *in every state of*

²This follows from Townsend (1979), but has been popularized in this context by Carlstrom & Fuerst (1997) and BGG.

nature since the banks are owned by risk-averse households. This implies the risk-neutral entrepreneurs bear all the aggregate risk, and the bank's optimality condition (5.12) holds both in and out of expectation:

$$\underbrace{r_t^d b_t^j}_{\text{Funding Cost}} = \underbrace{\left(1 - F_t(\bar{\omega}_{t+1}^j)\right) r_t^{b,j} b_t^j}_{\text{Non-Defaulting Payoff}} + \underbrace{(1 - \mu) \int_0^{\bar{\omega}_{t+1}^j} \omega_{t+1}^j dF_t(\omega_{t+1}^j) r_{t+1}^k k_t^j q_t}_{\text{Defaulting Payoff}} \quad (5.17)$$

where

$$v_t(\bar{\omega}_{t+1}) \equiv \frac{\Upsilon_t(\bar{\omega}_{t+1})}{P_{t+1}} = \int_0^{\bar{\omega}_{t+1}^j} \omega_{t+1}^j dF_t(\omega_{t+1}^j) \frac{r_{t+1}^k}{\pi_{t+1}} k_t^j q_t. \quad (5.18)$$

We can simplify this expression (and the resulting entrepreneur's optimization problem) by defining the following terms. First let $G_t(\bar{\omega}_{t+1}^j)$ be defined as the expected productivity value for defaulting entrepreneurs.

$$G_t(\bar{\omega}_{t+1}^j) = \int_0^{\bar{\omega}_{t+1}^j} \omega_{t+1}^j dF_t(\omega_{t+1}^j) \quad (5.19)$$

Also let $\Gamma_t(\bar{\omega}_{t+1}^j)$ be defined as the expected share of entrepreneurial profits going to the bank gross of auditing costs.

$$\Gamma_t(\bar{\omega}_{t+1}^j) = \left(1 - F_t(\bar{\omega}_{t+1}^j)\right) \bar{\omega}_{t+1}^j + G_t(\bar{\omega}_{t+1}^j) \quad (5.20)$$

Now I can combine (5.17) with (5.14), (5.16), (5.19) and (5.20) to rewrite the bank's individual rationality constraint as follows:

$$r_t^d \left(q_t k_t^j - p_t^n \bar{n} - a_t^j \right) = \left(\Gamma_t(\bar{\omega}_{t+1}^j) - \mu G_t(\bar{\omega}_{t+1}^j) \right) r_{t+1}^k k_t^j q_t. \quad (5.21)$$

We can now formally state the problem faced by entrepreneur j . To keep the debt-contract tractable, I assume the entrepreneur is instructed by the household to maximize their period $t + 1$ income (as a random fraction, $1 - \gamma_t$, is transferred to households) and in exchange

entrepreneurs receive consumption insurance from household's. Entrepreneurs who succeed, are contractually obligated to pay-back their secured loan, prior to transferring funds to homeowners. Since homeowners own the banks, they yield to the seniority of the debt structure.

Entrepreneur j therefore seeks to maximize the gross expected income (of which the fraction $(1 - \gamma_t)$ is transferred to their household), subject to (5.21):

$$\begin{aligned} \max_{k_t^j, \bar{\omega}_{t+1}^j} \mathbb{E}_t \left\{ \left(1 - \Gamma_t(\bar{\omega}_{t+1}^j) \right) r_{t+1}^k k_t^j q_t \right\} \\ \text{subject to } r_t^d \left(q_t k_t^j - p_t^n \bar{n} - a_t^j \right) = \left(\Gamma_t(\bar{\omega}_{t+1}^j) - \mu G_t(\bar{\omega}_{t+1}^j) \right) r_{t+1}^k k_t^j q_t \end{aligned}$$

Where I use (5.16) and (5.20) to write $\mathbb{E}_t \left\{ \int_{\bar{\omega}_{t+1}^j}^{\infty} \omega_{t+1}^j dF(\omega_{t+1}^j) r_{t+1}^k k_t^j q_t - (1 - F_t(\bar{\omega}_{t+1}^j)) r_t^{b,j} b_t^j \right\} = \mathbb{E}_t \left\{ (1 - \Gamma_t(\bar{\omega}_{t+1}^j)) r_{t+1}^k k_t^j q_t \right\}$. The solution to this optimization problem pins down the cut-off value $\bar{\omega}_{t+1}^j$ and the entrepreneur's demand for capital k_t^j .

$$\mathbb{E}_t \left\{ \left(1 - \Gamma_t(\bar{\omega}_{t+1}^j) \right) r_{t+1}^k - \frac{\Gamma'_t(\bar{\omega}_{t+1}^j)}{\Gamma'_t(\bar{\omega}_{t+1}^j) - \mu G'_t(\bar{\omega}_{t+1}^j)} \left[r_t^d - \left(\Gamma_t(\bar{\omega}_{t+1}^j) - \mu G_t(\bar{\omega}_{t+1}^j) \right) r_{t+1}^k \right] \right\} = 0 \quad (5.22)$$

The problem is identical in nature to the problem entrepreneurs face in BGG who show the optimal debt contract has the property that the default rate, $F_t(\bar{\omega}_{t+1}^j)$, and external finance premium, r_t^k / r_{t-1}^d , move inversely with net-worth. In this model, the net-worth component is replaced with the collateral value and wage earnings, $p_t^n \bar{n} + w_t^E$, implying that $\partial \bar{\omega}_{t+1}^j / \partial p_t^n < 0$ - finance premiums and default rates will move in the opposite direction of collateral prices. From (5.13), this implies finance premiums will move in the opposite direction of home prices.

Aggregate Wholesale Goods Production

The previous section describes the firm-level behavior in the wholesale goods producing sector, specifically it describes the debt-contracting problem faced by each producer. In this section, I describes the industry wide behavior. Since each entrepreneur (in the second period of their life) has access to the homogeneous production technology, $y_t^j = \omega_t^j \left(k_{t-1}^j \right)^\alpha \left(l_t^j \right)^{1-\alpha}$,

which can be aggregated over due to constant returns to scale. The aggregate goods production technology in any given period t is specified as

$$y_t = z_t (k_{t-1})^\alpha (l'_t)^{1-\alpha} \quad (5.23)$$

where z_t is an exogenous technology process which affects all entrepreneurs equally. I assume this technology follows a first-order autoregressive process.

$$\ln(z_t) = \rho_z \ln(z_{t-1}) + \varepsilon_t^z \quad \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z) \quad (5.24)$$

Wholesale producers use this technology to produce the wholesale good which is purchased by retailers. This wholesale goods market is perfectly competitive and therefore the wholesale price, P_t^W , is taken as given. We can now consider the determination of factor demands for capital and labor.

The labor aggregate in the production function is a composite of labor supplied by the household, l_t , and labor supplied by this periods young entrepreneurs, l_t^E ,

$$l'_t = (l_t^E)^{\alpha^E} (l_t)^{1-\alpha^E}, \quad (5.25)$$

where α^E captures the share of goods producing income received by entrepreneurs. This implies the wage paid to the household's labor and the wage paid to entrepreneurial labor are given by:

$$W_t = P_t^W (1 - \alpha) (1 - \alpha_t^E) \frac{y_t}{l_t} \quad (5.26)$$

$$W_t^E = P_t^W (1 - \alpha) \alpha_t^E \frac{y_t}{l_t^E}. \quad (5.27)$$

Similarly, the marginal product of capital is given by:

$$mp_t^k = \alpha \frac{y_t}{k_{t-1}} \quad (5.28)$$

The gross nominal return on holding a unit of capital from period $t - 1$ to period t is defined by

$$r_t^k = \frac{u_t mp_t^k P_t^W - a(u_t) P_t + (1 - \delta^k) Q_t}{Q_{t-1}}, \quad (5.29)$$

where u_t is the utilization rate chosen by the producers and $a(u_t)$ are the real resources spent to support this utilization rate:

$$a(u_t) = \frac{\bar{m} \bar{p}^k}{\chi^a} \left[e^{\chi^a (u_t - 1)} - 1 \right]. \quad (5.30)$$

The only way the utilization rate impacts entrepreneurs is through (5.29). Therefore, entrepreneurs solve the problem:

$$\max_{u_t} r_t^k \quad (5.31)$$

Hence, from this point on, u_t represents the optimal utilization rate and satisfies the first order condition:

$$mp_t^k p_t^W = \bar{m} \bar{p}^k \bar{p}^W e^{\chi^a (u_t - 1)}, \quad (5.32)$$

where the specification of $a(u_t)$ ensures that $\bar{u} = 1$ so steady state utilization costs are zero.

Notice that in (5.32), the first order condition for choosing u_t to maximize r_t^k , and in (5.22), the optimal choice of $\bar{\omega}^j$ are both independent of each entrepreneur's liquid assets. Hence, the distribution of the liquid assets across entrepreneurs is not needed to determine aggregate capital stock and loans.³ Hence, the aggregate income of entrepreneurs in period t is $(1 - \Gamma_{t-1}(\bar{\omega}_t)) \frac{r_t^k}{\pi_{t+1}} k_{t-1} q_{t-1}$. Entrepreneurs have the ability to transform their endowment \bar{n} - into non-tradable housing. Recall however, this endowment was leveraged last period to

³See equations 2.15 and 2.16 in Christiano et al. (2013) for more details on the aggregation across entrepreneurs.

secure a loan in the amount $p_{t-1}^n \bar{n}$. Hence, entrepreneurs who are able, choose to payback the secured loan with interest $p_{t-1}^n \bar{n} r_{t-1}^d$ and then convert \bar{n} into housing services one for one. If they don't payback the secured loan then they default on this contract and the bank takes possession of the collateral \bar{n} .

I assume in the aggregate, all the entrepreneurs who did not default on their unsecured loan, payback their secured loan and divide the rest of their income between transfers to households and liquid assets next period so that a_t evolves according to:

$$a_t = \gamma \left[(1 - \Gamma_{t-1}(\bar{\omega}_t)) \frac{r_t^k}{\pi_t} k_{t-1} q_{t-1} - (1 - F_{t-1}(\bar{\omega}_t)) p_{t-1}^n \bar{n} \frac{r_{t-1}^d}{\pi_t} \right] + w_t^E. \quad (5.33)$$

A more elaborate specification would make the entrepreneur's decision of whether to pay-back their secured loan a strategic choice of the entrepreneur; whereas, this specification implies the only source of default on secured loan is due to insufficient income. An appealing aspect of this description is the existence of a single default rate in the economy.⁴ Moreover, this specification captures the essence of securing debt with mortgage backed securities. When the default rate of mortgages rise, financial firms (investment banks for example) who rely on these payments to fund their debt (which further fund MBS purchases and nonfinancial firms projects) are more likely to default. These risk cause secured loan markets to dry-up. Interpreting the entrepreneur as an investment bank who funds non-financial firms in a market without imperfections yields such an interpretation.

5.2.1.4 Retail Goods Producers

Retailers are monopolistically competitive firms who purchase the generic wholesale good at price P_t^W and transform it into a unique variety. Due to the differentiated nature of each firm's, $f \in [0, 1]$, output they face a downward sloping demand for their variety of the final

⁴That is to say, the default rate on unsecured loans is the same as the default rate on secured loans. I choose this as a starting point although this assumption can be relaxed.

good:

$$y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\theta_t} y_t \quad (5.34)$$

where variations in θ_t appear as “Cost Push,” or non-technology supply shocks in the log-linearized New-Keynesian Phillips Curve that emerges from the retailer’s first order condition. I assume as in Smets & Wouters (2007):

$$\ln(\theta_t) = (1 - \rho_\theta)\bar{\theta} + \rho_\theta \ln(\theta_{t-1}) + \varepsilon_t^\theta \quad \varepsilon_t^\theta \sim \mathcal{N}(0, \sigma_\theta). \quad (5.35)$$

Given the market power of retail firms, each firm f would like to sell their variety for a rent maximizing mark-up over marginal cost. However, to introduce nominal price rigidities, I assume firms also bear a convex cost of adjusting their price relative to the rate of inflation as in Rotemberg (1982). More specifically, retailers seek to maximize the discounted sum of their real dividend stream:

$$\max_{\{P_t(f)\}_{t=0}^\infty} \sum_{t=0}^\infty \mathbb{E}_0 \left\{ \mathcal{M}_{0|t} \left[\frac{P_t(f)}{P_t} y_t(f) - \frac{P_t^W}{P_t} - \frac{\phi}{2} \left[\frac{P_t(f)}{P_{t-1}(f) \pi_{t-1}^\varphi \bar{\pi}_t^{1-\varphi}} - 1 \right]^2 y_t \right] \right\} \quad \text{subject to} \quad (5.34). \quad (5.36)$$

In a symmetric equilibrium where $P_t(f) = P_t \forall f$, the resulting first order condition defines the typical forward-looking aggregate supply relationship relating inflation to real marginal cost. In other words, the New-Keynesian Phillips Curve emerges from the firm’s problem.

$$\begin{aligned} \theta_t p_t^W \left(\frac{P_t(f)}{P_t} \right)^{-\theta_t-1} - (\theta_t - 1) \left(\frac{P_t(f)}{P_t} \right)^{-\theta_t} &= \phi \left[\frac{\pi_t(f)}{\pi_{t-1}^\varphi \bar{\pi}_t^{1-\varphi}} - 1 \right] \frac{\pi_t(f)}{\pi_{t-1}^\varphi \bar{\pi}_t^{1-\varphi}} \frac{P_t}{P_t(f)} \\ &\quad - \phi \mathbb{E}_t \left\{ \frac{y_{t+1} \mathcal{M}_{t,t+1}}{y_t \mathcal{M}_{t,t}} \left[\frac{\pi_{t+1}(f)}{\pi_t^\varphi \bar{\pi}_t^{1-\varphi}} - 1 \right] \frac{\pi_{t+1}(f)}{\pi_t^\varphi \bar{\pi}_t^{1-\varphi}} \frac{P_t}{P_t(f)} \right\} \end{aligned} \quad (5.37)$$

5.2.1.5 New Housing Production

I assume new housing is produced in a purely competitive market and free from financial frictions. In particular, housing producers combine labor l_t^h with housing specific technology,

z_t^h in the production technology,

$$h_t^{new} = z_t^h (l_t^h)^{1-\alpha^h} \quad (5.38)$$

where

$$\ln(z_t^h) = \rho_{zh} \ln(z_{t-1}^h) + \varepsilon_t^{zh} \quad \varepsilon_t^{zh} \sim \mathcal{N}(0, \sigma_{zh}). \quad (5.39)$$

I model housing specific technology independent of technology in the goods producing sector since much of the economic growth over the last two decades has been IT-driven and housing production is a non IT-intensive industry. Moreover, this specification allows for the technology process in the goods producing sector, z_t , to potentially play a significant role in determining output without implying a counterfactual negative correlation between home prices and GDP (see for example Davis & Heathcote (2005)). The resulting demand for labor from the housing sector takes the form:

$$W_t^h = P_t^h (1 - \alpha^h) \frac{h_t^{new}}{l_t^h}. \quad (5.40)$$

5.2.1.6 Investment Goods Production

Perfectly competitive firms purchase y_t^i units of the final retail good at a price of P_t and use the linear, time-varying technology:

$$i_t = z_t^i y_t^i \quad (5.41)$$

to transform these retail goods into investment goods, which they sell to capital goods producers for a market determined price P_t^i . The representative investment goods producing firm solves the following problem:

$$\max_{y_t^i} P_t^i z_t^i y_t^i - P_t y_t^i, \quad (5.42)$$

which pins down the relative price of investment as:

$$\frac{P_t^i}{P_t} = \frac{1}{z_t^i}. \quad (5.43)$$

The purpose of explicitly including this sector in the model is to examine the role investment-specific technology play in shaping the business cycle. During the estimation the price of investment goods relative to consumption goods, $p_t^i = P_t^i/P_t$ is treated as an observable variable which disciplines the estimation of the z_t^i process:

$$\ln(z_t^i) = \rho_{zi} \ln(z_{t-1}^i) + \varepsilon_t^{z^i} \quad \varepsilon_t^{z^i} \sim \mathcal{N}(0, \sigma_{z^i}). \quad (5.44)$$

5.2.1.7 Capital Goods Production

In period t , the capital goods producing firm purchases any non-depreciated capital from period $t-1$ at the market price Q_t and produces new capital using the investment good which is purchased at the price P_t^i . The sum of these two components comprises the total capital stock at the end of period t , which is sold to entrepreneurs at the price Q_t . The rate at which the capital goods producers can transform the investment good and old, undepreciated capital into new capital is constrained by the technology:

$$k_t = \left(i_t - S \left(\frac{i_t}{z_t^k k_{t-1}} \right) z_t^k k_{t-1} \right) + (1 - \delta^k) k_{t-1} \quad (5.45)$$

where $S(\cdot)$ captures cost of adjusting the level of investment. This function has the properties that $S(\delta^k) = S'(\delta^k) = 0$ and $S''(\delta^k) > 0$. In practice, these derivatives can be matched with the function:

$$S \left(\frac{i_t}{z_t^k k_{t-1}} \right) = \frac{\chi^S}{2} \left[\frac{i_t}{z_t^k k_{t-1}} - \delta^k \right]^2 \quad \text{where } \chi^S > 0. \quad (5.46)$$

In the financial accelerator model of Bernanke et al. (1999) and Christiano et al. (2013), this function plays a critical role in the propagation of downturns by endogenously decreasing the price of capital when investment falls. This drop in asset prices decreases the net worth

of entrepreneurs in these models which ultimately leads to further reductions in investment, starting the cycle over. This is the celebrated *financial accelerator* mechanism.

Meanwhile the term z_t^k in (5.45) captures variations in the efficiency at which the investment goods can be transformed into installed capital:

$$\ln(z_t^k) = \rho_{zk} \ln(z_{t-1}^k) + \varepsilon_t^{z^k} \quad \varepsilon_t^{z^k} \sim \mathcal{N}(0, \sigma_{zk}). \quad (5.47)$$

In Justiniano et al. (2011), movements in z_t^k make the largest contribution to output, investment and labor dynamics at business cycle frequencies. However, the bulk of these movements correspond to financial distress. Therefore it is perhaps not surprising that when these shocks are included in the estimation of the enriched BGG/financial accelerator model of Christiano et al. (2013), movements in σ_t^ω overtake movements in z_t^k as the most important shocks at business cycle frequencies. The framework presented in this paper allows these shocks to compete with housing demand disturbances in driving the business cycle. The capital goods producer solves:

$$\max_{k_t, i_t} Q_t k_t - Q_t(1 - \delta^k)k_{t-1} - P_t^i i_t \quad \text{subject to} \quad (5.45), \quad (5.48)$$

which implies the following Tobin's Q relationship:

$$Q_t = \frac{P_t^i}{1 - S' \left(\frac{i_t}{z_t^k k_t} \right)}. \quad (5.49)$$

5.2.1.8 Central Bank

The central bank is charged with setting monetary policy. The monetary policy instrument is the rate on 1-period deposits, r_t^d , which is assumed to follow a linear (in logs) feedback

rule of the type specified by Taylor (1993):

$$\frac{r_t^d}{\bar{r}^d} = \left(\frac{r_{t-1}^d}{\bar{r}^d} \right)^{\rho_r} \left(\frac{\pi_t}{\bar{\pi}_t} \right)^{\psi_\pi} \left(\frac{y_t}{y_{t-1}} \right)^{\psi_y} \epsilon_t^r \epsilon_{t,4}^r \quad (5.50)$$

where,

$$\ln(\epsilon_t^r) = \varepsilon_t^r \quad \varepsilon_t^r \sim \mathcal{N}(0, \sigma_r) \quad (5.51)$$

$$\ln(\epsilon_t^{r,4}) = \rho_{r,4} \ln(\epsilon_{t-1}^{r,4}) + \varepsilon_{t-4}^{r,4} \quad \varepsilon_t^{r,4} \sim \mathcal{N}(0, \sigma_{r,4}). \quad (5.52)$$

There are three senses in which this rule evolves stochastically. First, there is a typical monetary policy shock, ϵ_t^r which is completely unforecastable by agents. However, I also allow for the possibility that agents learn the central bank is going to deviate from their policy rule in the future through the exogenous term $\epsilon_t^{r,4}$. Although the solution method employed to estimate the model parameters doesn't allow for explicitly incorporating agents' knowledge of the zero lower bound on nominal interest rates, these news shocks allow for deviations which are not a surprise to agents.

A third sense in which the policy rule evolves stochastically arises from the central bank's inflation target, $\bar{\pi}_t$ which evolves according to an AR(1) process:

$$\ln(\bar{\pi}_t) = (1 - \rho_{\bar{\pi}})\bar{\pi} + \rho_{\bar{\pi}}\ln(\bar{\pi}_{t-1}) + \varepsilon_t^{\bar{\pi}} \quad \varepsilon_t^{\bar{\pi}} \sim \mathcal{N}(0, \sigma_{\bar{\pi}}). \quad (5.53)$$

This shock is introduced as a means of capturing the downward trend in inflation in the early part of the sample. To this end, I set $\rho_{\bar{\pi}} = 0.975$ and $\sigma_{\bar{\pi}} = 0.0001$ (Christiano et al., 2013).

5.2.1.9 Market Clearing

Sections 5.2.1.1 - 5.2.1.8 describe the optimal behavior of all agents in the economy. A symmetric competitive equilibrium is defined as a sequence of quantities, prices and Lagrange multipliers (shadow prices) which satisfy all optimality conditions, policy rules and market

clearing conditions. In particular, symmetry across households implies that:

$$l_t(i) = l_t \tag{5.54}$$

$$W_t(i) = W_t \tag{5.55}$$

$$l_t^h(i) = l_t^h \tag{5.56}$$

$$W_t^h(i) = W_t^h. \tag{5.57}$$

Symmetry across banks implies:

$$b_t^z = b_t \tag{5.58}$$

$$d_t^z = d_t \tag{5.59}$$

$$n_t^z = n_t. \tag{5.60}$$

Symmetry across entrepreneurs implies:

$$b_t^j = b_t \tag{5.61}$$

$$k_t^j = k_t. \tag{5.62}$$

Similarly, symmetry across retailers implies:

$$P_t(f) = P_t \tag{5.63}$$

$$y_t(f) = y_t. \tag{5.64}$$

The market clearing condition in the collateralized loan market requires that at all times:

$$n_t = \bar{n}. \tag{5.65}$$

Using this requirement, the housing market clearing condition can be expressed as:

$$h_t = h_t^{new} + F_{t-1}(\bar{\omega}_t)z^r\bar{n} + (1 - \delta^h)h_{t-1}. \quad (5.66)$$

Using the collateralized loan market clearing again, the goods market clearing condition can be expressed by the resource constraint:

$$y_t = c_t + p_t^i i_t + F_{t-1}(\bar{\omega}_t)\chi^r\bar{n} + \mu v_{t-1}(\bar{\omega}_t) + a(u_t) + ac_t, \quad (5.67)$$

where ac_t defines the real-resource cost of adjusting prices and wages:

$$ac_t = \frac{\phi}{2} \left[\frac{\pi_t}{\pi_{t-1}^\varphi \bar{\pi}_t^{1-\varphi}} - 1 \right]^2 y_t + \frac{\phi_W}{2} \left[\frac{\pi_t^W}{\pi_{t-1}^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right]^2 l_t + \frac{\phi_W}{2} \left[\frac{\pi_t^{W^h}}{\pi_{t-1}^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right]^2 l_t^h. \quad (5.68)$$

Since ac_t is of second-order, in a linear approximation we have that $ac_t \approx 0$.

It is useful for the purpose of model inference to define gross domestic product in this multi-sector model:

$$gdp_t = c_t + p_t^i i_t + p_t^h h_t^{new}. \quad (5.69)$$

5.3 Model Inference

In this section I describe the econometric strategy employed to estimate the model's parameters. I discuss the data used in this analysis, the choice of priors with an emphasis on the relative tightness of the prior specifications and finally the specification of news shocks and examination of model fit.

5.3.1 Data

I use quarterly data on 11 variables spanning 1990:Q1 to 2013:Q4. 7 variables are standard macroeconomic time-series including: consumption, non-residential investment, the relative price of non-residential investment goods, the federal funds rate, inflation, the real wage

rate and hours worked. Since there are multiple sectors in the model, including the real wage rate and hours worked in both sectors would require bringing in more variables to the analysis, limiting the number of financial variables I could introduce without inducing stochastic singularity. To avoid this issue I compute the aggregate real wage rate, w_t^a , and hours worked, l_t^a , as a Tornquist-Thiel (or Divisia) index number:

$$\ln\left(\frac{l_t^a}{l_{t-1}^a}\right) = \frac{1}{2}(s_t^h + s_{t-1}^h)\ln\left(\frac{l_t^h}{l_{t-1}^h}\right) + \frac{1}{2}(s_t + s_{t-1})\ln\left(\frac{l_t}{l_{t-1}}\right) \quad (5.1)$$

$$\ln\left(\frac{w_t^a}{w_{t-1}^a}\right) = \frac{1}{2}(s_t^h + s_{t-1}^h)\ln\left(\frac{w_t^h}{w_{t-1}^h}\right) + \frac{1}{2}(s_t + s_{t-1})\ln\left(\frac{w_t}{w_{t-1}}\right). \quad (5.2)$$

where $s_t = w_t l_t / (w_t l_t + w_t^h l_t^h)$ is the share of income earned in the goods producing sector and $s_t^h = 1 - s_t$ is the share of income earned in the housing sector. For the data component, I use BLS aggregate measures of hours worked and wages earned in the private sector.

The remaining 4 variables consist of housing and financial market variables. The 2 housing market variables include residential fixed investment and real home prices, measured using the S&P/Case-Shiller National Composite Home Price Index. The remaining 2 variables are financial variables. As Christiano et al. (2013) show, including financial variables in the estimation is critical to capturing the role credit frictions play in the business cycle. Most critical of these variables is the external finance premium, $r_t^k/r_{t-1}^d - 1 = \mu^{\frac{v_{t-1}(\bar{\omega}_t)}{b_{t-1}}}$ which is measured empirically as the difference between BAA-rated corporate bonds and the 10-year U.S. government bond rate. Again, similar to Christiano et al. (2013) I inform my estimation on the measure of credit using data on credit to non-financial firms from the Flow of Funds dataset. In particular, $b_t + p_t^n \bar{n}$ is measured as the (first log difference in) the ‘credit market instruments’ measure of total liabilities for nonfarm, nonfinancial corporate and noncorporate business.

5.3.2 Parameters: Calibration and Prior Specification

The model has a total of 54 parameters. I will divide these parameters into 3 groups: calibrated parameters, parameters with tight priors, parameters with loose priors. The logic of the partition is as follows. Several pairs of parameters are likely to not be jointly identified and therefore one of the pair must be calibrated prior to estimation. Also, several parameters such as capital's share in the production function and the depreciation rate of capital are standard in macroeconomic models and therefore can be easily calibrated. Moreover, the model variables are by construction in log-deviations from steady state and therefore are mean zero. Hence, for the data to align with the construction of the model variables all data is demeaned. That being said, the information contained in the first moment need not be disposed. Instead, this information can be brought into the model by specifying the values of parameters which determine the model's steady state. This information thus plays a role in the calibration and specification of priors.

5.3.2.1 Calibrated Parameters

It is well known the discount rate β and depreciation rates δ^h and δ^k are not jointly identified. To avoid this potential pitfall, I use the consensus value of $\delta^k = 0.025$. At the same time, I set \bar{n} to ensure the steady state return on capital is $\bar{r}^k = 1.025$. Moreover, I fix β so that $\bar{r}^d = \bar{\pi}/\beta = 1.015$ where $\bar{\pi}$ represents the mean of the central bank's inflation target. These two facts imply a steady-state difference between the risky loan rate and the policy rate equals to 4.00% annually, the average spread between the BAA-corporate bond rate and the federal funds rate, according to data obtained from the St. Louis Fed's FRED database. The housing depreciation parameter, δ^h is calibrated to 0.0123 which implies, along with the other means of the prior distributions, that the steady state ratio of housing wealth to annual GDP equals 1.4. In other words, $\delta^h = \frac{\bar{p}^h(h^{new} + F(\omega)z^r\bar{n})}{1.4 \times 4 \times gdp}$. Following Christiano et al. (2013) I fix the inflation target at $\bar{\pi} = 1.0051$ so that inflation averages 2.4% per year. As noted in the text, to capture the downward trend in inflation during the early part of the

sample, I set $\rho_{\bar{\pi}} = 0.975$ and $\sigma_{\bar{\pi}} = 0.0001$.

Table 5.1: Model Calibrated Parameters

Parameter Name	Parameter	Calibrated Value ^(a)
Steady State Inflation Target	$\bar{\pi}$	1.0051
Inflation Target Shock	$\rho_{\bar{\pi}}$	0.9750
	$\sigma_{\bar{\pi}}$	0.0001
Household Discount Factor	β	0.9902
Capital Depreciation Rate	δ^k	0.0250
Housing Depreciation Rate	δ^h	0.0123
Entrepreneur's Housing Secured Assets	\bar{n}	$r^k = 1.025$
Dispersion of Entrepreneur Productivity	$\bar{\sigma}_{\sigma\omega}$	0.1596
CES across Retail Goods	θ	6
CES across Labor Types	θ_l	21
Inverse Frisch Elasticity of Labor	ν_l	1
Labor's Share in Goods Production	$1 - \alpha$	0.60
Labor's Share in Housing Production	$1 - \alpha_h$	0.70
Entrepreneur's Share of Labor Input	α_E	0.01
Disutility of Goods Labor Supply	$\bar{\eta}_l$	$\bar{l} = 1$
Disutility of Housing Labor Supply	$\bar{\eta}_l^h$	$\bar{l}^h / (\bar{l} + \bar{l}^h) = 0.065$
Bank's Liquidation Technology	z^r	1
Bank's Monitoring Cost	μ	0.1617

^(a) Parameters which are defined by a given equality condition are adjusted to satisfy the given condition at every iteration.

Along these same lines, the slopes of the price and wage Phillips curves are given by:

$$\kappa_{\pi} = \frac{\theta - 1}{\phi}$$

$$\kappa_{\pi W} = \frac{\theta_l - 1}{\phi_W}.$$

Therefore, the elasticity of substitution and the parameter governing the cost of price adjustment are not both identified. Thus, I estimate ϕ and ϕ_W while calibrating θ and θ_l . More specifically, I set $\theta = 6$ which implies a steady state mark-up of 20% on retail goods and $\theta_l = 21$ which implies a steady state mark-up of 5% on wages.

As for the production technologies, I set $\alpha = 0.4$ which implies labor's share of income in the goods producing sector is 60%. Following Iacoviello & Neri (2010), I set $\alpha_h = 0.3$. Finally, as in Bernanke et al. (1999), I set $\alpha_E = 0.01$. The household's preferences over labor are specified so that the curvature on the disutility of labor, $\nu_l = 1$. There are no natural units of hours worked in the model. Hence, I normalize the steady state hours worked in the goods producing sector to 1 and use the empirical fact that the average share of hours worked in construction relative to all sectors is $\bar{l}^h / (\bar{l}^h + \bar{l}) = 0.065$. This value is found in U.S. data produced by the BLS. I adjust $\bar{\eta}_l$ and $\bar{\eta}_l^h$ so that the household's preferences are consistent with this steady state. As for the banking parameters, z^r and $\bar{\chi}^r$ are likely to be weakly identified jointly. Therefore, I normalize $z^r = 1$ and estimate $\bar{\chi}^r$.

5.3.2.2 Prior Specification

The remaining 38 parameters are estimated using Bayesian methods. Of these parameters, 24 govern of the persistence and standard deviation of the exogenous shock processes. Since the central question of this paper is the relative importance of housing secured debt instruments and therefore the importance of shocks which drive home prices and the business cycle, I remain agnostic about the prior importance of all shocks. For all of these processes, I specify a Beta prior on the auto-correlation terms with mean 0.8 and a standard deviation of 0.1. For the standard deviation of the white noise shocks which drive the model, I take a similar agnostic stand and specify an Inverse-Gamma prior with mean 0.001 and standard deviation 0.01.

The 14 remaining parameters allow for considerably more information to be imparted into the prior. Beginning with the costs of price and wage adjustment, there are wide ranging estimates (see for example Keen & Wang (2007)). One obvious way to interpret the value is to use the implied equivalence between the Calvo and Rotemberg approach to nominal rigidities. In particular, the two approaches yield identical New-Keynesian Phillips Curves, when the slope terms are calibrated so that: $\frac{(1-\xi)(1-\beta\xi)}{\xi} = \frac{\theta-1}{\phi}$. This equality implies that for a given probability of prices remaining fixed $\xi \in [0, 1]$, the corresponding cost of price adjustment ϕ

lies in $[0, \infty)$. To keep the supports of the priors for ϕ and ϕ_W compact, I specify a Beta prior for ξ and ξ_W and apply the above transformation to find the corresponding cost of price and wage adjustment. The prior means are consistent with prices and wages remain unchanged for on average 2 quarters and 4 quarters respectively.

The prior for the steady state fraction of liquid assets retained by entrepreneurs, $\bar{\gamma}$, monitoring costs μ , repossession cost $\bar{\chi}^r$ and cross-sectional variance of productivity shocks across entrepreneurs $\bar{\sigma}^\omega$, are calibrated to match the following empirical facts (with all other parameters set to their prior mean): The average ratio of liquid assets to secured loans is equal to 0.42, the average ratio of secured loans to all firm liabilities is 0.21, the average annual business failure rate is 0.04 and residential investment's share of GDP is 0.0541.⁵

The model is able to exactly match these facts and for this reason I calibrate $\mu = 0.1617$ and $\bar{\sigma}^\omega = 0.1596$, which can be interpreted from the Bayesian perspective as specifying extremely tight priors.⁶ For the remaining parameters, the implied values from this moments matching exercise implies the following specifications of the prior distributions: $\bar{\gamma}$ is distributed as a Beta with mean 0.9103 and standard deviation 0.10 and $\bar{\chi}^r$ is distributed Log-Normally with mean 0.3109 and standard deviation 0.10. Although many of these variables are used as observables in the estimation, all of the data are demeaned. Hence, matching the model's steady state with these sample means does not contaminate the priors.

The remaining parameters do not determine any steady state outcomes. Therefore, I specify rather loose priors for these parameters so that the econometrics specification has considerable flexibility to match the dynamics in the data. I specify the elasticity of the adjustment cost function, χ^a , as a Log-Normal with mean 1.0 and standard deviation 1.0. Similarly, the elasticity of the capital adjustment cost function χ^S , is specified as a Log-Normal with mean 5.0 and standard deviation 5.0. Similarly, I set the prior for the price indexation, wage

⁵The housing preference shock's mean $\bar{\eta}_h$, is set so that $p^n = 1$ in steady state. This normalization is necessary since \bar{n} and p^n are not separately identifiable given this information.

⁶Calibrating these parameters, as opposed to estimating them, also leads to a considerably more efficient algorithm to solve for the model's steady state. This therefore speeds-up the estimation significantly.

indexation and habit parameters to Beta distribution with mean 0.5 and standard deviation 0.40.

Finally, regarding the specification of monetary policy, I set the priors for the parameters ρ_r , ϕ_π and ψ_g which allow for considerable smoothing but contemporaneous reactions to inflation and real economic activity which are largely consistent with Taylor's (1993) values. In particular, I specify a Beta prior for ρ_r with mean 0.8 and standard deviation 0.1. Since ψ_π and ψ_g are assumed to be positive, I set Log-Normal priors with means 1.5 and 0.125 and standard deviations 0.05 and 0.01 respectively.

5.3.3 Empirical Results

Table 5.2: Model Parameters Priors and Posteriors - Economic Parameters

Parameter Name	Parameter	Prior	Prior		Posterior	
			Mean	StDv	Mode	StDv
Policy Smoothing	ρ_r	Beta	0.80	0.10	0.9353	0.0016
Policy Weight on Inflation	ψ_π	$\ln\mathcal{N}$	1.50	0.05	1.4095	0.0189
Policy Weight on Output Growth	ψ_g	$\ln\mathcal{N}$	0.125	0.01	0.1211	0.0022
Habit in Consumption	ϱ	Beta	0.50	0.40	0.6575	0.0255
Calvo Wage Stickiness	ξ_W	Beta	0.75	0.25	0.9620	0.0029
Wage Indexation	φ_W	Beta	0.50	0.40	0.0020	0.0050
Calvo Price Stickiness	ξ	Beta	0.50	0.25	0.6554	0.0139
Price Indexation	φ	Beta	0.50	0.40	0.0011	0.0214
Entrepreneur Wealth Transfer	γ	Beta	0.91	0.10	0.5932	0.0070
Curvature of Utilization Cost	χ^a	$\ln\mathcal{N}$	1.00	0.50	0.8205	0.2137
Curvature of Adjustment Cost	χ^S	$\ln\mathcal{N}$	5.00	1.00	5.6935	0.7182
Bank Liquidation Cost	$\bar{\chi}^r$	$\ln\mathcal{N}$	0.31	0.10	0.1782	0.0386

The parameters describing the preferences, price and wage setting, monetary policy and adjustment costs are all quite standard. In particular, there is significant evidence the Federal Reserve adjusts their policy rate slowly over time. In particular, the data suggests policy changes are fully implemented after about 14 quarters. Meanwhile, the contemporaneous responses to inflation and the real economy are in line with Taylor's (1993) historical values.

Since the prior distributions were centered at these values, it may seem the data can't clearly identify these parameters. However, the standard errors of the posterior estimates are considerably smaller than the standard errors of the prior distributions - suggesting the parameters are well identified. The point estimate of the consumption habit parameter is a bit smaller than the value found in Smets & Wouters (2007), but still lies within their 95% confidence region. Moreover, it is within one standard deviation of the estimates by Fuhrer (2000).

There is considerable evidence in the data of nominal price and wage rigidities. Using the mapping from the Rotemberg (1982) price/wage adjustment cost parameterization and the Calvo (1983) parameterization, wages remain fixed for more than 6 years, while prices are adjusted slightly more often than twice a year. There is very little evidence that these pricing decisions are made with reference to previous wage/price inflation measures. This suggests the Federal Reserve's target rate of inflation plays an important role in goods and labor market contracts. These extreme values for the parameters governing wage dynamics are similar to the findings of Smets & Wouters (2007) when they specify less informative prior distributions over the wage setting dynamics.⁷

The curvature of the investment adjustment cost function, χ^S , and the share of assets retained by entrepreneurs, γ , are both significantly smaller than those found in Christiano et al. (2013). The smaller value of the investment adjustment cost function is likely driven by the differences between the foreclosure accelerator specified in this paper and the financial accelerator embedded in Christiano et al. (2013). In the BGG financial accelerator model, adverse economic shocks are propagated over time through adjustment costs. As borrowers face higher margins between the return on their investment and the cost of external funds, they scale down their investment projects. When changing investment is costly this reduction in investment lowers the price of capital, which decreases the net worth of existing owners of capital – the borrowers. This reduction in the borrower's net worth further increases the

⁷See for example footnote (9) in their paper.

margin between the return on their investment and the cost of external funds. This feedback mechanism is the celebrated financial accelerator.

The exact strength of the financial accelerator depends critically on the curvature of the adjustment cost function. If adjusting the rate of investment is costless, the financial accelerator collapses into the model of Carlstrom & Fuerst (1997). The large value of χ^S found by Christiano et al. (2013) suggests the financial accelerator plays an important role in explaining business cycle dynamics. Does the smaller value found in this study contradict this conclusion? No, because the financial accelerator is augmented in this model with the foreclosure accelerator.

Although the value of the capital stock plays some role in determining the net-worth of the entrepreneurs, in the foreclosure accelerator so too does the value of housing-secured assets. When an adverse shock occurs which elicits a drop in p_t^n , borrowers can not issue as much housing-secured debt and instead must rely on costly unsecured external financing. This leads to higher default rates on unsecured loans (due to the unfavorable loan terms) which ultimately leads the borrower to default on their secured debt, forcing the bank to liquidate the collateral backing the secured debt. This liquidation occurs on the housing market, which further drops home prices, leading to less secured debt relative to unsecured debt. This feedback mechanism differs from the financial accelerator in that it does not require falling capital values. Thus, the smaller value of χ^S found in this study does not contradict the importance of financial frictions in propagating business cycles. Instead, the smaller adjustment cost function curvature combined with the smaller value of γ simply suggests the foreclosure accelerator and the financial accelerator *both* play a prominent role in driving the business cycle.

Table (5.3) describes the prior and posterior distributions for the 13 exogenous shock processes which drive the model. Consistent with the findings of Ireland (2004a), the neutral technology shock is by far the most persistent disturbance. Mark-up shocks, housing and investment-specific technology disturbances and the bank liquidation shocks also show con-

Table 5.3: Model Parameters Priors and Posteriors - Shocks

Parameter Name	Parameter	Prior	Prior		Posterior	
			Mean	StDv	Mode	StDv
Policy	σ_r	IG	0.001	0.01	0.0018	0.0001
Policy News	$\rho_{r,4}$	Beta	0.80	0.10	0.9486	0.0088
	$\sigma_{r,4}$	IG	0.001	0.01	0.0006	0.0001
Consumption Preference	ρ_{η^c}	Beta	0.80	0.10	0.5250	0.0061
	σ_{η^c}	IG	0.001	0.01	0.0182	0.0010
Housing Demand	ρ_{η^h}	Beta	0.80	0.10	0.9457	0.0087
	σ_{η^h}	IG	0.001	0.01	0.0804	0.0135
Consumption Labor Supply	ρ_{η_l}	Beta	0.80	0.10	0.5353	0.0131
	σ_{η_l}	IG	0.001	0.01	0.2095	0.0301
Housing Labor Supply	$\rho_{\eta_l^h}$	Beta	0.80	0.10	0.7276	0.0194
	$\sigma_{\eta_l^h}$	IG	0.001	0.01	0.0390	0.0006
Mark-Up	ρ_{θ}	Beta	0.80	0.10	0.9715	0.0032
	σ_{θ}	IG	0.001	0.01	0.0390	0.0006
Bank Liquidtion	ρ_{χ^r}	Beta	0.80	0.10	0.9563	0.0025
	σ_{χ^r}	IG	0.001	0.01	1.9043	0.4707
Risk	ρ_{σ^ω}	Beta	0.80	0.10	0.7576	0.0193
	σ_{σ^ω}	IG	0.001	0.01	0.0176	0.0008
Neutral Technology	ρ_z	Beta	0.80	0.10	0.9872	0.0171
	σ_z	IG	0.001	0.01	0.0060	0.0002
Housing Technology	ρ_{z^h}	Beta	0.80	0.10	0.9633	0.0131
	σ_{z^h}	IG	0.001	0.01	0.0082	0.0002
Invesment-Specific Technology	ρ_{z^i}	Beta	0.80	0.10	0.9616	0.0090
	σ_{z^i}	IG	0.001	0.01	0.0043	0.0001
Investment-Efficiency Technology	ρ_{z^k}	Beta	0.80	0.10	0.9349	0.0063
	σ_{z^k}	IG	0.001	0.01	0.0023	0.0004

siderable persistence. Interestingly, the risk shock shows considerably less serial correlation than that found in Christiano et al. (2013). Moreover, the estimated standard deviations of the risk shock are considerably smaller than those of the bank liquidation and housing-demand disturbances. Of course, size of the standard deviations do not necessarily correlate to the relative importance of these shocks in explaining business cycle fluctuations. To analyze this question, I turn to impulse response functions, historical decompositions and forecast-error variance decompositions.

5.4 Dissecting the Great Recession

Interest in understanding the source of business cycles has significantly increased since the Great Recession. Much of this interest is due to the desire to explain why the recent recession was more severe and prolonged than all of the other postwar recessions. This has led to a horse-race of sorts. Dating back to Kydland & Prescott (1982), technology shocks have been an elegant and simple explanation of business cycles. However, the relative importance of total factor productivity in explaining output at business cycle frequencies has declined in the New-Keynesian framework (see for example Ireland (2004b); Smets & Wouters (2007)). This has led to a resurgence in developing concise explanations of business cycles.

In a series of papers, Alejandro Justiniano, Giorgio Primiceri and Andrea Tambalotti (JPT) have introduced various investment specific technology shocks into otherwise standard medium scale DSGE models. These shocks impact either the rate at which consumption goods can be transformed into investment goods (2010) or the rate at which investment goods can be transformed into installed, usable capital (2011). Very similar in spirit to Kydland & Prescott (1982), JPT find simple and elegant explanations of the business cycle in each paper.

In JPT (2010), they find that when a medium-scale DSGE model with investment-specific technology (IST) shocks is confronted with the data, these IST shocks explain the majority of the variation in output. However, the estimation strategy employed implies these shocks are unconstrained. However, variations in the IST process have direct implications for the price of investment goods relative to consumption goods. In other words, the real price of investment goods can be brought to bear on the estimation to discipline the exogenous IST process. This is precisely the exercise performed in JPT (2011). However, the authors add another shock which directly impacts the rate at which the investment good can be transformed into installed capital (these are called Marginal Efficiency of Investment shocks, or MEI). Thus, it is perhaps not surprising that once the real price of investment disciplines the

IST shocks, the MEI shocks take-over in importance in explaining business cycles. Moreover, JPT (2011) point out that these shocks are highly correlated with interest-rate spreads, suggesting they serve as a plausible explanation of the financially driven Great Recession . Given that the transformation of goods into installed capital occurs in frictionless markets in JPT's papers, the resulting shocks which drive the business cycle are likely proxying for a more fundamental friction which inhibits the rate at which entrepreneurs (those who demand capital/investment goods) can transform consumption goods into inputs for production. Interestingly the financial accelerator literature, and more specifically the work of Carlstrom & Fuerst (1997), Kiyotaki & Moore (1997) and Bernanke et al. (1999), provides an endogenous explanation for why the process of transforming savings into investment may be impaired. Christiano et al. (2013) subsequently embed the Bernanke et al. (1999) financial friction into a medium-scale DSGE model. The resulting model leads to a novel explanation of business cycles - risk shocks.

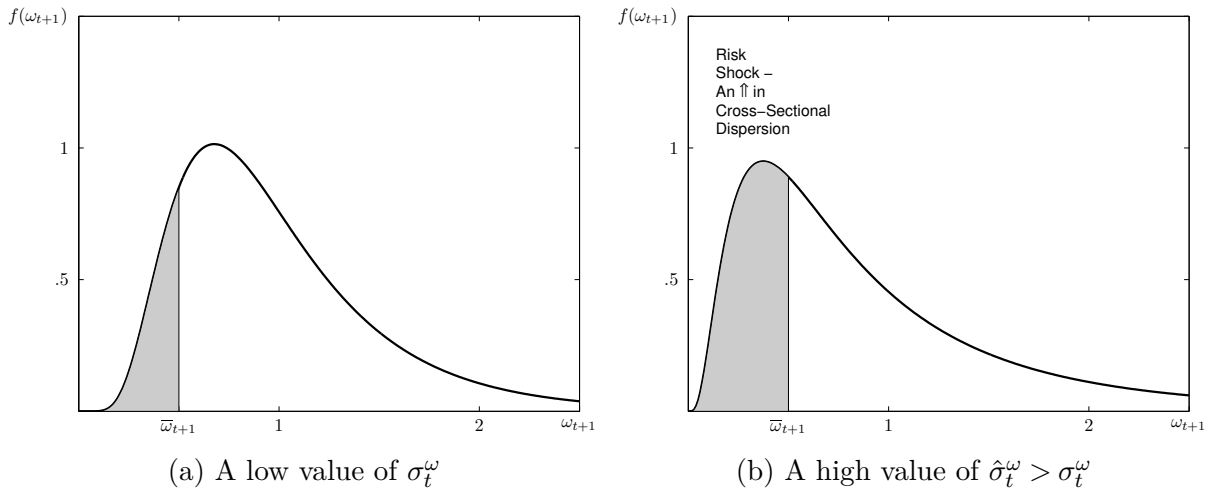


Figure 5.2: Illustrating risk shocks. The shaded gray area denotes the probability of default, or mathematically $\int_0^{\bar{\omega}_{t+1}} f(\omega_{t+1}) d\omega_{t+1} = F_t(\bar{\omega}_{t+1})$.

Risk shocks are represented in this model, as in CMR, by the exogenous process for σ_t^ω . For a concrete example, suppose there is an increase in σ_t^ω to $\hat{\sigma}_t^\omega > \sigma_t^\omega$. This increase the cross-sectional dispersion of the idiosyncratic productivity values across the j entrepreneurs, ω_{t+1}^j . As shown in Figure (5.2), this can be represented graphically by a “flattening-out” of the

distribution of potential ω_{t+1} draws from which each entrepreneur j receives their productivity value. What this means for the entrepreneur is that for a given threshold productivity value $\bar{\omega}_{t+1}$ there is now a greater probability the entrepreneur will receive a productivity draw $\omega_{t+1} < \bar{\omega}_{t+1}$. More simply, the default probability increases since $\hat{F}_t(\bar{\omega}_{t+1}) > F_t(\bar{\omega}_{t+1})$. Since default is costly due to the costly-state verification the interest rate the bank charges on unsecured external financing increases to cover the expected higher cost. This induces the entrepreneur to decrease their borrowing of external funds, leading credit to contract.

The combination of increasing finance premiums, falling credit and ultimately falling output make risk shocks a prime candidate to explain business cycles. In fact, the inclusion of finance premiums and credit in the estimation is precisely why CMR find that risk shocks explain the bulk of the movement in output at business cycle frequencies. They show this by estimating the model without these financial market variables and find that JPT's MEI shocks explain the majority of business cycle movements. The model presented in this paper, and the foreclosure accelerator mechanism in particular, provides an alternative framework (which nests these previous models as special cases) to examine the source of business cycle movements. In contrast to these previous estimation results, I find that home prices play a significant role in explaining the behavior of finance premiums, credit and output. However, the debt-contract allows for neutral technology shocks explain the majority of the movements in GDP without implying empirically inconsistent relationships between financial variables. Finally, in contrast to CMR (2013), risk shocks play virtually no role in driving real variables and only explain the high frequency movements in finance premiums.

5.4.1 What are plausible sources of business cycles?

In what follows, I present impulse response functions from the model estimated in this paper, the model estimated in CMR (2013) and variations of these two models to gain some intuition for the different econometric conclusions reached in this paper.

Risk shocks

Figure (5.3) displays the response of output, home prices, finance premiums and various forms of credit to a contractionary risk shock as illustrated in Figure (5.2). As noted in CMR (2013), an exogenous increase in risk causes output and credit to fall, and finance premiums rise. These qualitative features are robust across both models. However, examining the behavior of home prices, qualitative differences emerge between these various models. In particular, home prices increase following this increase in risk.

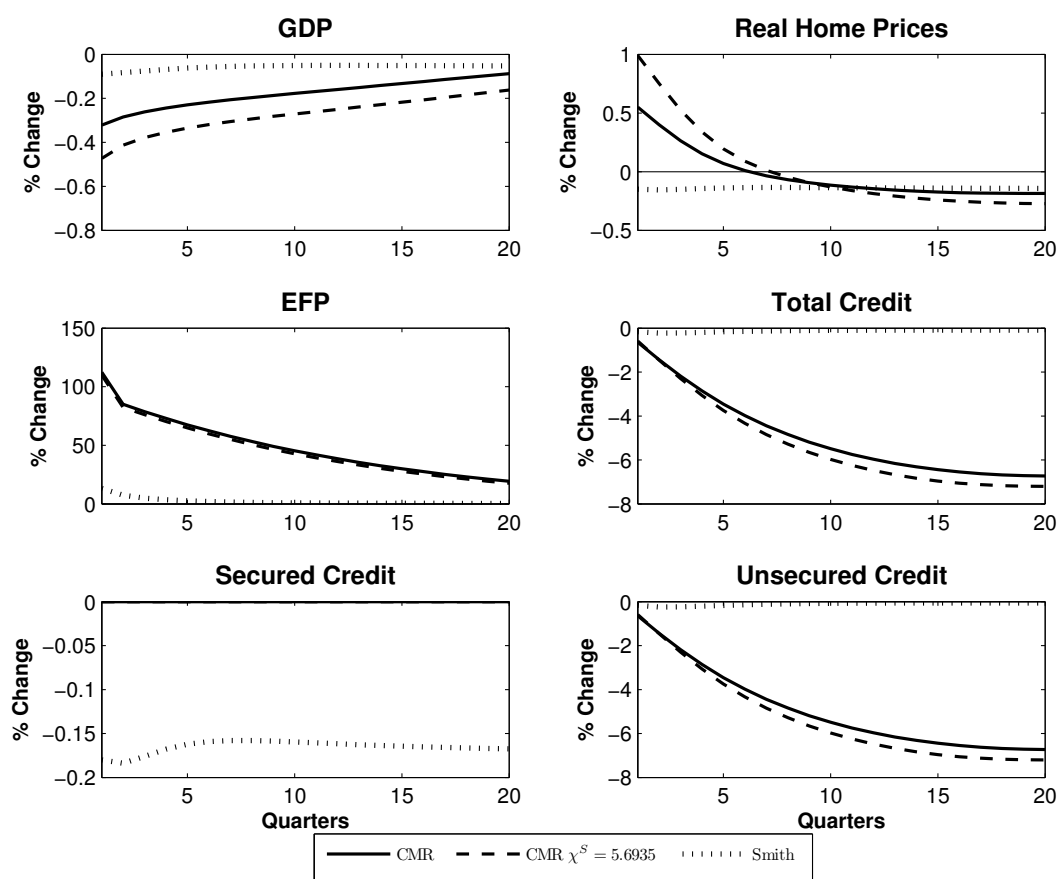


Figure 5.3: Impulse Responses to a Contractionary Risk Shock

To highlight the implausible relationship between home prices and finance premiums implied by the CMR (2013) model, consider for a moments the correlations between home prices and finance premiums when the models are only driven by risk shocks. The CMR model predicts the correlation between home prices and the external finance premium is 10% if $\chi^S = 10.78$

and rises to 28% when $\chi^S = 5.69$. The value in the data over the sample period is 21% and falls to -61% when computed since 2000. The value in the model estimated in this paper is -39%. Thus, once home prices are included in the estimation, it seems that risk shocks can no longer be the primary driver of finance premiums in the CMR model.

Neutral technology shocks

Figure (5.4) displays the response of output, home prices, finance premiums and various forms of credit to a contractionary TFP technology shock. As expected, a drop in TFP causes GDP to fall, along with home prices and credit in all models. However, the behavior of finance premiums is qualitatively different across models, especially across CMR's model with alternative values of the curvature of the investment adjustment cost function.

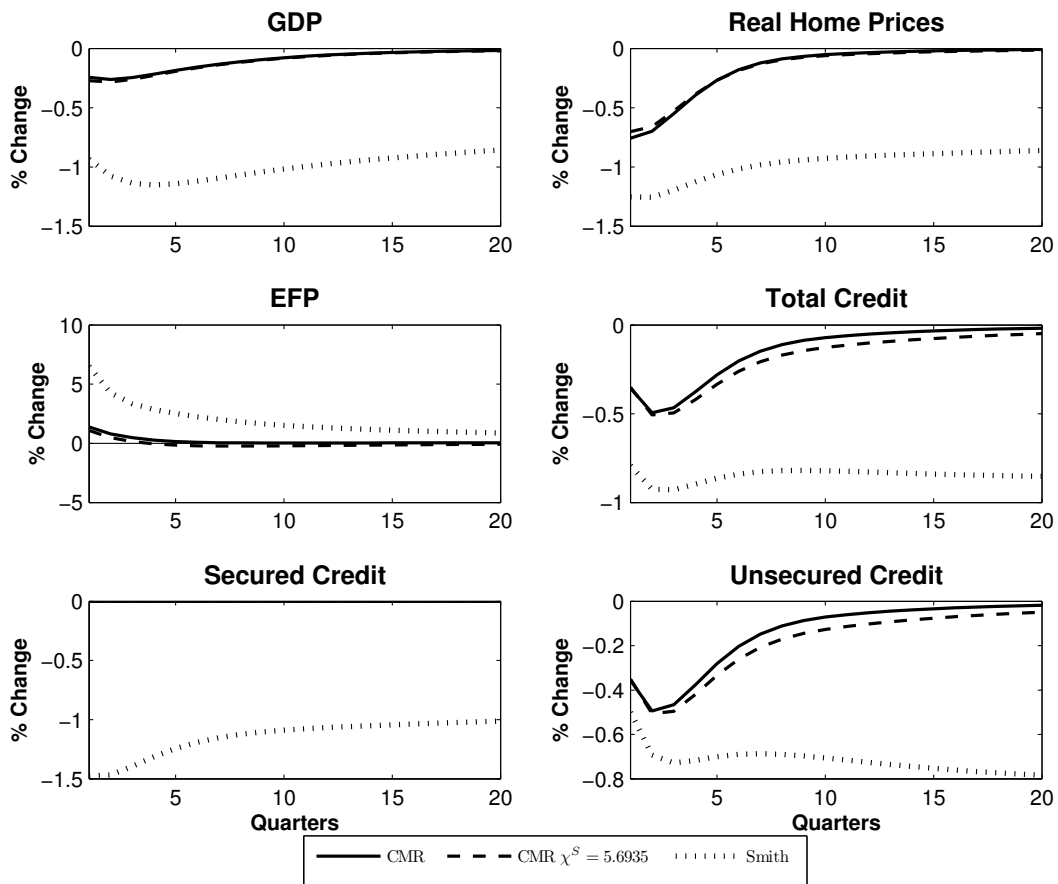


Figure 5.4: Impulse Responses to a Contractionary Technology Shock

On impact, finance premiums rise across all model specifications. However, in the CMR

model, the lower level of aggregate technology induces borrowers to scale down their investment projects. This decrease in the leverage position of borrowers, all other things equal, leads to lower finance premiums. When investment adjustment costs are large, scaling down the investment project leads to a drop in the price of capital, which ultimately decreases the borrowers net worth, leading to higher finance premiums. In the foreclosure accelerator the drop in technology lowers income which decreases the demand for housing, leading to a drop in home prices. This decreases the amount of secured credit which borrowers can issue. This alone is sufficient to drive up finance premiums regardless of the curvature of the investment adjustment cost function in the model presented in this paper.

Housing demand shocks

Figure (5.5) displays the response of output, home prices, finance premiums and various forms of credit to a contractionary housing demand shock. Although the exogenous process driving housing demand disturbance is identical in all models, there is a considerable amount of endogenous amplification and persistence in the model presented in this paper relative to the CMR models. In particular, the drop in home prices leads to an immediate rise in the finance premium. Through the foreclosure accelerator, this leads to more defaults on secured loans, for which the collateral is liquidated as housing. This further drowns down home prices in a feedback loop which leads to a large and considerable fall in output.

The CMR model predicts that a drop in home prices actually decreases finance premiums. This reaction is qualitatively robust to moderate and large investment adjustment cost elasticities. Once again, this response implies a counter-factual correlations between home prices, GDP and finance premiums. In summary, housing demand shocks don't provide a plausible explanation of business cycles through the lens of the CMR model since they fail to capture the observed relationship between finance premiums and home prices. Conversely, in the model presented in this paper, housing demand shocks have features consistent with the Great Recession whereby a drop in home prices leads to a large prolonged recession.

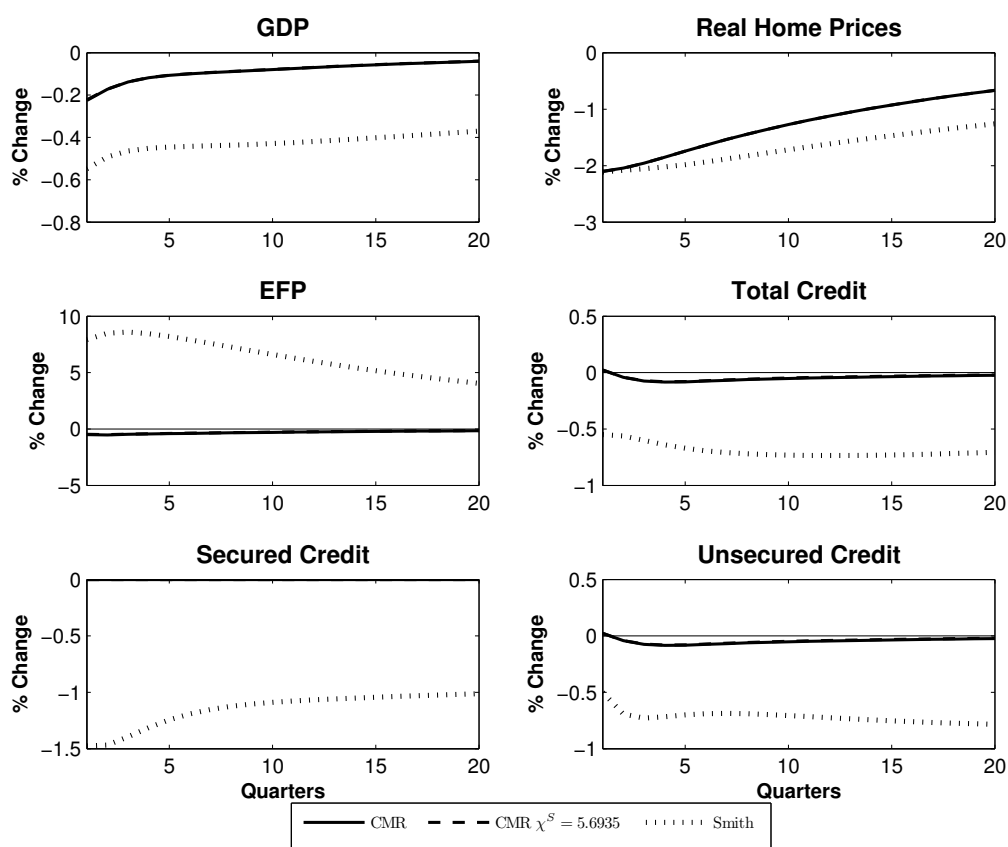


Figure 5.5: IRFs to a Contractionary Housing Demand Shock

5.4.2 What drives credit and output?

The 2008 financial crisis and resulting recession are intimately linked. The typical narrative rests on interrelated credit and housing booms which fueled one another. As borrowers received more favorable loan terms, housing prices continued to rise. While, at the same time, increasing home prices allowed intermediaries to issue debt secured by the very assets they were purchasing/providing - mortgage backed securities. This relationship led to increasing GDP and lower than average home-default rates which put more pressure on home prices to rise. Beginning to disentangle these rich feedback effects call for first understanding which shocks can produce the relationships observed between output, home prices, finance premiums and credit over the last decade.

Figure (5.6) reports historical decompositions from the estimated model. Interestingly, risk

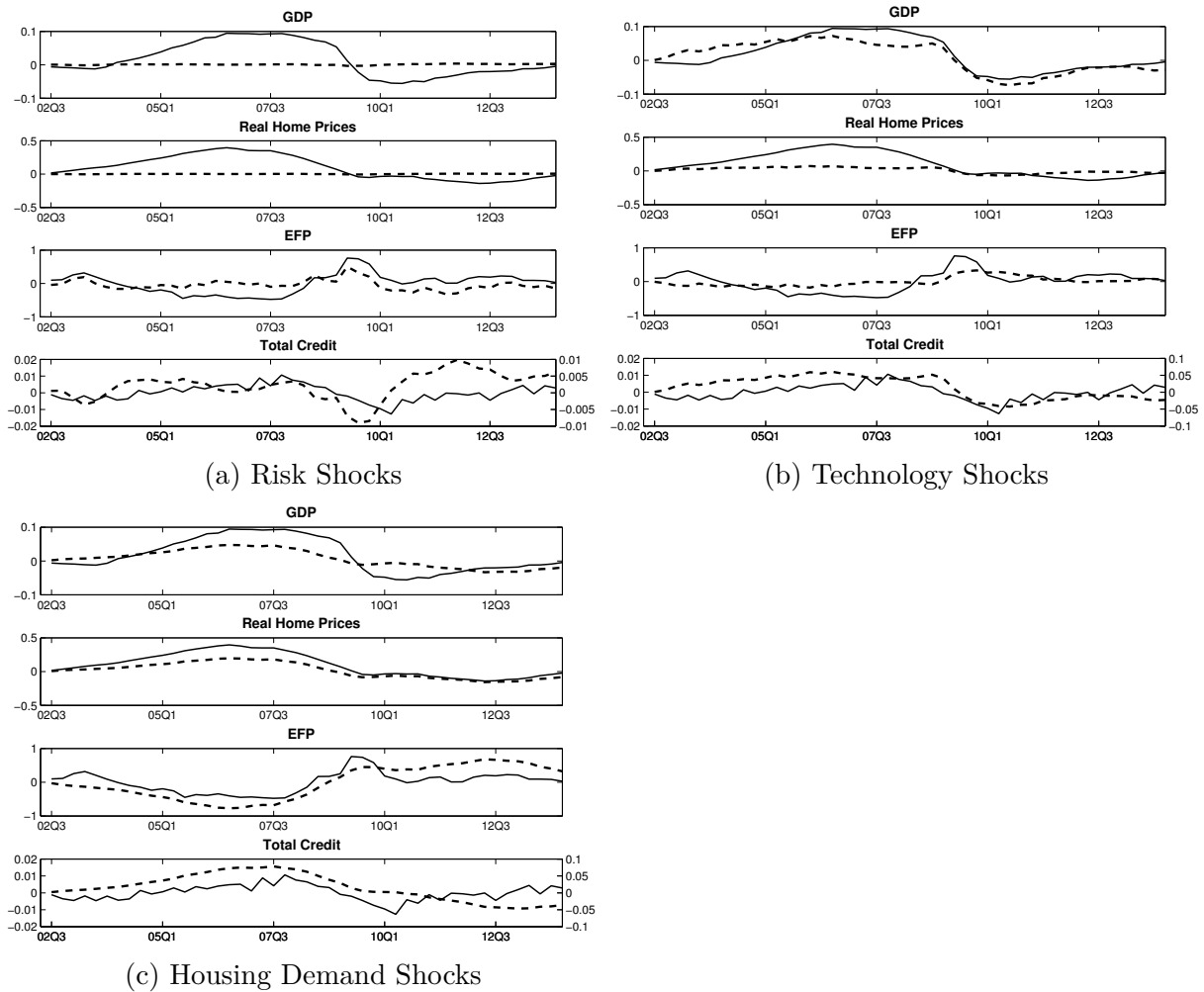


Figure 5.6: Historical Decompositions: The solid line denotes the actual data which is reproduced when all shocks are activated. The dashed line denotes the counterfactual time series of each variable when driven by each shock denoted by the subcaption.

shocks fail to generate any significant movement of real output and home prices. Moreover, the generated time series are *negatively* correlated with the actual time series: -10% for GDP and -16% for real home price. However, the counterfactual risk shocks time-series does reproduce some of the high frequency movements in the external finance premiums. that being said, it clearly misses the lower frequency movements in the finance premiums. For example, the historically low finance premiums from 2005-2007 were clearly driven by factors other than risk shocks.

One leading candidate for the exceptionally low and ultimate rise in finance premiums are

housing demand shocks. These shocks reproduce the majority of the movements in real home prices. More concretely, the resulting series are 99% correlated. Given the role played by housing secured-debt in this foreclosure accelerator model, it is therefore not surprising that by capturing the rise and fall in home prices housing demand shocks are able to capture the opposite behavior of finance premiums. Therefore, the low frequency movements in finance premiums are driven by home prices and ultimately the housing demand shocks which drive the price of housing. This is further confirmed by the forecast-error variance decompositions in Table (5.4).

Table 5.4: Forecast Error Variance Decompostions

Horizon (Quarters)	GDP			EFP			Home Prices			Credit		
	$\varepsilon^{\sigma^\omega}$	ε^z	ε^{η^h}	$\varepsilon^{\sigma^\omega}$	ε^z	ε^{η^h}	$\varepsilon^{\sigma^\omega}$	ε^z	ε^{η^h}	$\varepsilon^{\sigma^\omega}$	ε^z	ε^{η^h}
1	0	50	17	45	11	16	0	19	56	1	30	14
2	0	55	12	40	11	25	0	21	57	2	31	13
3	0	58	10	35	11	34	0	21	61	2	32	13
4	0	60	11	30	11	41	0	20	65	2	33	14
8	0	63	12	19	10	60	0	20	73	1	38	24
20	0	66	13	9	7	78	1	30	64	1	41	28

The procyclical relationship between residential investment and home prices provides a clear understanding as to why housing demand shocks generate a GDP time series which is 86% percent correlated with actual GDP. However, these shocks fail to capture the quantitative behavior of real output. Instead, Figure (5.6) shows that technology shocks play a critical role in understanding the sharp drop in output in 2009.

The impulse response functions in Figure (5.4) show that unlike the CMR (2013) model, the foreclosure accelerator model can generate a drop in output via a productivity shock without decreasing finance premiums. This difference, along with the counterfactual rise in home prices which accompanies risk shocks, is the primary reason why technology shocks play an important role in driving output in this model and risk shocks play that role in the CMR (2013) model. The time series produced by technology shocks also boasts the highest

correlation with credit at 60%. The housing demand series is not far behind at 48% while the risk shocks time series of credit is only 23% correlated with the actual time series of credit.

5.4.3 Was the Great Recession the efficient response to technology shocks?

The above findings suggest that technology shocks played a significant role in shaping the path of output and credit in the wake of the great recession. In the spirit of Kydland & Prescott (1982), this calls into question the extent to which countercyclical policy was necessary in response to the recent recession. To assess the extent to which the fall in output was ‘inefficient,’ and hence should be offset by policy makers, I compute the financial frictions output gap. More specifically, I feed the filtered technology shocks through the model presented in this paper and an identical model without financial frictions. The frictionless model is identical, except the entrepreneur’s first order condition is replaced by a typical Euler equation for capital. I then compute the difference between GDP in the full model and the frictionless model - the financial frictions output gap.

Figure (5.7) illustrates the results of this exercise which are also repeated by feeding all non-financial shocks through each model so that the number of shocks is always identical in both models. Two immediate conclusions emerge from this exercise. First, the majority of the fall in the output gap at the onset of the recession was due to technology shocks. Second, the adverse technology shocks are primarily transferred through the financial sector, which results in a large negative output gap, even when technology shocks are the only source of the business cycle. Thus, the endogenous financial frictions, amplify and propagated the downturn, therefore justifying considerable intervention into financial markets to limit the inefficiencies associated with the recession.

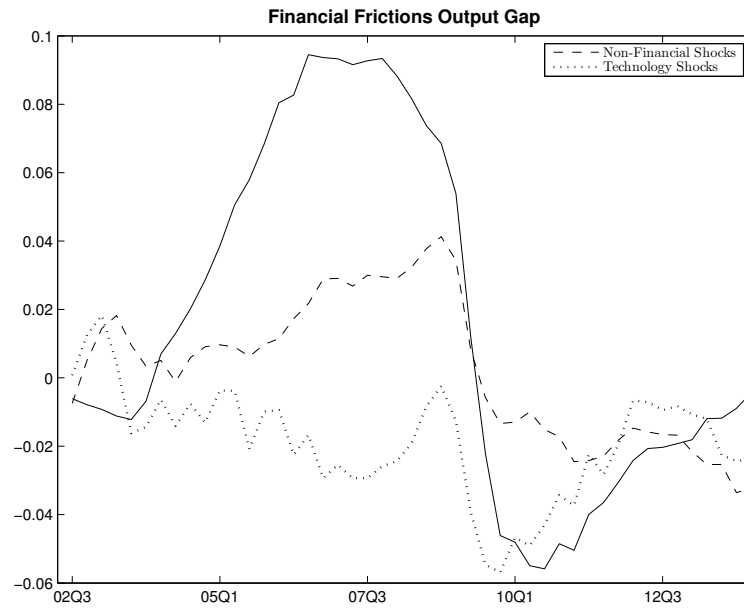


Figure 5.7: Historical Decompositions: The solid line denotes the actual data. The dashed line denotes the counterfactual difference between GDP in a model with and without financial frictions. The dashed line denotes the same difference when each economy is only driven by technology shocks.

5.4.4 Was loose monetary policy to blame for the housing boom?

In a search for better understanding what caused the 2008 financial crisis, some have blamed the Federal Reserve for keeping interest rates too low for too long in the years preceding the crisis. For example, John Taylor provided testimony to congress in March of 2012 to this extent. During his testimony, Taylor (2012a) cited loose policy as a primary cause of most recent business cycle:

In 2003-2005 the Federal Reserve deviated from the policies it followed in most of the 1980s and 1990s by holding interest rates too low for too long and thereby setting off excesses in housing and other markets which helped bring on the most recent boom and bust.

Examining historical decompositions provides a unique opportunity to perform a counterfactual experiment to examine the empirical support for this critique of Federal Reserve policy. Figure (5.8) illustrates the results of an experiment where I first produce the time series

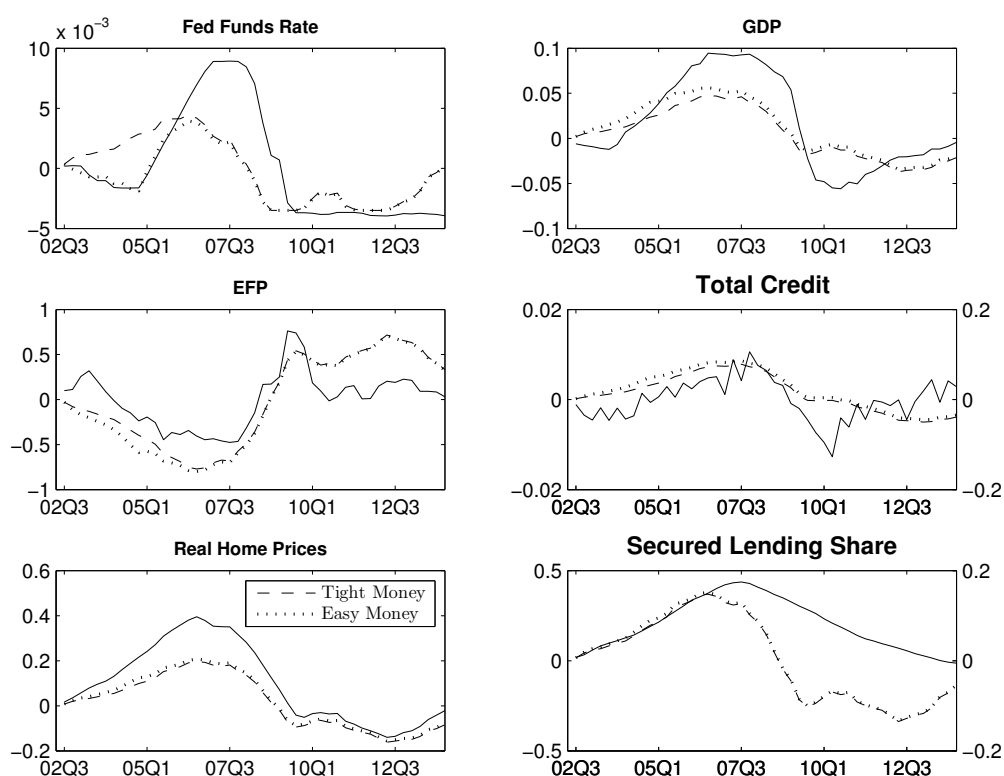


Figure 5.8: Historical Decompositions: The solid line denotes the actual data which is reproduced when all shocks are activated. The dashed line denotes the counterfactual time series of each variable when driven by housing demand disturbances. The dotted line denotes the counterfactual time series of each variable with housing demand and expansionary monetary policy shocks

of several variables assuming the economy is only driven by housing demand shocks. these shocks are the primary driver of home prices which, according to Taylor's (2012b) critique, the Fed failed to appropriately contain. Interestingly, the path of interest rates during the 2003-2005 period Taylor (2012a) cites features policy rates on the estimated Taylor rule above the observed Fed Funds rate.

To precisely understand the extent to which these deviations from the Taylor rule led to a larger housing bubble, I then consider another counterfactual experiment in which the Taylor rule is perturbed by expansionary monetary policy shocks from 2003-2005 which reproduce the observed path of the Fed Funds rate. The resulting path of 'easy money' policy rates yields marginally higher levels of output and credit than otherwise would have occurred under the Taylor rule. However, the overall impact on home prices, finance premiums and

secured lending is negligible. In summary, tighter money policies from 2003-2005 could not have prevented the forthcoming financial crisis and recession. Instead, the ensuing credit crunch was driven by a collapse in home prices which greatly reduced the share of secured lending causing finance premiums on unsecured debt to soar.

5.4.5 How contractionary is the zero lower bound?

In addition to deviations from Taylor's (1993) rule in the years leading up to the Great Recession, there were large deviations from Taylor's rule in the following years. However, instead of policy being too loose, these deviations were contractionary in nature and driven by the lower bound of zero on nominal interest rates, not discretion. To assess the extent to which the zero lower bound constrained monetary policy makers in the wake of the Great Recession, I again turn to historical decompositions. In this exercise I consider aggregate demand disturbances in general, which includes shocks to preferences over housing and preferences over consumption goods. This model verifies the findings in the nonlinear analysis in Fernández-Villaverde et al. (2012) that preference shocks are likely to push the economy to the zero lower bound.

Figure (5.9) displays model simulations when the only driving shocks are the aggregate demand disturbances. The first observation is how remarkably similar the counterfactual path of policy rates looks to the effective federal funds rate, at least until 2008. At the point where the implied level of the gross nominal interest rate is one, the zero lower bound binds which prevents policy makers from further accommodation. What is especially revealing about the time series which satisfy the zero lower bound is the additional contraction.

GDP reacts to the the implicit monetary tightening inherent in the zero lower bound by contracting sharply. The inclusion of the zero lower bound also allows for preference shocks to account for the spike in the external finance premium. This ultimately leads to a larger contraction in credit, which brings the counterfactual time series closer in line with the observed behavior of credit. In all, the zero lower bound appears to have played a significant role in explaining the sharp contraction in output, increase in finance premiums and drop

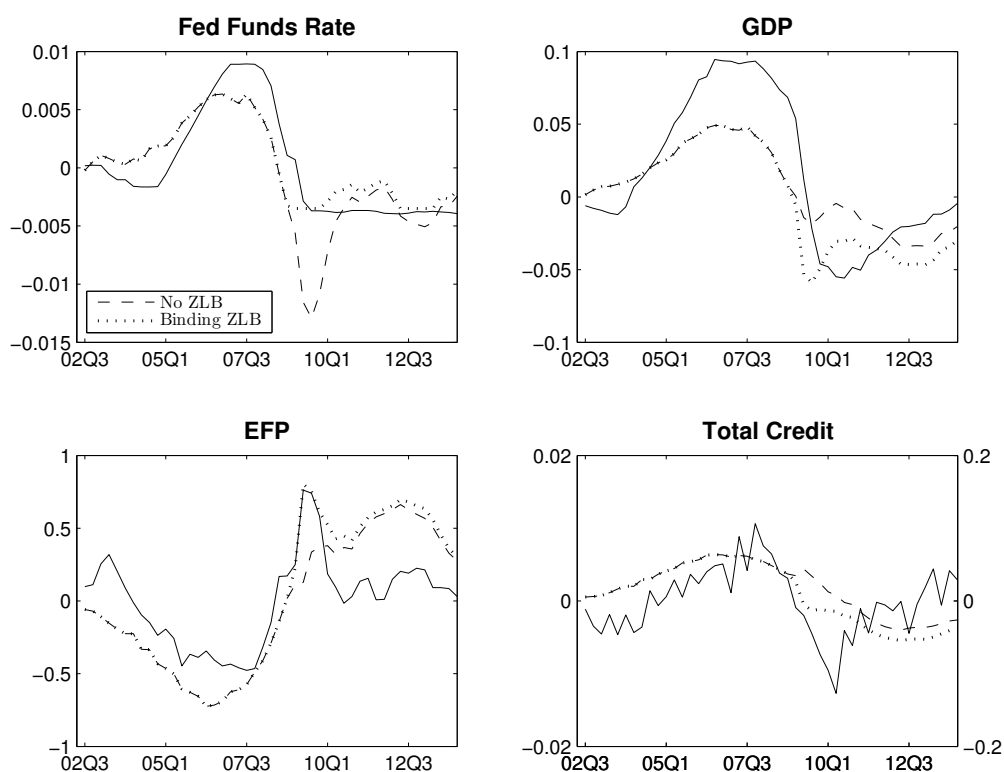


Figure 5.9: Historical Decompositions: The solid line denotes the actual data which is reproduced when all shocks are activated. The dashed line denotes the counterfactual time series of each variable when driven by housing/consumption demand disturbances. The dotted line is the same time series when the ZLB binds.

in credit. Risk shocks are noticeably absent from this explanation as they don't drive the economy to the zero lower bound. Figure (5.6) reveals that risk shocks were actually expansionary from 2009 onwards. As counter-intuitive as this seems, it is likely the result of the unconventional policy actions taken by the Federal Reserve and the TARP equity injections which decreased systemic risk by stabilizing large financial firms.

5.5 Conclusion

Despite the fact that most narratives of the 2008 financial crisis and ensuing Great Recession feature housing markets and secured debt markets, current equilibrium models don't typically feature both (and often feature neither) markets. The implication is a large gap between the narrative description of the worst postwar recession and the sources of the downturn attributed by structural equilibrium models. This paper bridges that gap. By including

integrated housing and financial markets in an otherwise standard medium scale DSGE model, I provide an interpretation of the last decade that is more consistent with the casual observation that housing and debt markets are highly integrated.

Unlike the recent findings of Christiano et al. (2013), I find that risk shocks account for essentially none of the variation in real GDP. These shocks essentially capture high frequency movements in finance premiums but otherwise have little predictive power regarding output, home prices, finance premiums and credit. Instead, housing demand and total factor productivity disturbances explain the bulk of the variation of housing, financial and goods markets. This is due in large part to the novel debt contract which augments the financial accelerator with a foreclosure accelerator mechanism. This framework is better suited to capture the observed relationships between home prices, finance premiums and output. Although technology shocks in frictionless models imply there is no role for countercyclical policy, I show the endogenous behavior of the modeled financial frictions played a significant role in transmitting the technology shocks to the broader economy. The implication being that countercyclical policy, especially intervention into financial markets, is well justified.

Along these same policy lines, I find the claim that the Federal Reserve kept interest rate too low for too long and fueled the housing bubble has little empirical footing. Instead, the housing expansion was largely driven by the interaction of exogenous housing demand disturbances and the foreclosure accelerator mechanism. Counterfactual simulations show that tighter monetary policy would have had a marginal effect on most variables. Conversely, the deviations from the interest rate rule following the Great Recession played a significant role in the large contraction of output and the spike in finance premiums in 2008-2009.

References

- Bernanke, B. S., Gertler, M., & Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In J. B. Taylor & M. Woodford (Eds.), *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics* chapter 21, (pp. 1341–1393). Elsevier.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3), 383–398.
- Carlstrom, C. T. & Fuerst, T. S. (1997). Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review*, 87(5), 893–910.
- Christiano, L., Motto, R., & Rostagno, M. (2013). *Risk Shocks*. NBER Working Papers 18682, National Bureau of Economic Research, Inc.
- Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1), 1–45.
- Davis, M. A. & Heathcote, J. (2005). Housing and the business cycle. *International Economic Review*, 46(3), 751–784.
- Erceg, C. J., Henderson, D. W., & Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46(2), 281–313.
- Fernández-Villaverde, J., Gordon, G., Guerrón-Quintana, P. A., & Rubio-Ramírez, J. (2012).

- Nonlinear Adventures at the Zero Lower Bound*. NBER Working Papers 18058, National Bureau of Economic Research, Inc.
- Fuhrer, J. C. (2000). Habit Formation in Consumption and Its Implications for Monetary-Policy Models. *American Economic Review*, 90(3), 367–390.
- Iacoviello, M. (2005). House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review*, 95(3), 739–764.
- Iacoviello, M. & Neri, S. (2010). Housing market spillovers: Evidence from an estimated dsge model. *American Economic Journal: Macroeconomics*, 2(2), 125–64.
- Ireland, P. N. (2004a). Money’s role in the monetary business cycle. *Journal of Money, Credit and Banking*, 36(6), 969–83.
- Ireland, P. N. (2004b). Technology shocks in the new keynesian model. *The Review of Economics and Statistics*, 86(4), 923–936.
- Justiniano, A., Primiceri, G., & Tambalotti, A. (2010). Investment shocks and business cycles. *Journal of Monetary Economics*, 57(2), 132–145.
- Justiniano, A., Primiceri, G., & Tambalotti, A. (2011). Investment Shocks and the Relative Price of Investment. *Review of Economic Dynamics*, 14(1), 101–121.
- Keen, B. & Wang, Y. (2007). What is a realistic value for price adjustment costs in new keynesian models? *Applied Economics Letters*, 14(11), 789–793.
- Kiyotaki, N. & Moore, J. (1997). Credit cycles. *Journal of Political Economy*, 105(2), 211–48.
- Kydland, F. E. & Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6), 1345–70.
- Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90(6), 1187–1211.

- Smets, F. & Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3), 586–606.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1), 195–214.
- Taylor, J. B. (2012a). *Monetary Policy Going Forward: Why a Sound Dollar Boosts Growth and Employment*. Testimony before the joint economic committee.
- Taylor, J. B. (2012b). Monetary Policy Rules Work and Discretion Doesn't: A Tale of Two Eras. *Journal of Money, Credit and Banking*, 44(6), 1017–1032.
- Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2), 265–293.

5.A Appendix: The Stationary, Non-Linear Model

This section presents the system of N non-linear equations in N variables which defines the model economy. All nominal variables inherit a unit root from the conduct of monetary policy which induces a non-stationary price level through inflation targeting.

5.A.1 Household

$$\lambda_t = \mathbb{E}_t \left\{ \frac{\eta_t^c}{c_t - \varrho c_{t-1}} - \beta \varrho \frac{\eta_{t+1}^c}{c_{t+1} - \varrho c_t} \right\} \quad (5.A.1)$$

$$\lambda_t p_t^h = \frac{\eta_t^h}{h_t} + \beta(1 - \delta^h) \mathbb{E}_t \left\{ \lambda_{t+1} p_{t+1}^h \right\} \quad (5.A.2)$$

$$\lambda_t = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} r_t^d}{\pi_{t+1}} \right\} \quad (5.A.3)$$

$$\begin{aligned} \phi_W \left[\frac{\pi_t^W}{\pi_{t-1}^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right] \frac{\pi_t^W}{\pi_{t-1}^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} &= (1 - \theta_l) w_t + \frac{\eta_l l_t^{\nu_l}}{\lambda_t} \theta_l \\ &+ \beta \phi_W \mathbb{E}_t \left\{ \frac{\lambda_{t+1} l_{t+1}}{\lambda_t l_t} \left[\frac{\pi_{t+1}^W}{\pi_t^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right] \frac{\pi_{t+1}^W}{\pi_t^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} \right\} \end{aligned} \quad (5.A.4)$$

$$\begin{aligned} \phi_W \left[\frac{\pi_t^{W,h}}{\pi_{t-1}^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right] \frac{\pi_t^{W,h}}{\pi_{t-1}^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} &= (1 - \theta_l) w_t^h + \frac{\eta_l^h l_t^h(i)^{\nu_l}}{\lambda_t} \theta_l \\ &+ \beta \phi_W^h \mathbb{E}_t \left\{ \frac{\lambda_{t+1} l_{t+1}}{\lambda_t l_t} \left[\frac{\pi_{t+1}^{W,h}}{\pi_t^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right] \frac{\pi_{t+1}^{W,h}}{\pi_t^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} \right\} \end{aligned} \quad (5.A.5)$$

$$\pi_t^W = \frac{w_t}{w_{t-1}} \pi_t \quad (5.A.6)$$

$$\pi_t^{W,h} = \frac{w_t^h}{w_{t-1}^h} \pi_t \quad (5.A.7)$$

$$\mathcal{M}_{t-1|t} = \beta \frac{\lambda_t}{\lambda_{t-1}} \quad (5.A.8)$$

5.A.2 The Financial Sector

$$\mathbb{E}_t \left\{ \mathcal{M}_{t|t+1} \frac{r_t^d}{\pi_{t+1}} p_t^n \right\} = \mathbb{E}_t \left\{ \mathcal{M}_{t|t+1} \left(p_{t+1}^H z^r - \chi^r \right) \right\} \quad (5.A.9)$$

$$d_t = b_t + p_t^n \bar{n} \quad (5.A.10)$$

5.A.3 The Entrepreneur

$$r_t^d \left(q_t k_t^j - p_t^n \bar{n} - a_t \right) - \left(\Gamma_t(\bar{\omega}_{t+1}^j) - \mu G_t(\bar{\omega}_{t+1}^j) \right) r_{t+1}^k k_t^j q_t = 0 \quad (5.A.11)$$

$$\mathbb{E}_t \left\{ (1 - \Gamma_t(\bar{\omega}_{t+1})) r_{t+1}^k - \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})} \left[r_t^d - (\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})) r_{t+1}^k \right] \right\} = 0 \quad (5.A.12)$$

$$q_t k_t - p_t^n \bar{n} - p_t b_t - a_t = 0 \quad (5.A.13)$$

$$a_t - \gamma \left[(1 - \Gamma_{t-1}(\bar{\omega}_t)) \frac{r_t^k}{\pi_t} k_{t-1} q_{t-1} - (1 - F_{t-1}(\bar{\omega}_t)) p_{t-1}^n \bar{n} \frac{r_{t-1}^d}{\pi_t} \right] - w_t^E = 0 \quad (5.A.14)$$

5.A.4 Relevant Distributions for the Entrepreneurs

$$x_{t+1} = \frac{\ln(\bar{\omega}_{t+1}) + .5(\sigma_t^\omega)^2}{\sigma_t^\omega} \quad (5.A.15)$$

$$G_t(\bar{\omega}_{t+1}) = \Phi^{\mathcal{N}}(x_{t+1} - \sigma_t^\omega) \quad (5.A.16)$$

$$\Gamma_t(\bar{\omega}_{t+1}) = \Phi^{\mathcal{N}}(x_{t+1} - \sigma_t^\omega) + \bar{\omega}_{t+1}(1 - \Phi^{\mathcal{N}}(x_{t+1})) \quad (5.A.17)$$

$$G'_t(\bar{\omega}_{t+1}) = \left(\frac{1}{\sigma_t^\omega \sqrt{2\pi}} \right) e^{-x_{t+1}^2/2} \quad (5.A.18)$$

$$\Gamma'_t(\bar{\omega}_{t+1}) = 1 - \Phi^{\mathcal{N}}(x_{t+1}) \quad (5.A.19)$$

$$F_t(\bar{\omega}_{t+1}) = \Phi^{\mathcal{N}}(x_{t+1}) \quad (5.A.20)$$

$$v_{t-1}(\bar{\omega}_t) = G_{t-1}(\bar{\omega}_t) \frac{r_t^k}{\pi_t} q_{t-1} k_{t-1} \quad (5.A.21)$$

5.A.5 The Wholesale Goods Producers

$$y_t = z_t (k_{t-1})^\alpha (l'_t)^{1-\alpha} \quad (5.A.22)$$

$$l'_t = l_t^{1-\alpha^E} \quad (5.A.23)$$

$$w_t = p_t^W (1 - \alpha)(1 - \alpha^E) \frac{y_t}{l_t} \quad (5.A.24)$$

$$w_t^E = p_t^W (1 - \alpha) \alpha^E y_t \quad (5.A.25)$$

$$mp_t^k = \alpha \frac{y_t}{k_{t-1}} \quad (5.A.26)$$

$$r_t^k = \frac{u_t mp_t^k p_t^W - a(u_t) + (1 - \delta^k) q_t}{q_{t-1}} \pi_t \quad (5.A.27)$$

$$a(u_t) = \frac{\overline{mp}^k}{\chi^a} \left[e^{\chi^a (u_t - 1)} - 1 \right] \quad (5.A.28)$$

$$mp_t^k = \overline{mp}^k e^{\chi^a (u_t - 1)} \quad (5.A.29)$$

5.A.6 Retail Goods Producers

$$\begin{aligned} \phi \left[\frac{\pi_t}{\pi_{t-1}^\varphi \bar{\pi}_t^{1-\varphi}} - 1 \right] \frac{\pi_t}{\pi_{t-1}^\varphi \bar{\pi}_t^{1-\varphi}} &= \theta p_t^W - (\theta - 1) \\ &+ \phi \mathbb{E}_t \left\{ \frac{y_{t+1} \mathcal{M}_{t,t+1}}{y_t \mathcal{M}_{t,t}} \left[\frac{\pi_{t+1}}{\pi_t^\varphi \bar{\pi}_t^{1-\varphi}} - 1 \right] \frac{\pi_{t+1}}{\pi_t^\varphi \bar{\pi}_t^{1-\varphi}} \right\} \end{aligned} \quad (5.A.30)$$

5.A.7 New Housing Production

$$h_t^{new} = z_t^h (l_t^h) (1 - \alpha^h) \quad (5.A.31)$$

$$w_t^h = p_t^h (1 - \alpha^h) \frac{h_t^{new}}{l_t^h} \quad (5.A.32)$$

5.A.8 Investment Goods Production

$$p_t^i = \frac{1}{z_t^i} \quad (5.A.33)$$

5.A.9 Capital Goods Production

$$k_t = \left(i_t - S \left(\frac{i_t}{z_t^k k_{t-1}} \right) z_t^k k_{t-1} \right) + (1 - \delta^k) k_{t-1} \quad (5.A.34)$$

$$S \left(\frac{i_t}{z_t^k k_{t-1}} \right) = \frac{\chi^S}{2} \left[\frac{i_t}{z_t^k k_{t-1}} - \delta^k \right]^2 \quad (5.A.35)$$

$$q_t = \frac{p_t^i}{1 - S' \left(\frac{i_t}{z_t^k k_t} \right)} \quad (5.A.36)$$

5.A.10 Monetary Policy

$$\frac{r_t^d}{\bar{r}^d} = \left(\frac{r_{t-1}^d}{\bar{r}^d} \right)^{\rho_r} \left(\frac{\pi_t}{\bar{\pi}_t} \right)^{\psi_\pi} \left(\frac{y_t}{y_{t-1}} \right)^{\psi_y} e^{\varepsilon_t^r} \quad (5.A.37)$$

5.A.11 Market Clearing

$$h_t = h_t^{new} + F_{t-1}(\bar{\omega}_t) z^r \bar{n} + (1 - \delta^h) h_{t-1} \quad (5.A.38)$$

$$y_t = c_t + p_t^i i_t + F_{t-1}(\bar{\omega}_t) \chi^r \bar{n} + \mu v_{t-1}(\bar{\omega}_t) + a(u_t) + ac_t \quad (5.A.39)$$

$$ac_t = \frac{\phi}{2} \left[\frac{\pi_t}{\pi_{t-1}^\varphi \bar{\pi}_t^{1-\varphi}} - 1 \right]^2 y_t + \frac{\phi_W}{2} \left[\frac{\pi_t^W}{\pi_{t-1}^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right]^2 l_t + \frac{\phi_W}{2} \left[\frac{\pi_t^{W^h}}{\pi_{t-1}^{\varphi_l} \bar{\pi}_t^{1-\varphi_l}} - 1 \right]^2 l_t^h \quad (5.A.40)$$

5.A.12 Exogenous Shocks

$$\ln(\eta_t^c) = \rho_{\eta^c} \ln(\eta_{t-1}^c) + \varepsilon_t^{\eta^c} \quad (5.A.41)$$

$$\ln(\eta_t^h) = (1 - \rho_{\eta^h}) \ln(\bar{\eta}^h) + \rho_{\eta^h} \ln(\eta_{t-1}^h) + \varepsilon_t^{\eta^h} \quad (5.A.42)$$

$$\ln(\eta_{l,t}) = (1 - \rho_{\eta_l}) \ln(\bar{\eta}_l) + \rho_{\eta_l} \ln(\eta_{l,t-1}) + \varepsilon_t^{\eta_l} \quad (5.A.43)$$

$$\ln(\eta_{l,t}^h) = (1 - \rho_{\eta_l^h}) \ln(\bar{\eta}_l^h) + \rho_{\eta_l^h} \ln(\eta_{l,t-1}^h) + \varepsilon_t^{\eta_l^h} \quad (5.A.44)$$

$$\ln(\chi_t^r) = (1 - \rho_{\chi^r}) \bar{\chi}^r + \rho_{\chi^r} \ln(\chi_{t-1}^r) + \varepsilon_t^{\chi^r} \quad (5.A.45)$$

$$\ln(\sigma_t^\omega) = (1 - \rho_{\sigma^\omega}) \ln(\bar{\sigma}^\omega) + \rho_{\sigma^\omega} \ln(\sigma_{t-1}^\omega) + \varepsilon_t^{\sigma^\omega} \quad (5.A.46)$$

$$\ln(z_t) = (1 - \rho_z) \bar{z} + \rho_z \ln(z_{t-1}) + \varepsilon_t^z \quad (5.A.47)$$

$$\ln(\theta_t) = (1 - \rho_\theta) \bar{\theta} + \rho_\theta \ln(\theta_{t-1}) + \varepsilon_t^\theta \quad (5.A.48)$$

$$\ln(z_t^h) = (1 - \rho_{z^h}) \ln(\bar{z}^h) + \rho_{z^h} \ln(z_{t-1}^h) + \varepsilon_t^{z^h} \quad (5.A.49)$$

$$\ln(z_t^i) = \rho_z^i \ln(z_{t-1}^i) + \varepsilon_t^{z^i} \quad (5.A.50)$$

$$\ln(z_t^k) = \rho_z^k \ln(z_{t-1}^k) + \varepsilon_t^{z^k} \quad (5.A.51)$$

$$\ln(\bar{\pi}_t) = (1 - \rho_{\bar{\pi}}) \bar{\pi} + \rho_{\bar{\pi}} \ln(\bar{\pi}_{t-1}) + \varepsilon_t^{\bar{\pi}} \quad (5.A.52)$$

$$\ln(\epsilon_t^r) = \varepsilon_t^r \quad (5.A.53)$$

$$\ln(\epsilon_t^{r,news}) = \rho_r^{news} \ln(\epsilon_{t-1}^{r,news}) + \epsilon_{t-4}^{r,news} \quad (5.A.54)$$